بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

MATRICES AND DETERMINANTS

LECTURE- HM 01

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Why Matrix ?

Solve for x and y

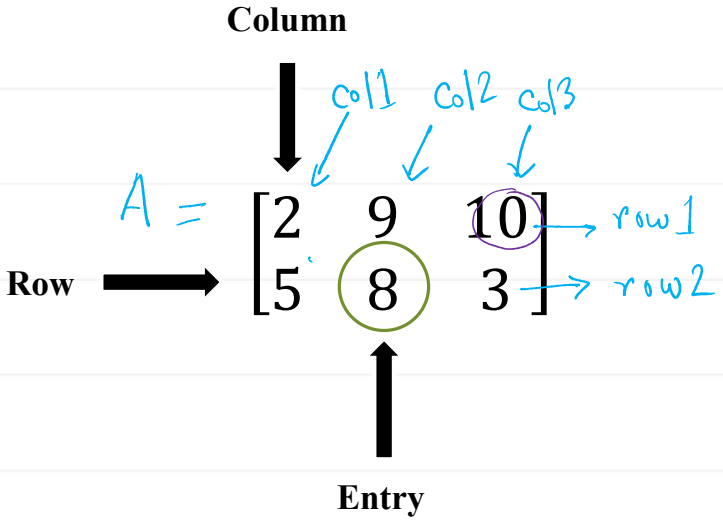
- x+y = 5
- x-y = 1

Solve for x, y and z

- 2x+y-2z = 10
- 3x+2y+2z = 1
- 5x+4y+3z = 4

MATRICES:

A **matrix(m*n)** is a collection of numbers arranged into a fixed number of rows(m) and columns(n) where the dimension of this matrix is m*n.



Dimension : row X col

2 X 3

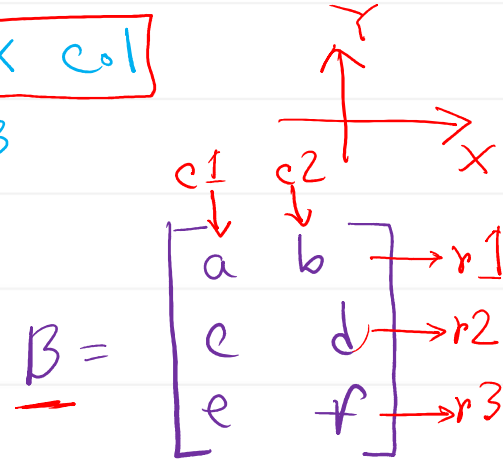
$A_{22} = 8$

$A_{21} = 5$

$A_{13} = 10$

r c

RC



3 X 2

$B_{22} = d$

$B_{32} = f$

TYPES OF MATRICES:

Types	Description	Example
A. Rectangular matrix:	$B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ row 2 × col 3	c1 c2 c3 r1 [1 7 1] r2 [16 13 6]
B. Row Matrix: / vector	Only 1 row 1 × n	1 × 3 r1 → [5 5 9]
C. Column Matrix: / vector	Only 1 col m × 1	3 × 1 $\begin{bmatrix} 8 \\ 7 \\ 19 \end{bmatrix}$

TYPES OF MATRICES:

Some important definitions

rectangular Matrix A $m \times n$

Square matrix: $m \times m$ $n \times n$

$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$

Main diagonal:

$\begin{bmatrix} 5 & 7 & 6 \\ 1 & 7 & 8 \\ 3 & 1 & 4 \end{bmatrix}$

Trace: $5 + 7 + 4 = 16$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Main diagonal, trace will exist if and only if the matrix is a square matrix.

Poll Question-01

$$\begin{bmatrix} 2 & 4 & 5 \\ 7 & X & -7 \\ -3 & 2 & 4 \end{bmatrix}$$ If the trace of this matrix is 7, what's the value of X?

(a) -1 (b) 5 (c) 1 (d) 4

$2 + X + 4 = 7$

$6 + X = 7$

$X = 1$

TYPES OF MATRICES:

Types	Description	Example
D. Square matrix:	$m \times m$	$\begin{bmatrix} 2 & 5 & 7 \\ 9 & 7 & 9 \\ 13 & 1 & 4 \end{bmatrix}$ *Main diagonal*
E. Diagonal matrix:	$\begin{bmatrix} -5 & 0 \\ 0 & 6 \end{bmatrix}$	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
F. Scalar matrix:	$\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$	$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
G. Identity or Unit matrix:	$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A \cdot I = A$ $a \cdot 1 = a$	$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Poll Question-02

Which one is scalar matrix?

(a) $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) a & c

TYPES OF MATRICES:

Types	Description	Example
H. Null matrix:		$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
I. Transpose matrix:	$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ 2 X 3	$\begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix}^T = \begin{bmatrix} 5 & 2 \\ 6 & 3 \end{bmatrix}$
J. Symmetric matrix:	$A^T = A$ $B^T = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 7 & -3 \\ -1 & -3 & 2 \end{bmatrix}$ $= B$	$B = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 7 & -3 \\ -1 & -3 & 2 \end{bmatrix}$
K. Skew Symmetric matrix:	$A^T = -A$ $C^T = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$ $-C = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$ $= -C$	$C = \begin{bmatrix} 0 & -6 \\ 6 & 0 \end{bmatrix}$

Poll Question-03

Which one is skew symmetric matrix?

(a) $\begin{bmatrix} 5 & 6 & 1 \\ -6 & 2 & 7 \\ -1 & 7 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -3 \\ -3 & 2 \\ 3 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & -2 \\ -3 & 2 & 0 \end{bmatrix}$

$B^T = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$

$-B = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 2 \\ 3 & -2 & 0 \end{bmatrix}$

Equality OF MATRICES:

The condition of equality of two matrices:

- The dimension of the matrices must be equal/exactly same. 3×2 3×2

- The entries of one matrix must be equal to the respective entries of another.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 4 & 6 \end{bmatrix}$$

$A \neq B$ $B \neq C$

Poll Question-04

If $\begin{bmatrix} x & 4 \\ 8 & 2y \end{bmatrix} = \begin{bmatrix} 2 & 2x \\ 8 & 3x \end{bmatrix}$, what's the value of x,y ?

(a) 4,3 (b) 2, 3 (c) 3,2 (d) 2,4

$x = 2$

$4 = 2x \Rightarrow x = 2$

$8 = 8$

$(2, 3)$

$2y = 3x$

$2y = 3(2)$

$\therefore y = 3$

ADDITION AND SUBTRACTION OF MATRICES:

The basic condition of adding or subtracting matrices is that the matrices must be of equal size/dimension.

Addition: Addition of respective entries

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} \qquad D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

$$A + D = \begin{bmatrix} a+1 & b+2 \\ c+3 & d+4 \end{bmatrix}$$

Subtraction: Subtraction of respective entries from the entries of the first one

EXERCISE 1.1:

6. Given, $A = \begin{bmatrix} 8 & 4 & -1 \\ 0 & 1 & 3 \\ 5 & 4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 6 & 2 \\ 1 & 3 & 7 \\ 5 & 4 & 1 \end{bmatrix}$, determine $A+B$ *and* $A-B$

$$A+B = \begin{bmatrix} 8+(-4) & 4+6 & -1+2 \\ 0+1 & 1+3 & 3+7 \\ 5+5 & 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} & & \end{bmatrix}$$

$$A-B = \begin{bmatrix} 8-(-4) & 4-6 & -1-2 \\ & & \end{bmatrix} =$$

ACTIVITY:

Determine A+B, A-B

1. $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & 2 \\ 1 & 6 & 9 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 6 & -4 \end{bmatrix}, B = \begin{bmatrix} 6 & 0 & -4 \\ 5 & 3 & -1 \end{bmatrix}$

H. W.

MULTIPLICATION OF MATRICES:

