১০ম শ্রেণি একাডেমিক প্রোগ্রাম ২০২০

উচ্চতর গণিত

লেকচার : HM-25

অধ্যায় ৮ : ত্রিকোণমিতি (৮.৩)

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

Last Class

$\left(n\frac{\pi}{2} + \theta\right)$

n বিজোড়

sin ⟷ cos

tan ⟷ cot

sec ⟷ cosec

n জোড়

অনুপাত পরিবর্তিত হবে না

চিহ্ন → quadrant থেকে

2nd	1st
sin	All
3rd	4th
tan	cos

SD

$\tan^2\theta = (\tan\theta)^2$

➤ $\theta = \frac{\pi}{3}$ হলে, প্রমাণ কর যে, $\sin 2\theta = 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ ।

L.H.S. $= \sin 2\theta$

$= \sin 2\left(\frac{\pi}{3}\right)$ $= \sin 120^\circ$ Calculator

$= \sqrt{3}/2$

M.H.S. $= 2\sin\theta\cos\theta$

$= 2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

$= 2 \cdot \sqrt{3}/2 \cdot 1/2$

$= \sqrt{3}/2$

R.H.S. $= \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2\tan\frac{\pi}{3}}{1+\left(\tan\frac{\pi}{3}\right)^2}$

$= \frac{2\sqrt{3}}{1+(\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{1+3} = \sqrt{3}/2$

Poll Question-01

❑ $\cos\left(\frac{19\pi}{3}\right) = ?$

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) $-\frac{\sqrt{3}}{2}$

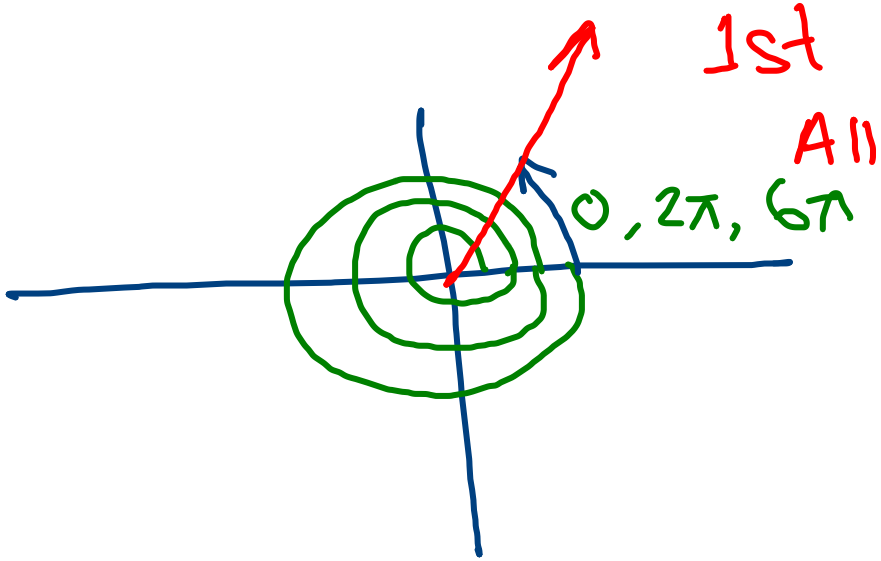
$\cos\left(\frac{19\pi}{3}\right)$

$\frac{19\pi}{3} = \frac{18\pi + \pi}{3}$

$= \cos\left(6\pi + \frac{\pi}{3}\right)$

$6\pi = 12\left(\frac{\pi}{2}\right)$

$= +\cos\frac{\pi}{3}$



$= 1/2$

$\frac{7\pi}{3} = \frac{6\pi+\pi}{3} = 2\pi + \frac{\pi}{3}$

$\frac{13\pi}{6} = \frac{12\pi+\pi}{6} = 2\pi + \frac{\pi}{6}$

➤ প্রমাণ কর যে, $\sin\frac{7\pi}{3}\cos\frac{13\pi}{6} - \cos\frac{5\pi}{3}\sin\frac{11\pi}{6} = 1$

L.H.S. $= \sin\frac{7\pi}{3}\cos\frac{13\pi}{6} - \cos\frac{5\pi}{3}\sin\frac{11\pi}{6}$

