



CLASS X ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : HM-24

Chapter 8.3 : Trigonometry



$$x = \sqrt{\frac{a^2}{c} + c} - \frac{b}{2}$$



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Trigonometric Ratio of Different Angles

- Acute Angle
 - $0 < \theta < \frac{\pi}{2}$
- Any Angle
 - 45
 - 110
 - 210
 - 330

Angles Ratios	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec} \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Trigonometric Ratio for Angle $(-\theta)$

$$0 < \theta < \frac{\pi}{2}$$

$$\sin \theta = \frac{y}{r}$$

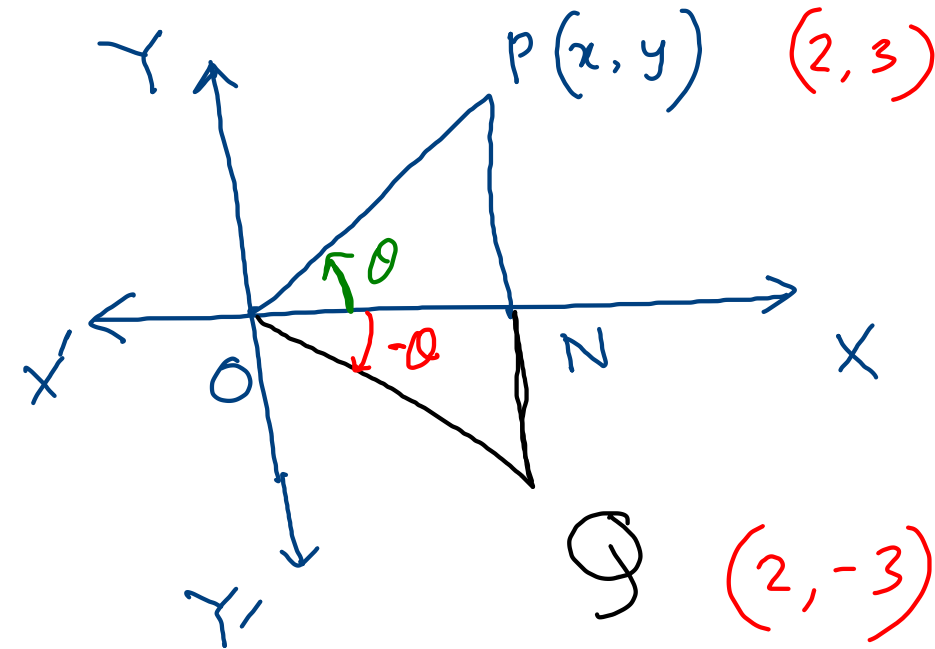
$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