First type:
Multiplication by constant value

System: Multiplying every entries by the constant

1. $A = \begin{bmatrix} 1 & -2 & 3 \\ 5 & 1 & -4 \end{bmatrix}$ determine 2A

$2A = \begin{bmatrix} 2\times1 & 2\times-2 & 2\times3 \\ 2\times5 & 2\times1 & 2\times-4 \end{bmatrix}$

$-A = (-1)A$

$= \begin{bmatrix} -1 & 2 & -3 \\ -5 & -1 & 4 \end{bmatrix}$

ACTIVITY:

1. $A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix}$, determine 3A+4B

2. $A = \begin{bmatrix} 8 & 4 & -1 \\ 0 & 1 & 3 \\ 5 & 4 & 8 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 6 & 2 \\ 1 & 3 & 7 \\ 5 & 4 & 1 \end{bmatrix}$, determine 3A-5B

H. W.

MULTIPLICATION OF MATRICES:

Second type:
Multiplication of two matrices

Condition: Column numbers of first matrix must be equal to the row numbers of second matrix

$$\begin{bmatrix} 2 & 5 & 2 \\ 1 & 0 & -2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 & 0 \\ -2 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

$A_{m \times n}$

$B_{n \times p}$

AB

MULTIPLICATION OF MATRICES:

If A is of the order m*n and B is of the order n*p, then AB will be of the order m*p

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ -2 & -1 \\ 0 & 4 \end{bmatrix}$$

EXERCISE 1.1:

1. $A = \begin{bmatrix} 5 & 2 & 9 \\ -2 & 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 7 \\ 1 & 2 \\ 0 & -5 \end{bmatrix}$, show that $AB \neq BA$

$5 \times 3 = 15$

$3 \times 5 = 15$

2×3 $\quad$ 3×2

$$AB = \begin{bmatrix} 5\times0+2\times1+9\times0 & 5\times7+2\times2+9\times-5 \\ -2\times0+5\times1+3\times0 & -2\times7+5\times2+3\times-5 \end{bmatrix}_{2\times2}$$

$A \rightarrow 4 \times 2$ $\quad$ $B \rightarrow 2 \times 3$

$AB \rightarrow 4 \times 3$

$$BA = \begin{bmatrix} 0 & 7 \\ 1 & 2 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 5 & 2 & 9 \\ -2 & 5 & 3 \end{bmatrix}$$

3×2 $\quad$ 2×3

$BA \rightarrow 2 \times 3$ $\quad$ 4×2

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$$= \begin{bmatrix} 0\times5+7\times-2 & 0\times2+7\times5 & 0\times9+7\times3 \\ 1\times5+2\times-2 & 1\times2+2\times5 & 1\times9+2\times3 \\ 0\times5+-5\times-2 & 0\times2+-5\times5 & 0\times9+-5\times3 \end{bmatrix}_{3\times3}$$

ACTIVITY:

1. Given $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, show that $(AB)C = A(BC)$ H. W.

$(AB)C = A(BC)$

2. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix}$, show that $(A+B)^2 \neq A^2 + 2AB + B^2$

$A \cdot A$ $\quad B \cdot B$

$x = 5 \qquad x^2 = 5^2 = 25$

H. W.

$A^2 = A \cdot A$ ✓

$A + B = [\quad]$

$(A+B)^2 = (A+B) \cdot (A+B)$

$A^3 = A^2 \cdot A$

Poll Question-05

If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $A^2 + 2A - kI = 0$, what's the value of k? where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(a) 11 (b) 10 (c) 8 (d) 13

$A^2 + 2A - kI = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & -6 \end{bmatrix} - \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$9 + 2 - k = 0$

$k = 11$

$A^2 = A.A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

$= \begin{bmatrix} 1\times1 + 2\times4 & 1\times2 + 2\times-3 \\ 4\times1 + -3\times4 & 4\times2 + -3\times-3 \end{bmatrix}$

$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

উদ্ভাস-উন্মেষ শিক্ষা
পরিবার


Thank
You