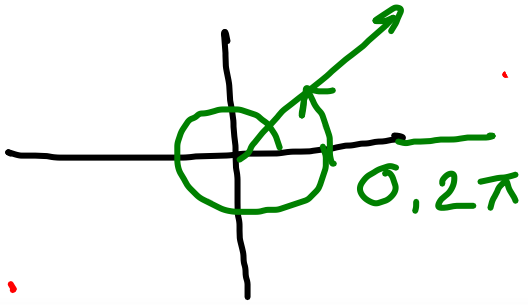
$= \sin\left(2\pi+\frac{\pi}{3}\right)\cos\left(2\pi+\frac{\pi}{6}\right) - \cos\left(2\pi-\frac{\pi}{3}\right).\sin\left(2\pi-\frac{\pi}{6}\right)$

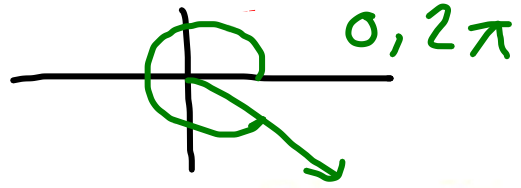
$= \left(+\sin\frac{\pi}{3}\right)\left(+\cos\frac{\pi}{6}\right) - \left(+\cos\frac{\pi}{3}\right)\left(-\sin\frac{\pi}{6}\right)$

$= \sqrt{3}/2 \cdot \sqrt{3}/2 - 1/2 \cdot (-1/2)$

$=$

$= 1 =$ R.H.S.





গাণিতিক সমস্যা

$\pi = 2\left(\frac{\pi}{2}\right)$

➢ মান নির্ণয়ঃ $\sin^2\frac{17\pi}{18} + \sin^2\frac{5\pi}{8} + \cos^2\frac{37\pi}{18} + \cos^2\frac{3\pi}{8}$

$$= \left(\sin\frac{17\pi}{18}\right)^2 + \left(\sin\frac{5\pi}{8}\right)^2 + \left(\cos\frac{37\pi}{18}\right)^2 + \left(\cos\frac{3\pi}{8}\right)^2$$

$$= \left\{\sin\left(\pi - \frac{\pi}{18}\right)\right\}^2 + \left\{\sin\left(\pi - \frac{3\pi}{8}\right)\right\}^2 + \left\{\cos\left(2\pi + \frac{\pi}{18}\right)\right\}^2 + \cos^2\frac{3\pi}{8}$$

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{3\pi}{8} + \cos^2\frac{\pi}{18} + \cos^2\frac{3\pi}{8}$$

$$= 1 + 1 = 2$$

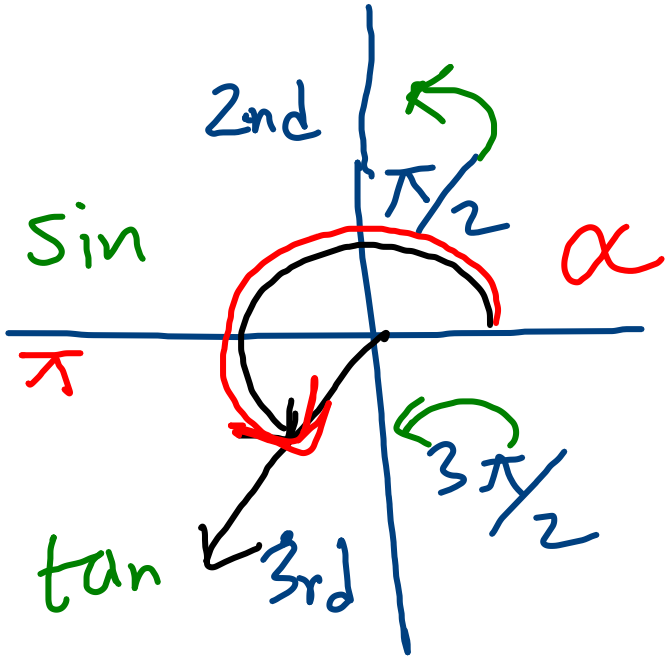
গাণিতিক সমস্যা

➤ $\sin\alpha = -\frac{\sqrt{3}}{2}$ হলে, α এর মান নির্ণয় কর যখন $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