$$\rightarrow OP = \sqrt{x^2 + y^2} = r$$

$$\rightarrow ON = x$$

$$PN = y$$



$\triangle OQN$

$$ON = x$$

$$QN = -y$$

$$OQ = r$$

$$\underline{\sin(-\theta)} = \frac{QN}{OQ} = \frac{-y}{r}$$

$$= -\frac{y}{r} = -\sin \theta$$

$$\cos(-\theta) = \frac{x}{r} = \cos \theta$$

$$\tan(-\theta) = \frac{-y}{x} = -\frac{y}{x} = -\tan \theta$$

$$\begin{aligned} OQ^2 &= ON^2 + QN^2 \\ &= x^2 + (-y)^2 \end{aligned}$$



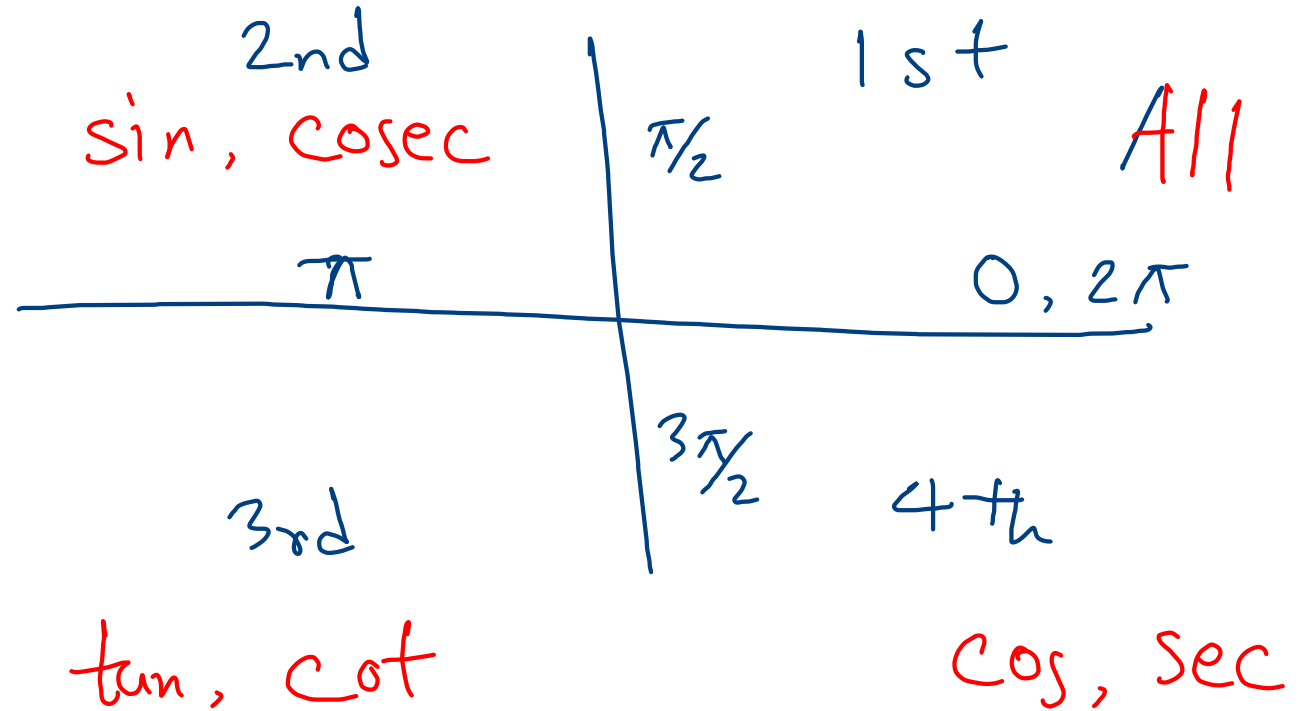
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Higher Math

Chapter 8.3 : Trigonometry

$$\cos(-\theta) = +\cos\theta$$



Poll Question 01

$$\cos\left(-\frac{\pi}{3}\right) = ?$$

$$\cos(-\theta) = \cos\theta$$

$$\cos \frac{\pi}{3}$$

$$= \frac{1}{2}$$

~~(a)~~ $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) $-\frac{\sqrt{3}}{2}$

Poll Question 02

$$\sin\left(-\frac{\pi}{3}\right) = ?$$

(a) $\frac{1}{2}$

(b) $-\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$

~~(d) $-\frac{\sqrt{3}}{2}$~~

$$= -\sin\frac{\pi}{3}$$
$$= -\frac{\sqrt{3}}{2}$$

$$\cos(-\theta) = \cos\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\tan(-\theta) = -\tan\theta$$

Trigonometric Ratio for Angle $\left(\frac{\pi}{2} - \theta\right)$

In $\triangle OPN$.

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$

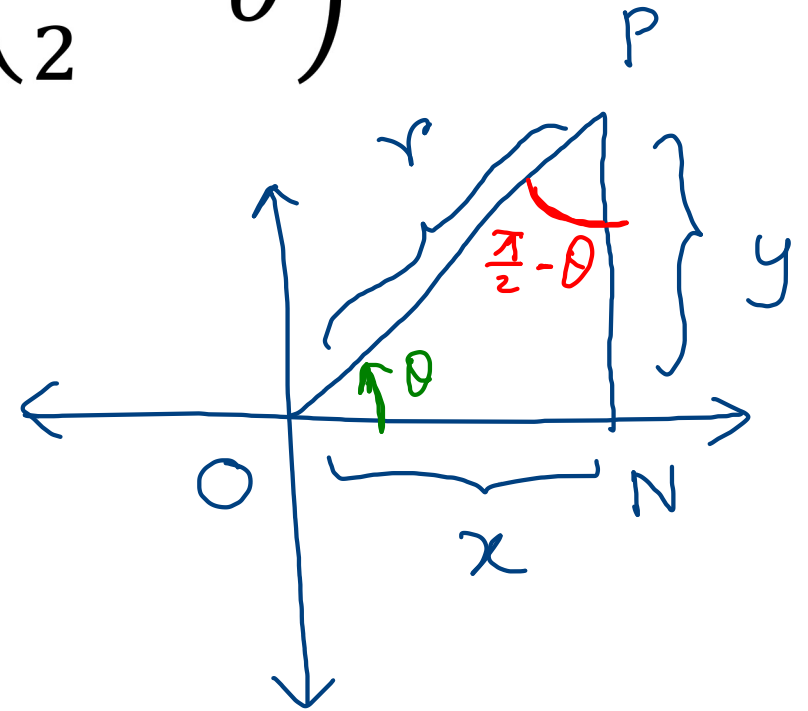
$$\cot \theta = \frac{x}{y}$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{x}{r} = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{y}{r} = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{x}{y} = \cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \frac{y}{x} = \tan \theta$$



$$\sec\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec} \theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2} - \theta\right) = \sec \theta$$

Trigonometric Ratio for Angle $\left(n\frac{\pi}{2} \pm \theta\right)$

$\left(1\frac{\pi}{2} - \theta\right)$

n is Odd

ratio changes

$\sin \longleftrightarrow \cos$

$\tan \longleftrightarrow \cot$

$\sec \longleftrightarrow \operatorname{cosec}$

n is Even

ratio doesn't
change

Trigonometric Ratio for Angle $\left(\frac{\pi}{2} + \theta\right)$

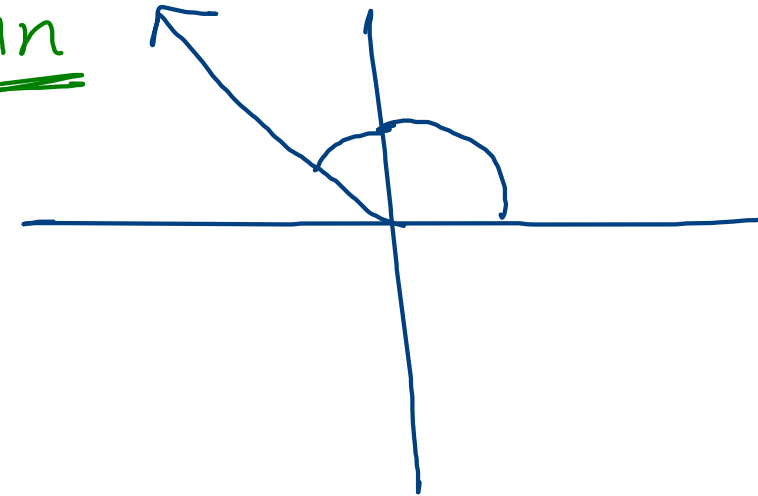
$$\sin\left(1 \cdot \frac{\pi}{2} + \theta\right) = + \cos \theta$$

$$\cos\left(1 \cdot \frac{\pi}{2} + \theta\right) = - \sin \theta$$

$$\tan\left(1 \cdot \frac{\pi}{2} + \theta\right) = - \cot \theta$$

Odd

sin

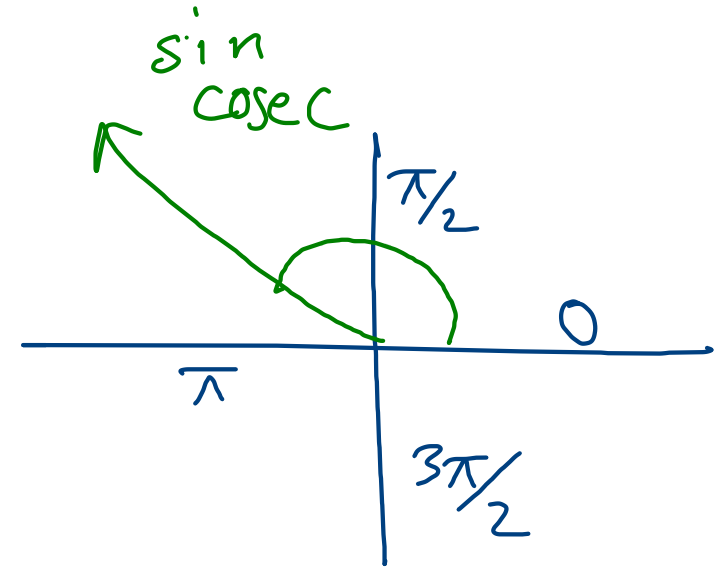


Trigonometric Ratio for Angle $(\pi - \theta)$

$$\sin \left(2\pi - \theta \right) = + \sin \theta$$

$$\cos \left(2\pi - \theta \right) = - \cos \theta$$

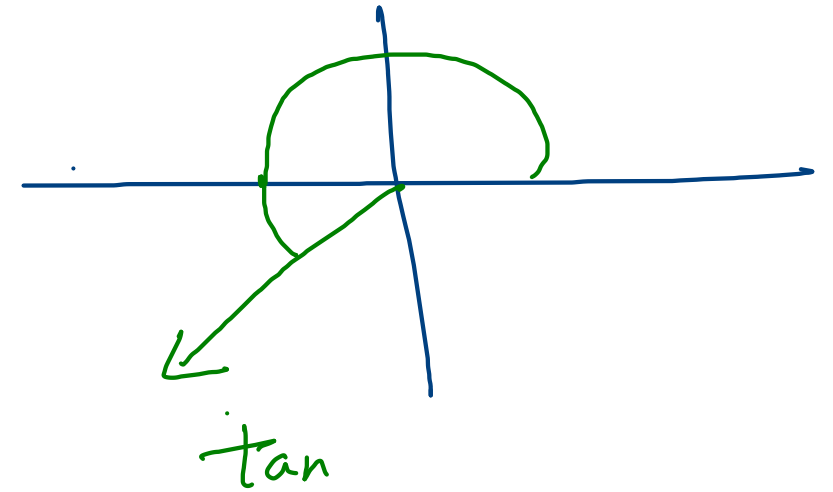
$$\tan \left(2\pi - \theta \right) = - \tan \theta$$



Trigonometric Ratio for Angle $(\pi + \theta)$

$$0 < \theta < \frac{\pi}{2}$$

$$\begin{aligned}\sin\left(2\frac{\pi}{2} + \theta\right) &= - \sin\theta \\ \cos\left(2\frac{\pi}{2} + \theta\right) &= - \cos\theta \\ \tan\left(2\frac{\pi}{2} + \theta\right) &= + \tan\theta\end{aligned}$$



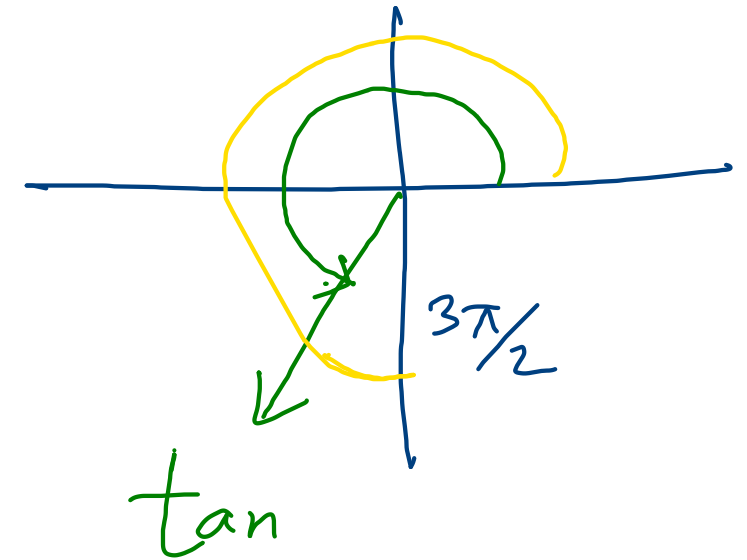
Trigonometric Ratio for Angle $\left(\frac{3\pi}{2} - \theta\right)$

$$0 < \theta < \frac{\pi}{2}$$

$$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = +\cot\theta$$



θ

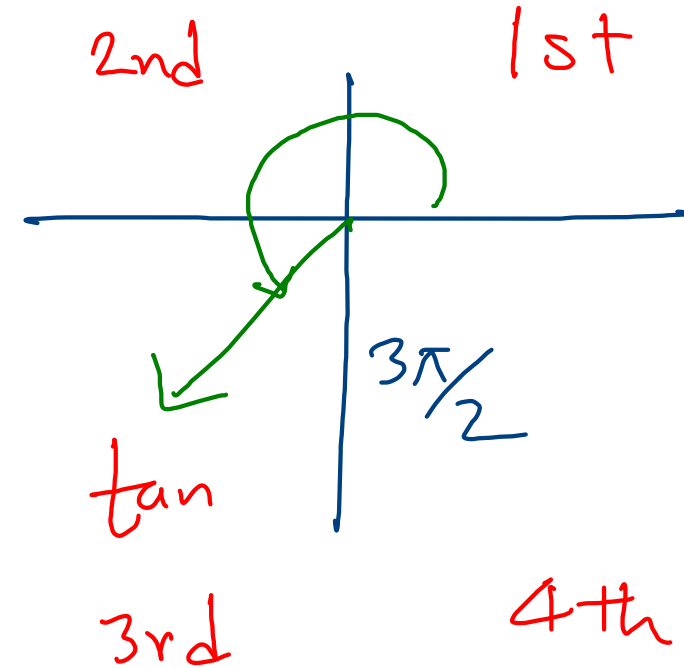
Poll Question 03

$$\tan\left(\frac{3\pi}{2} - \frac{\pi}{3}\right) = ?$$

- ~~(a) $\frac{1}{\sqrt{3}}$~~
- (b) $-\frac{1}{\sqrt{3}}$
- (c) $\sqrt{3}$
- (d) $-\sqrt{3}$

$$+ \cot \frac{\pi}{3} \\ = \frac{1}{\sqrt{3}}$$

$$\tan\left(3\pi/2 - \theta\right) = \cot \theta$$

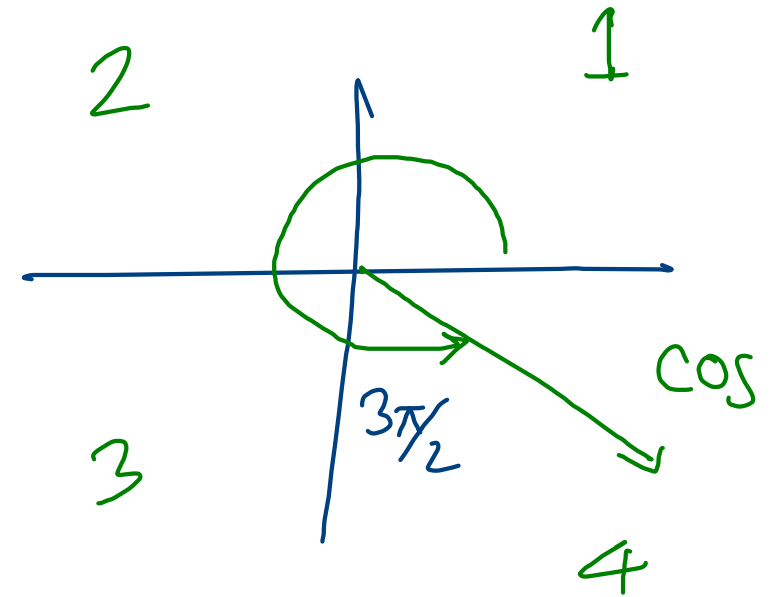


Trigonometric Ratio for Angle $\left(\frac{3\pi}{2} + \theta\right)$

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} + \theta\right) = +\sin\theta$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot\theta$$



Poll Question 04

$$\tan \left(\frac{3\pi}{2} + \frac{\pi}{3} \right) = ?$$

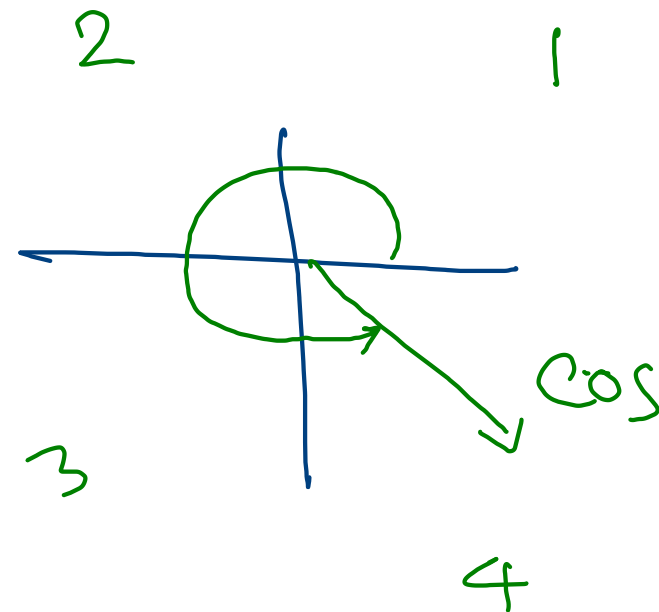
(a) $\frac{1}{\sqrt{3}}$

~~(b) $-\frac{1}{\sqrt{3}}$~~

(c) $\sqrt{3}$

(d) $-\sqrt{3}$

$$= -\cot \frac{\pi}{3} = -\frac{1}{\sqrt{3}}$$



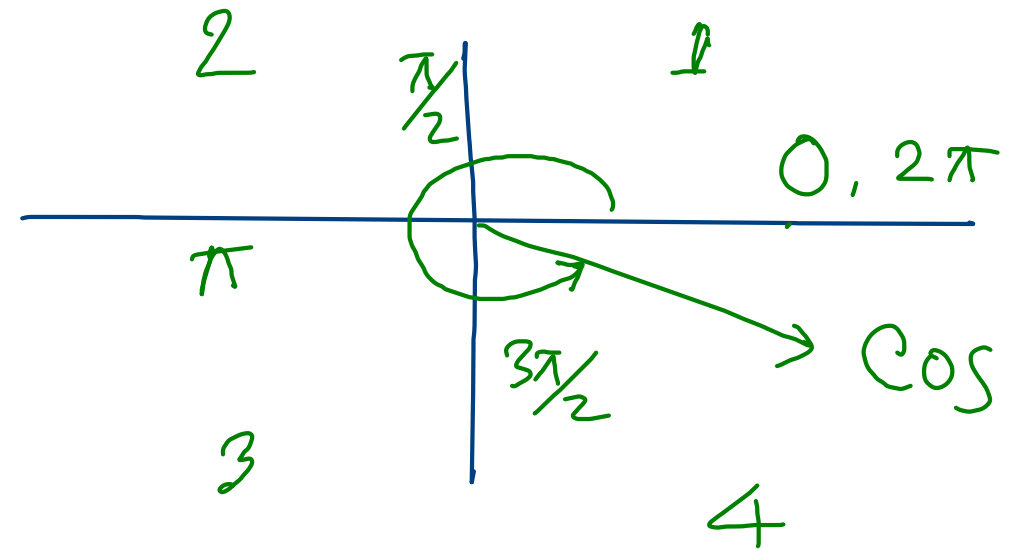
Trigonometric Ratio for Angle $(2\pi - \theta)$

$$\sin(2\pi - \theta) = - \sin \theta$$

$$\cos(2\pi - \theta) = + \cos \theta$$

$$\tan(2\pi - \theta) = - \tan \theta$$

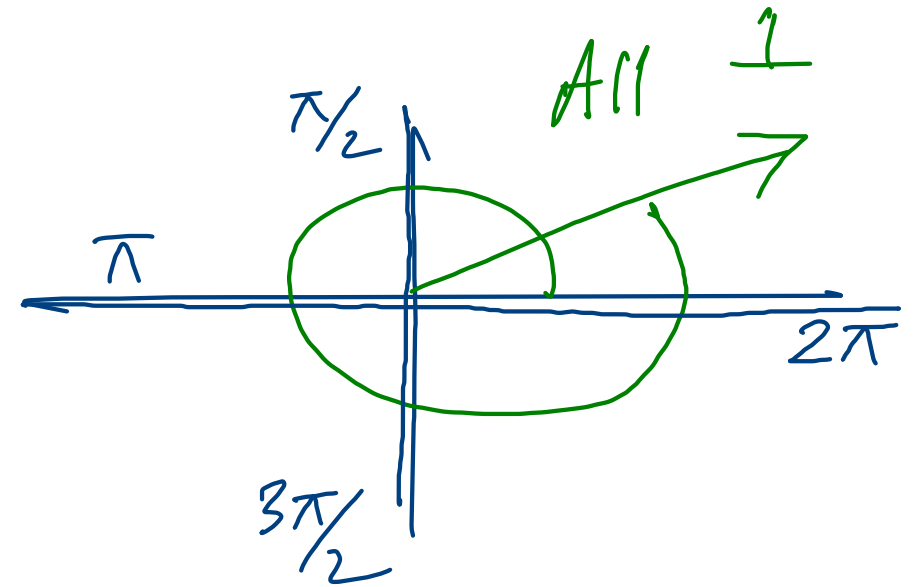
$$2\pi = \textcircled{4} \frac{\pi}{2}$$



Trigonometric Ratio for Angle $(2\pi + \theta)$

$$2\pi = 4\left(\frac{\pi}{2}\right)$$

$$\begin{aligned}\sin(2\pi + \theta) &= + \sin \theta \\ \cos(2\pi + \theta) &= + \cos \theta \\ \tan(2\pi + \theta) &= + \tan \theta\end{aligned}$$



Determine The Value

$$\sin\left(\frac{31\pi}{6}\right) = ?$$

obtuse $\rightarrow$ acute

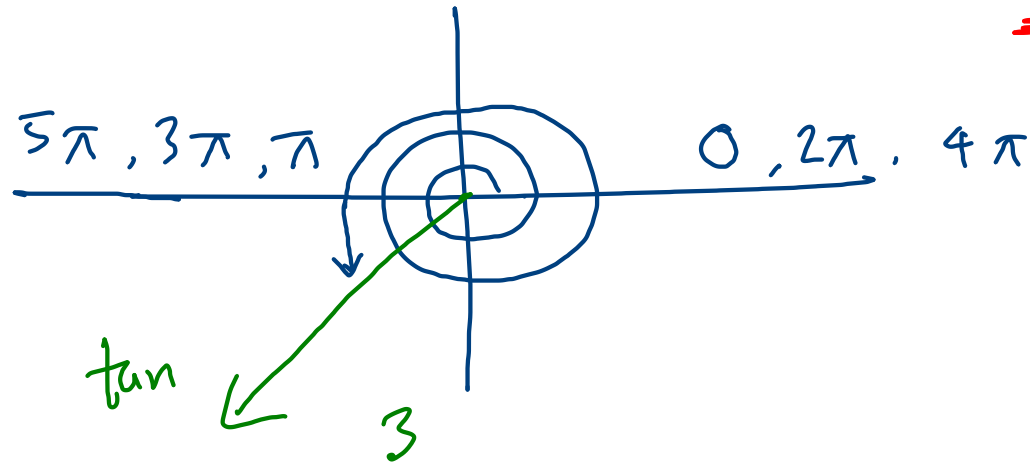
$$31 = 5 \times 6 + 1$$

$$= \sin\left(5\pi + \frac{\pi}{6}\right)$$

$$= \sin \frac{\pi}{6}$$

$$= -\frac{1}{2}$$

$$5\pi = \underline{10}\left(\frac{\pi}{2}\right)$$



Determine The Value

$$\cot\left(\theta - \frac{9\pi}{2}\right) = ?$$

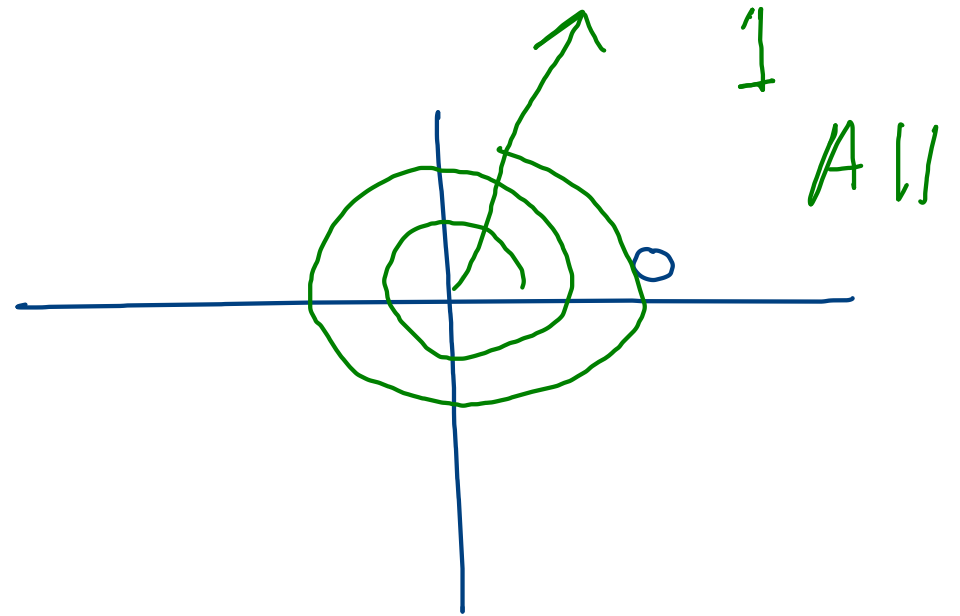
$$= \cot\left\{-\left(9\pi/2 - \theta\right)\right\}$$

$$= -\cot\left(9\pi/2 - \theta\right)$$

$$= -\left(+\tan\theta\right)$$

$$= -\tan\theta$$

$$\cos(-\theta) = \cos\theta$$



Poll Question 05

$$\tan\left(\frac{11\pi}{6}\right) = ?$$

- (a) $\frac{1}{\sqrt{3}}$
- (b) $-\frac{1}{\sqrt{3}}$
- (c) $\sqrt{3}$
- (d) $-\sqrt{3}$

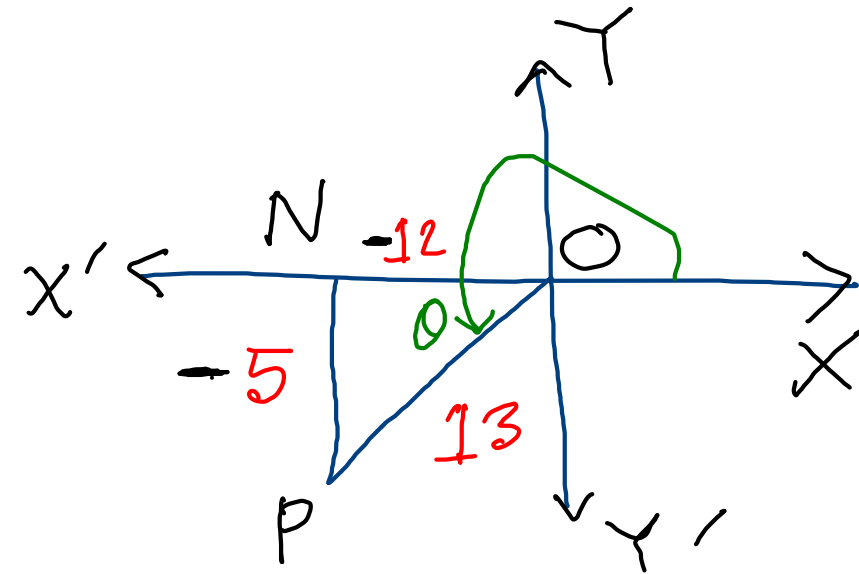
$\tan \theta = \frac{5}{12}$ and $\cos \theta$ is negative, prove that, $\frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta} = \frac{51}{26}$

$$\text{hyp} = \sqrt{5^2 + 12^2} = 13$$

$$\cos \theta = \frac{-12}{13}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\frac{13}{12}$$

$$\sin \theta = \frac{-5}{13}$$



$$\tan \theta = \frac{\text{op}}{\text{adj}}$$

$$\frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta}$$

$$= \frac{\sin \theta + \cos \theta}{\sec \theta + \tan \theta}$$

$$= \frac{-\frac{5}{13} + \frac{-12}{13}}{-\frac{13}{12} + \frac{5}{12}}$$

✓✓
If $0 < \theta < \frac{\pi}{2}$, solve the equation $\sin \theta + \cos \theta = \underline{\underline{\sqrt{2}}}$.

$$\sin \theta = \sqrt{2} - \cos \theta$$

$$\text{or, } \sin^2 \theta = (\sqrt{2} - \cos \theta)^2$$

$$\text{or, } \sin^2 \theta = (\sqrt{2})^2 - 2\sqrt{2}\cos \theta + \cos^2 \theta$$

$$\text{or, } 1 - \cos^2 \theta = 2 - 2\sqrt{2}\cos \theta + \cos^2 \theta$$

$$2\cos^2 \theta - 2\sqrt{2}\cos \theta + 1 = 0$$

$$(\sqrt{2}\cos \theta - 1)^2 = 0$$

$$\sqrt{2}\cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$= \cos \frac{\pi}{4}$$

$$\theta = \frac{\pi}{4}$$

✓
If $0 < \theta < 2\pi$, solve the equation $\sin^2 \theta - \cos^2 \theta = \cos \theta$.

$$1 - \cos^2 \theta - \cos^2 \theta = \cos \theta$$

$$2\cos^2 \theta + \cos \theta - 1 = 0$$

$$2\cos^2 \theta + 2\cos \theta - \cos \theta - 1 = 0$$

$$2\cos \theta (\cos \theta + 1) - 1(\cos \theta + 1) = 0$$

$$(\cos \theta + 1)(2\cos \theta - 1) = 0$$

either

$$\cos \theta + 1 = 0$$

$$\cos \theta = -1$$
$$= \cos \pi$$

$$\theta = \pi$$

or

$$2\cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$= \cos \frac{\pi}{3}$$

$$= \cos \left(2\pi - \frac{\pi}{3}\right)$$

$\tan \theta = -\sqrt{3}$ and $\frac{\pi}{2} < \theta < 2\pi$. Find the value of θ ?

$$\tan(\pi - x) = -\tan x$$

$$\tan(2\pi - x) = -\tan x$$

$$\frac{\sqrt{3}}{1} = \tan \frac{\pi}{6}$$

$$\theta = \pi - \frac{\pi}{6}$$

$$\theta = 2\pi - \frac{\pi}{6}$$

Home Task

- ☐ Practice examples thoroughly.
- ☐ Try exercise math that matches with example.

(It'll help in next class.)

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$$X = \frac{c \rho P \sqrt{2}}{2} S$$

$$X = \frac{c \rho P \sqrt{2}}{2} S$$

$$E = mc^2$$

$$x = \sqrt{\frac{a^2}{c} + c} - \frac{b}{2}$$



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