$\sin\alpha = -\sqrt{3}/2$

$\sin \pi/3 = \sqrt{3}/2$

$\sin\left(\pi + \frac{\pi}{3}\right) = -\sqrt{3}/2$



$\alpha = \pi + \frac{\pi}{3} = 4\pi/3$

Poll Question-02

❑ $\cos\theta = -\frac{1}{2}, \theta = ?$

(a) $0 < \theta < 90$

(b) $90 < \theta < 180$

(c) $180 < \theta < 270$

(d) b & c

$$\cos(-\theta) = \cos\theta, \quad 0 < \theta < \pi/2$$

গাণিতিক সমস্যা

➢ সমাধান করঃ $4(\cos^2\theta + \sin\theta) = 5$; [যখন $0 < \theta < 2\pi$]

4 টি quad

বা, $4(1 - \sin^2\theta + \sin\theta) = 5$

বা, $4 - 4\sin^2\theta + 4\sin\theta = 5$

বা, $4\sin^2\theta - 4\sin\theta + 1 = 0$

বা, $(2\sin\theta - 1)^2 = 0$

বা, $2\sin\theta - 1 = 0$

বা, $\sin\theta = 1/2$

$\sin\theta = 1/2$

$\sin\frac{\pi}{6} = 1/2$

$\sin(\pi - \frac{\pi}{6}) = 1/2$

মনে মনে

sin · All

$\therefore \theta = \pi/6, \pi - \frac{\pi}{6}$

$= \pi/6, 5\pi/6$

Poll Question-03

❑ $\sin\theta - \cos\theta = 0$; [যখন $0 < \theta < \frac{\pi}{2}$] হলে $\theta = ?$

1st

(a) 0

(b) $\frac{\pi}{2}$

(c) অসংজ্ঞায়িত

(d) $\frac{\pi}{4}$

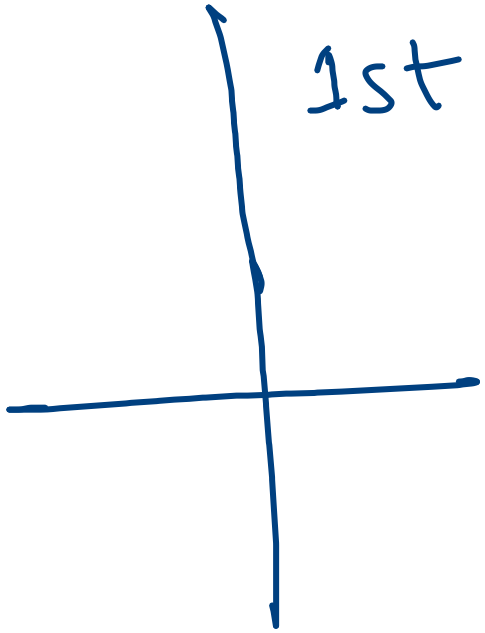
$\sin\theta = \cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$= \tan\frac{\pi}{4}$

$\theta = \pi/4$



গাণিতিক সমস্যা

➢ সমাধান করঃ $\tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$; [যখন $0 < \theta < \frac{\pi}{2}$] → 1st

$$\tan\theta + \frac{1}{\tan\theta} = \frac{4}{\sqrt{3}}$$

$$\frac{\tan^2\theta + 1}{\tan\theta} = \frac{4}{\sqrt{3}}$$

$$\sqrt{3}(\tan^2\theta + 1) = 4\tan\theta$$

$$\sqrt{3}\tan^2\theta - 4\tan\theta + \sqrt{3} = 0$$

$\tan\theta = \frac{1}{\sqrt{3}}$ $\qquad$ $\tan\theta = \sqrt{3}$

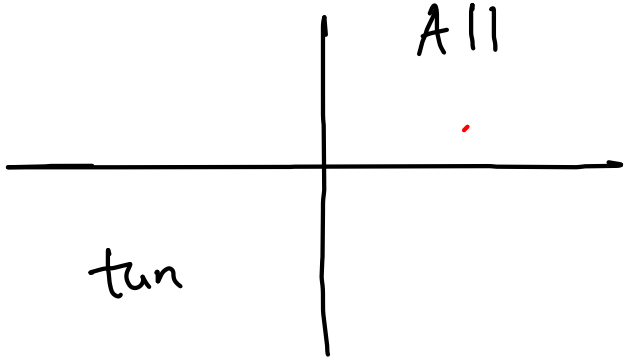
$= \tan\frac{\pi}{6}$ $\qquad$ $= \tan\frac{\pi}{3}$

$\theta = \pi/6$ $\qquad$ $\theta = \pi/3$

$\therefore \quad \theta = \pi/6, \ \pi/3$

$0 < \theta < \pi/2$

1st



গাণিতিক সমস্যা

4 টি quad

➤ সমাধান করঃ $\sec^2\theta + \tan^2\theta = \frac{5}{3}$; [যখন $0 < \theta < 2\pi$]

$1 + \tan^2\theta + \tan^2\theta = 5/3$

$1 + 2\tan^2\theta = 5/3$

$3(1 + 2\tan^2\theta) = 5$

$3 + 6\tan^2\theta = 5$

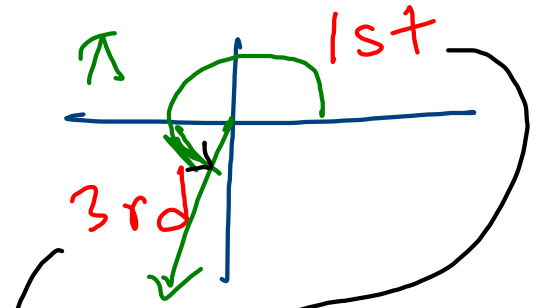
$6\tan^2\theta = 5 - 3 = 2$

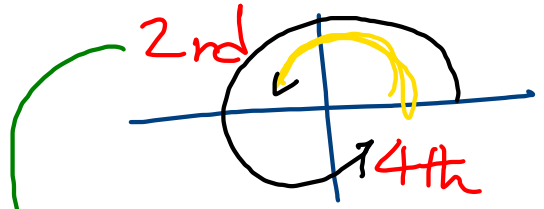
$\tan^2\theta = \frac{2}{6} = 1/3$

$\tan\theta = \pm 1/\sqrt{3}$

$\tan\theta = \frac{1}{\sqrt{3}}$ ← $\tan\frac{\pi}{6}$

$\tan\theta = -\frac{1}{\sqrt{3}}$





$\theta_1 = \pi/6$

$\theta_2 = \pi - \frac{\pi}{6}$

$\theta_3 = \pi + \frac{\pi}{6}$

$\theta_4 = 2\pi - \frac{\pi}{6}$

$\theta = \pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6$

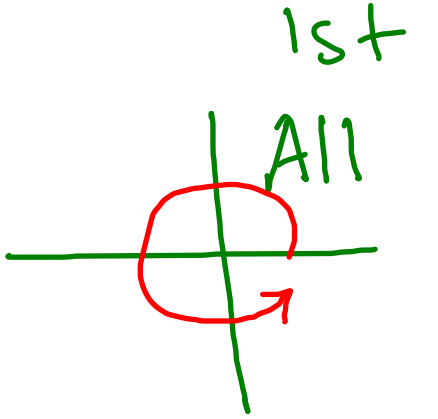
গাণিতিক সমস্যা

➤ সমাধান করঃ $2\sin x\cos x = \sin x$; [যখন $0 \le x \le 2\pi$]

4 টা quad

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$



হয় $\sin x = 0$

$= \sin(0)$,

$\sin(\pi)$, $\sin(2\pi)$

$x = 0 , \pi , 2\pi$

বা, $2\cos x - 1 = 0$

$\cos x = 1/2$

$x = \pi/3 , 2\pi - \frac{\pi}{3}$

$x = 0, \pi, 2\pi, \pi/3, 5\pi/3$

$$\cos(\theta - 1350^\circ)$$

$$= \cos\{-(1350^\circ - \theta)\}$$

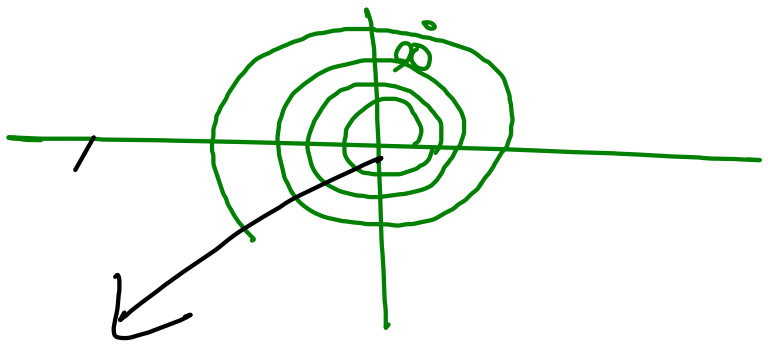
$$= \cos(1350^\circ - \theta)$$

$$= \cos(15 \times 90^\circ - \theta)$$

$$= -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$1350^\circ = 15 \times 360^\circ$$



$$\cot\theta = \cot\pi$$

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে