Class 11 Academic Program-2020

PHYSICS 1ST PAPER

Lecture : P-05

Chapter 02 : Vector

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

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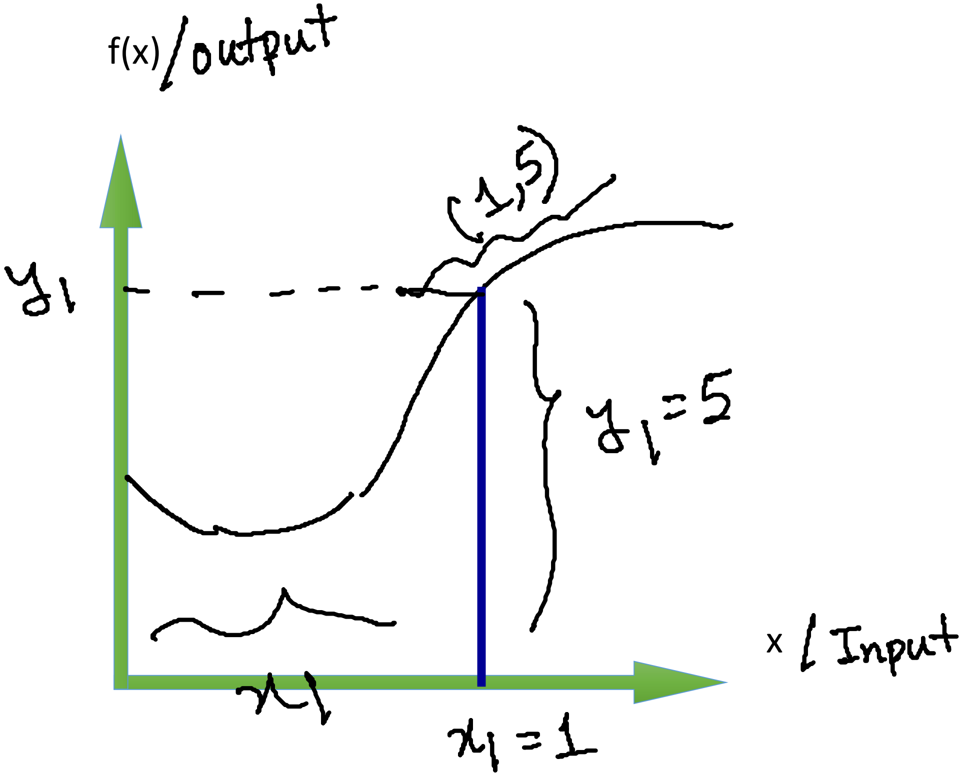
Topics

- Introduction to Calculus
- Differentiation
- Partial Differentiation
- Gradient
- Divergence
- Curl

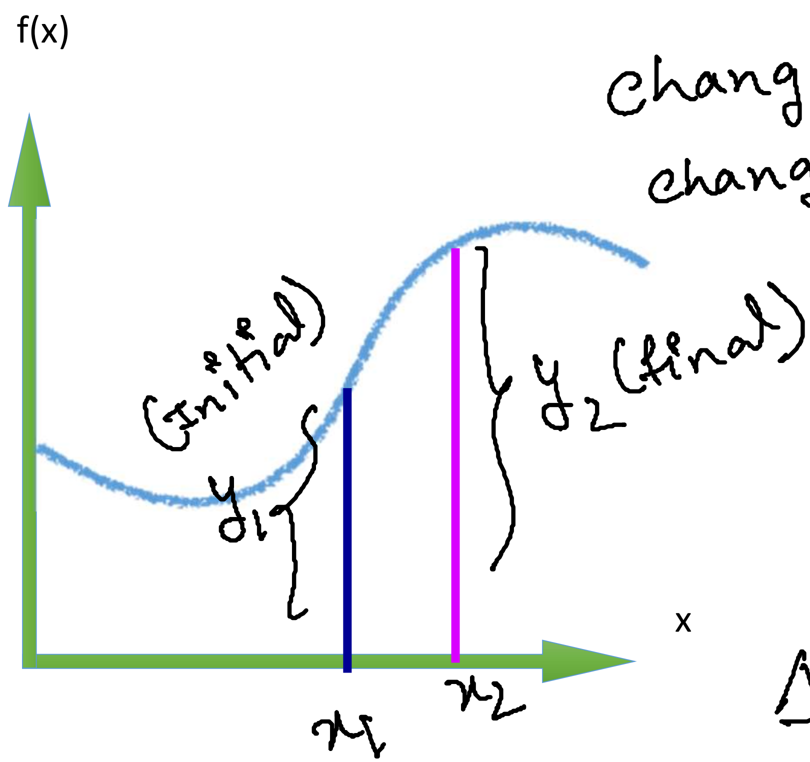
Differentiation : Rate of Change

Function (Input) = output

$f(x) = y$



Differentiation : Rate of Change



change of $x = x_2 - x_1$
change of $y = y_2 - y_1$

$(x_2 - x_1) = \Delta x$
$y_2 - y_1 = \Delta y$

Δx (change) $\longrightarrow$ Δy (change)

1 unit (change) $\longrightarrow$ $\frac{\Delta y}{\Delta x}$ = the rate of change

$\boxed{\Delta x \text{ (very small)}}$

$\frac{\Delta y}{\Delta x} = \boxed{\frac{dy}{dx}}$ = d d x of y

of y with respect to x

Practice Problem

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Input, output

$f(x) = x^n = y$

$\frac{dy}{dx} = \frac{d}{dx}(x^n) = n x^{n-1}$

$n=3 \quad \frac{dy}{dx} = \frac{d}{dx}(x^3) = 3x^{3-1} = 3x^2$

$n=-3 \quad \frac{dy}{dx} = \frac{d}{dx}(x^{-3}) = -3x^{-3-1} = -3x^{-4}$

$n=1 \quad \frac{dy}{dx} = \frac{d}{dx}(x^1) = 1x^{1-1} = 1.x^0 = 1.1 = 1$

Practice Problem

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x) \text{ where c is a constant}$$

$y = 4x^3$

$$\frac{d}{dx}(4x^3) = 4\frac{d}{dx}(x^3) = 4 \times 3 \times x^{3-1} = 4x^2$$

$$\frac{d}{dx}\left(\frac{x^2}{2}\right) = \frac{1}{2}\frac{d}{dx}(x^2) = \frac{1}{2} \times 2 \times x^{2-1}$$

$$\frac{d}{dx}(\text{constant}) = 0$$

$$\frac{d}{dx}(4) = 0 \quad \Big| \quad \frac{d}{dx}(\pi) = 0$$

$$\frac{d}{dx}(\sin x + x^3)$$
$$= \frac{d}{dx}(\sin x) + \frac{d}{dx}(x^3)$$

Practice Problem

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}(4x^2+3x) = \frac{d}{dx}(4x^2) + \frac{d}{dx}(3x)$$
$$= 4\frac{d}{dx}(x^2) + 3\frac{d}{dx}(x^1)$$
$$= 4 \times 2 \times x^{2-1} + 3 \times 1 \times x^{1-1}$$
$$= 8x + 3$$

Practice Problem

Differentiate $f(x) = 0.1x^2 + 3$ with respect to x.

$$y = f(x) = 0.1x^3 + 3$$

$$\frac{dy}{dx} = \frac{d}{dx}(0.1x^2 + 3)$$

$$= \frac{d}{dx}(0.1\boxed{x^2}) + \frac{d}{dx}(3)$$

constant

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$n=2$

$$= 0.1\frac{d}{dx}(x^2) + 0$$

$$= 0.1 \times 2 \times x^{2-1} + 0 = 0.2x + 0$$

$$= \frac{x}{5} \text{ (Answer)}$$

Poll Question - 01

What is the rate of change of $f(x) = x^2 + 3$ at $x=0$?

(a) 0

(b) 3

(c) none of the above

constant

$$\frac{d}{dx}(x^2+3) = \frac{d}{dx}(x^2) + \frac{d}{dx}(3)$$

$$= 2x^{2-1} + 0$$

$$= 2x$$

$x = 0$ point, $2x$

$= 2\times0$

$= 0$

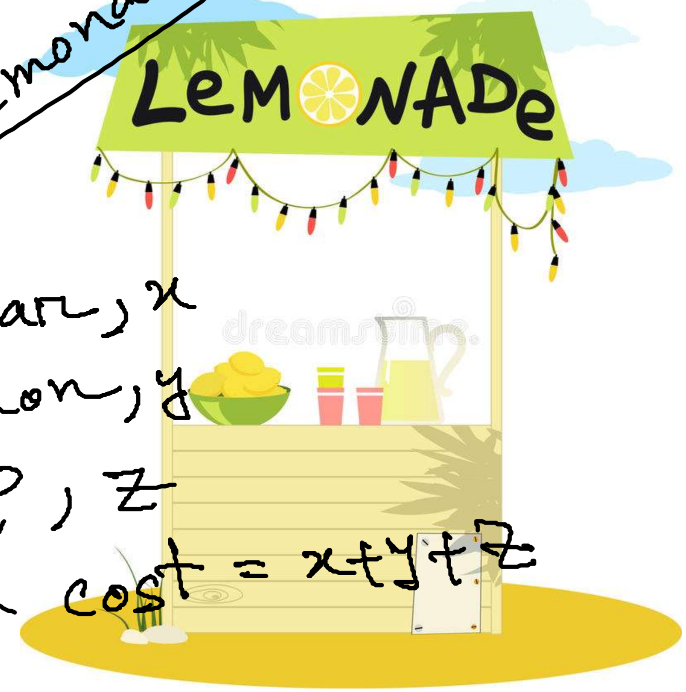
Function (Single Input) = output

Multivariable Function

Function (more than one input) = output

Real life example

Lemonade



Profit = Income − Cost

$= 1000 - (x+y+z)$

Income

\$1000

Sugar, x
Lemon, y
Cup, z

total cost $= x+y+z$

Profit $= f(x, y, z) = 1000 - (x+y+z)$

multi-varible function

$$\boxed{f(x,y) = x^2 y}$$

$\frac{d}{dx}(4x^3) = 4\frac{d}{dx}(x^3)$

constant

Partial Differentiation

$f(x,y,z) = xy + z^2 y$

$\frac{\partial}{\partial x} f(x,y,z)$ = the rate of change of $f(x,y,z)$ with respect $\boxed{x}$ [y, z unchanged]

constant

constant

$\frac{\partial}{\partial x}\{xy + z^2 y\} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial x}(z^2 y)$

$= y\frac{\partial}{\partial x}(x^1) + \frac{\partial}{\partial x}(\text{constant})$

$= y \times 1 \times x^{1-1} + 0$

$= y \times 1 \times x^0 + 0$

$= y$

$$f(x,y,z) = x^2 y + y z^2$$

constant

$$\frac{\partial}{\partial y} f(x,y) = \frac{\partial}{\partial y}(x^2 y) + \frac{\partial}{\partial y}(y z^2)$$

$$= x^2 \frac{\partial}{\partial y}(y) + z^2 \frac{\partial}{\partial y}(y)$$

$$= x^2 \times 1 + z^2 \times 1 \quad [x \text{ \& } y \text{ constant}]$$

$$\frac{\partial}{\partial z} f(x,y,z) = ? \quad \frac{\partial}{\partial z}(x^2 y + y z^2)$$

$$= \frac{\partial}{\partial z}(x^2 y) + \frac{\partial}{\partial z}(y z^2)$$

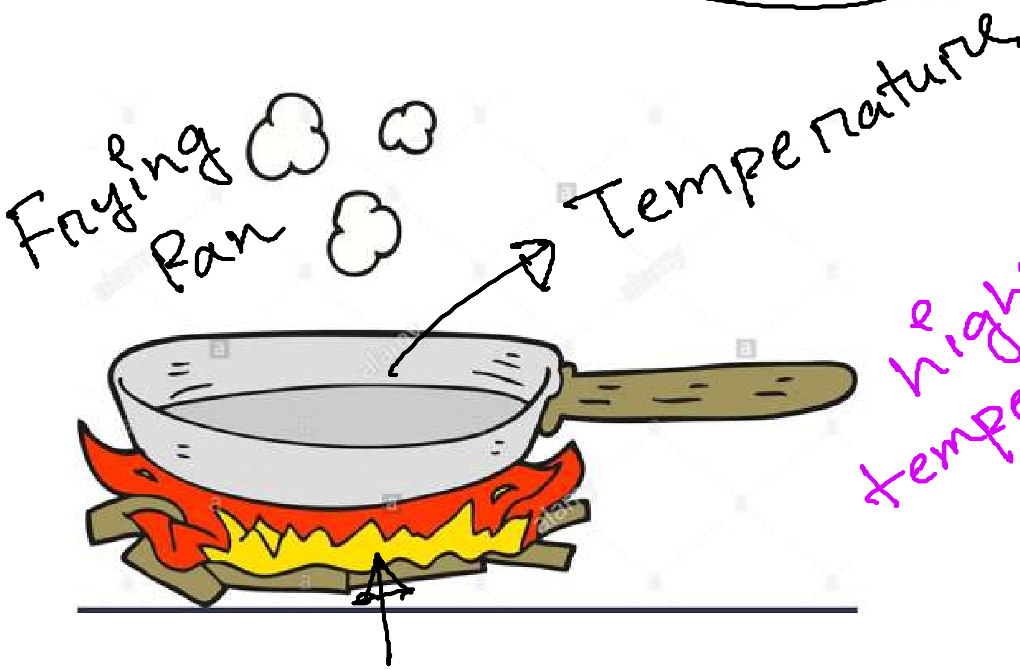
$$= 0 + y \frac{\partial}{\partial z}(z^2)$$

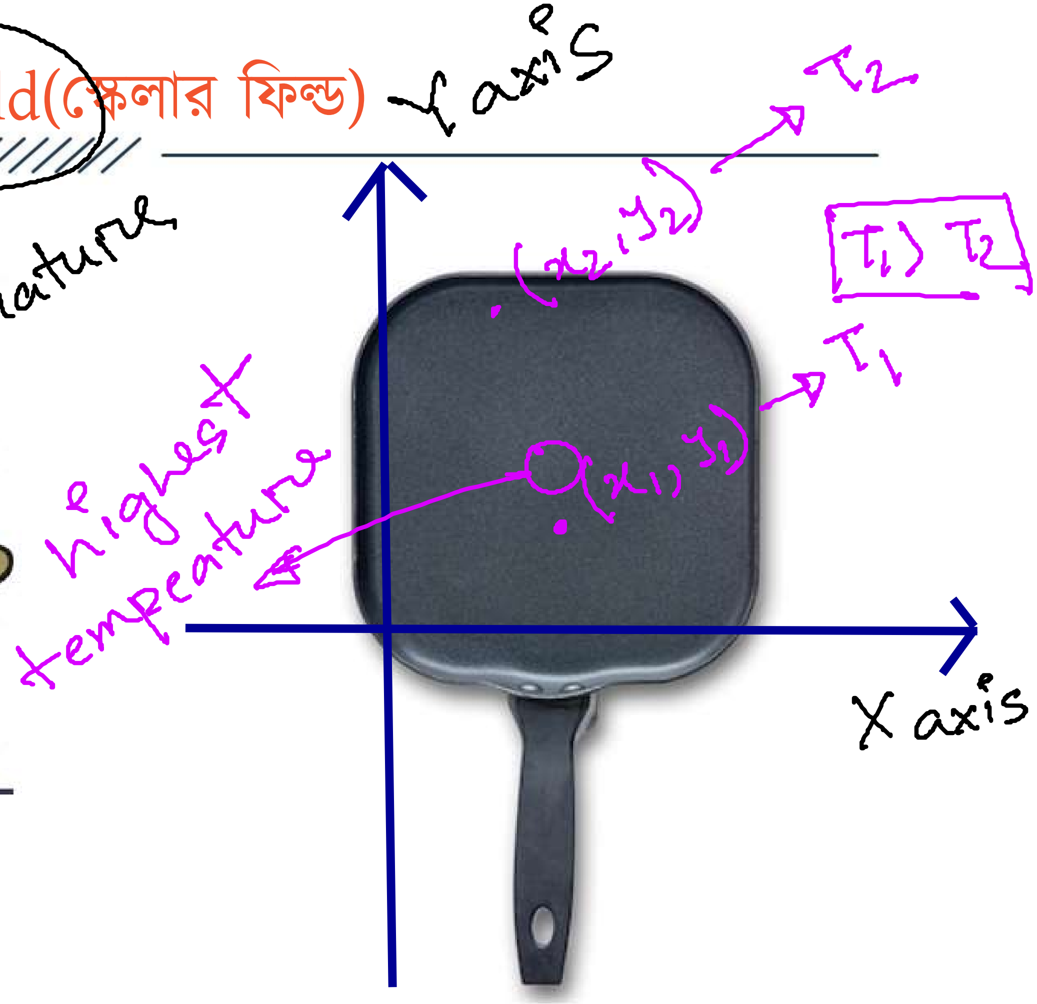
$$= y \times 2z$$

Practice Problem (HW)

Find the partial derivatives w.r.t x and y of the function $f(x,y) = -0.1x^2 - 0.1y^2 + 32.4$

Scaler Field(স্কেলার ফিল্ড)

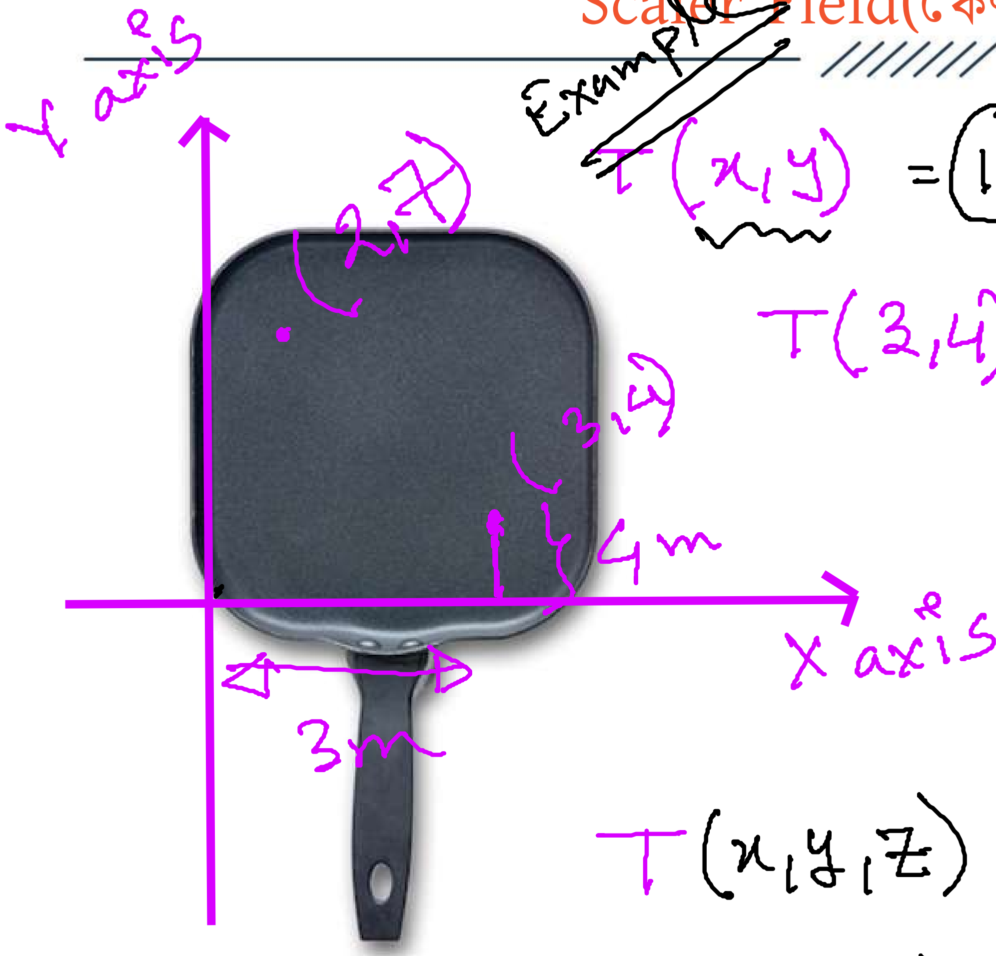




When change of space → change of scaler quantity

Scaler Field(স্কেলার ফিল্ড)

Example



$T(x,y) = 100 - x^2 - y^2$

$T(3,4) = 100 - 3^2 - 4^2$

$= 75^\circ C$

$T(2,7) = 100 - 2^2 - 7^2$

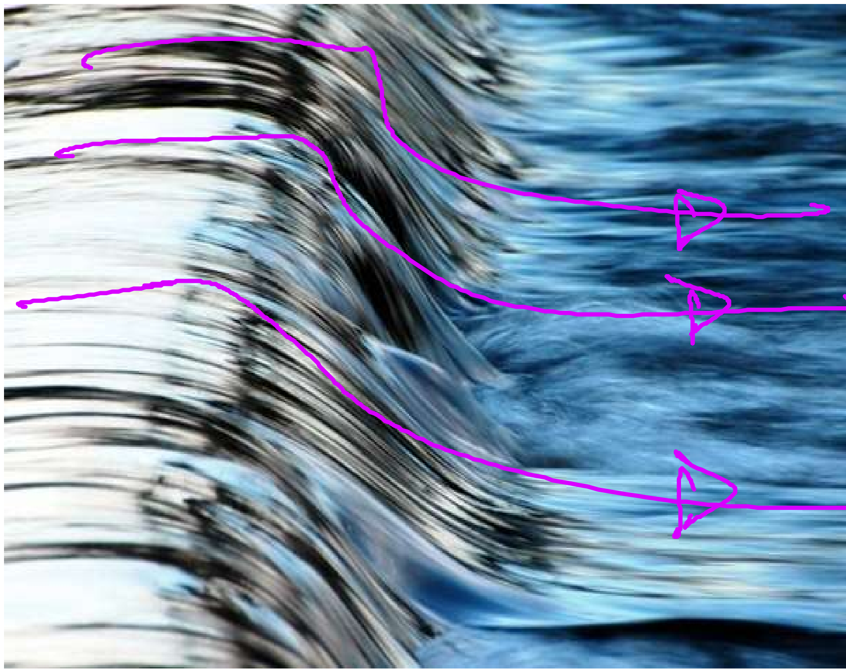
$T(x,y,z) = \text{scaler}$

$\Rightarrow T(x,y,z) = xy + z^2$ (This is also an example)

Vector Field(ভেক্টর ফিল্ড)

Example

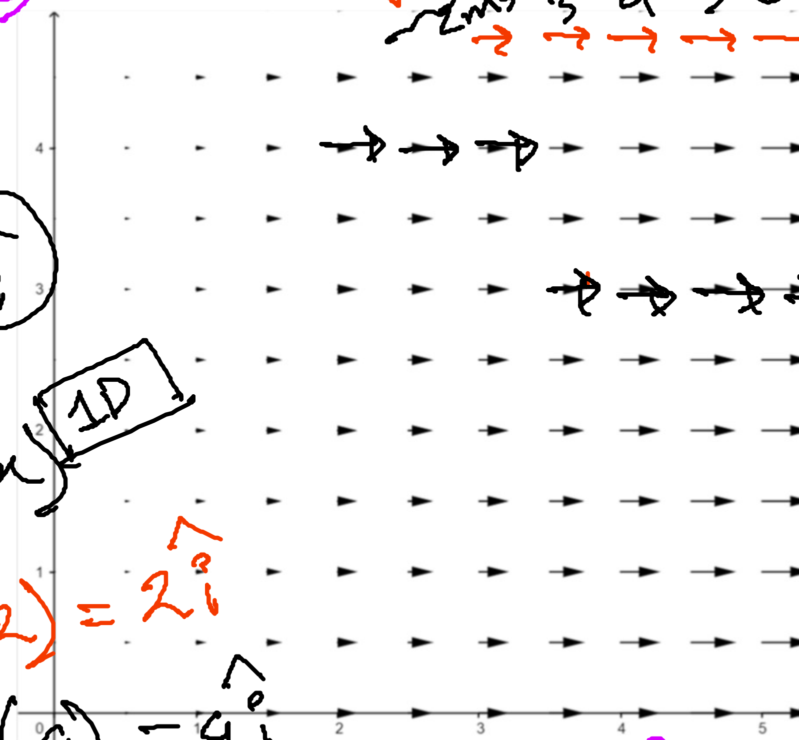
flow of fluid



Y axis

velocity

ms^{-1} 3 4 5 6



X axis

$\vec{v} = x\hat{i}$

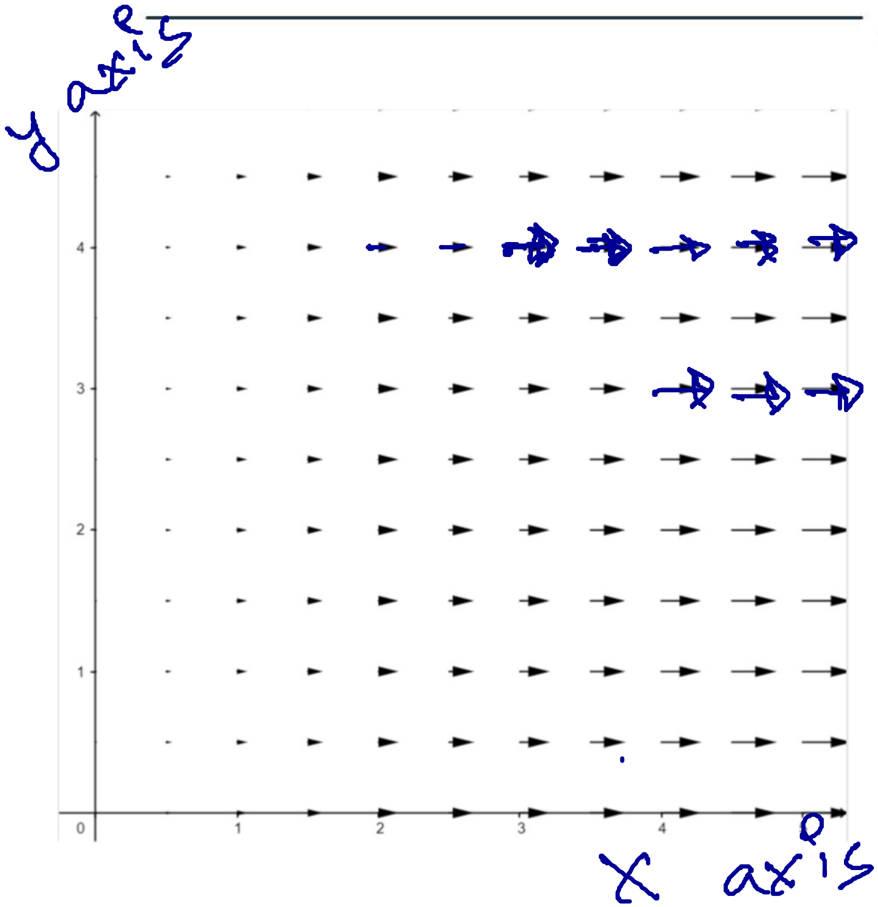
$\vec{v} = \vec{f}(x)$ 1D

$\vec{v} = \vec{f}(2) = 2\hat{i}$

$\vec{v} = \vec{f}(4) = 4\hat{i}$

Vector Field(ভেক্টর ফিল্ড)

Example:



$$\vec{V} = \vec{f}(x, y, z)$$

$$= xz\hat{i} + y\hat{j} + zx\hat{k}$$

$$\vec{V} = \vec{f}(1, 2, 3)$$

$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

Nabla operator, $\vec{\nabla}$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

(vector operator)

Application coming up

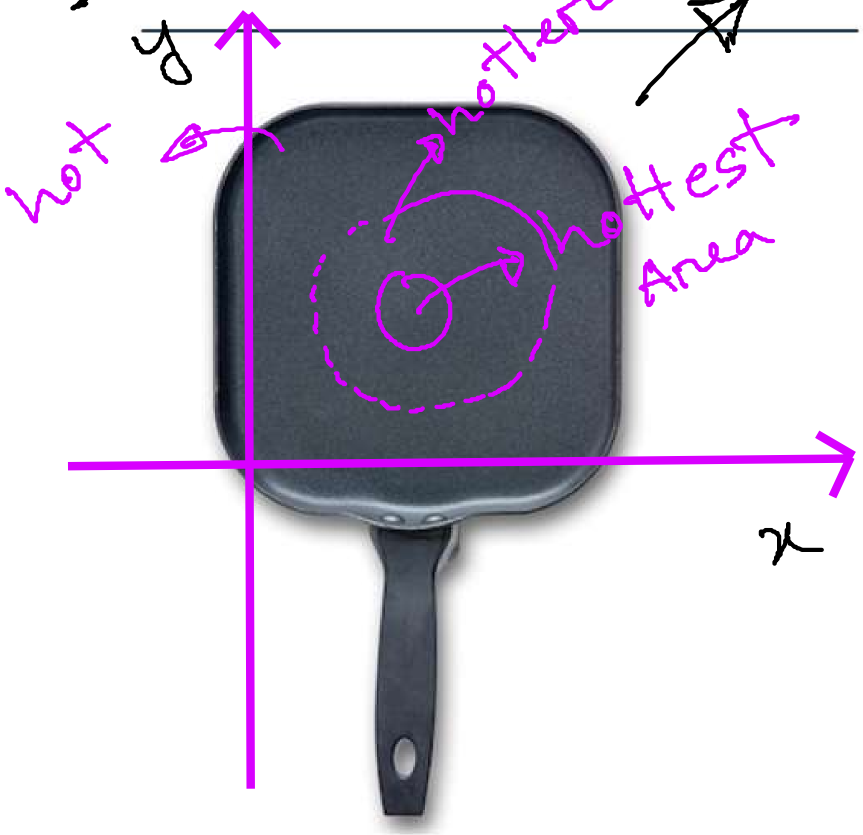
$(x)^2 \rightarrow x \times x$

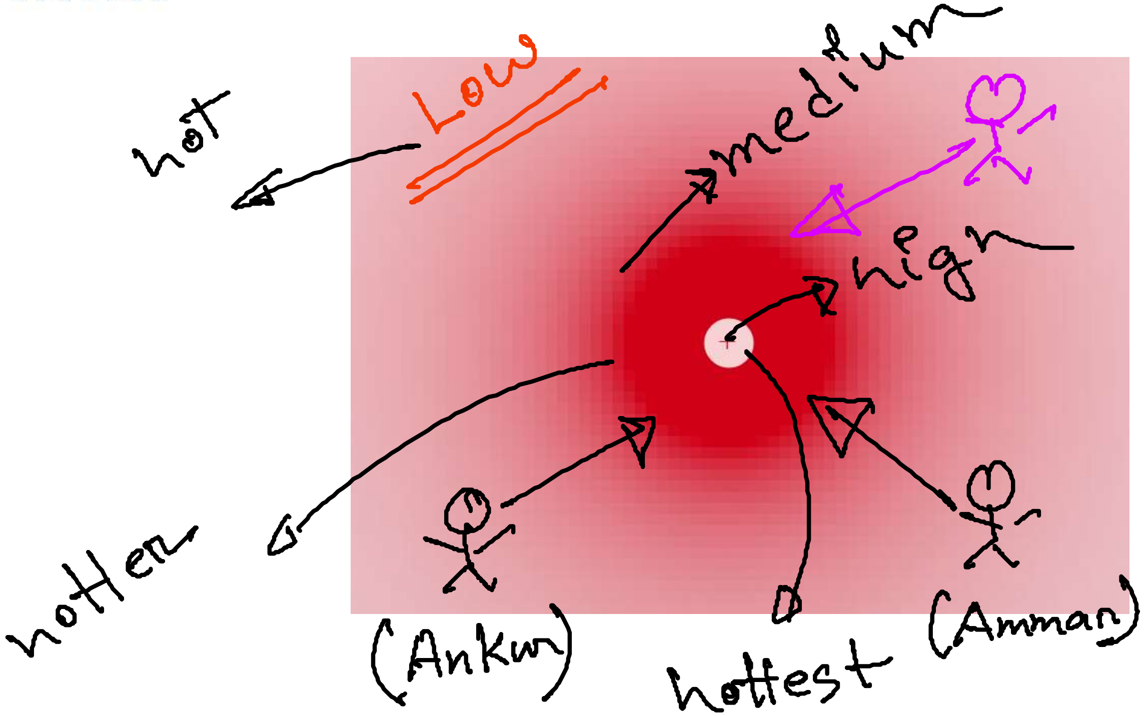
square

Gradient

$\vec{\nabla}$ (scaler field) = gradient of scaler field

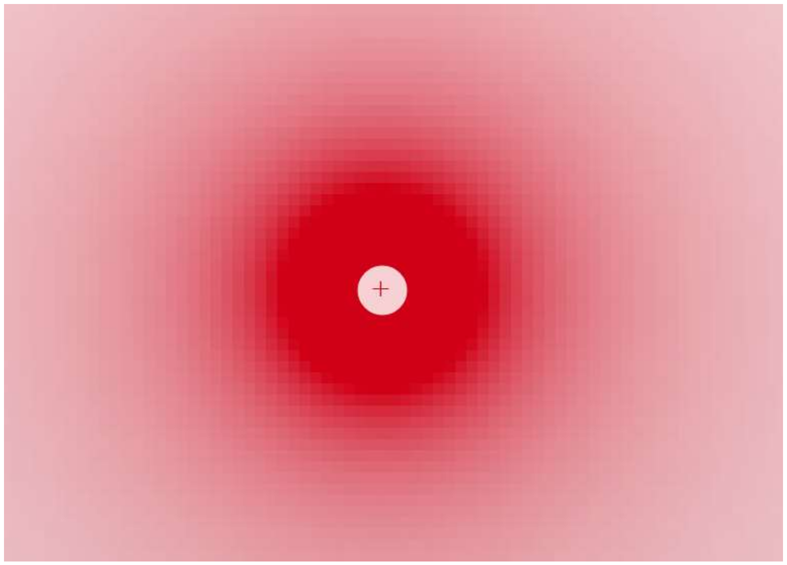
Gradient(গ্রেডিয়েন্ট)





grad (scaler field) = vector

Gradient(গ্রেডিয়েন্ট)



gradient

→ Direction to the maximum rate of change

→ maximum rate of change

Practice Problem

Find the gradient of scaler field $f(x, y, z) = x^2y + xyz$

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(x^2y + xyz)$$

$$= \frac{\partial}{\partial x}(x^2y + xyz)\hat{i} + \frac{\partial}{\partial y}(x^2y + xyz)\hat{j} + \frac{\partial}{\partial z}(x^2y + xyz)\hat{k}$$

$$= \left\{\frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial x}(xyz)\right\}\hat{i} + \left\{\frac{\partial}{\partial y}(x^2y) + \frac{\partial}{\partial y}(xyz)\right\}\hat{j} + \left\{\frac{\partial}{\partial z}(x^2y) + \frac{\partial}{\partial z}(xyz)\right\}\hat{k}$$

$$= (2xy + yz.1)\hat{i} + \{x^2.1 + xz.1\}\hat{j} + \{0 + xy\}\hat{k}$$

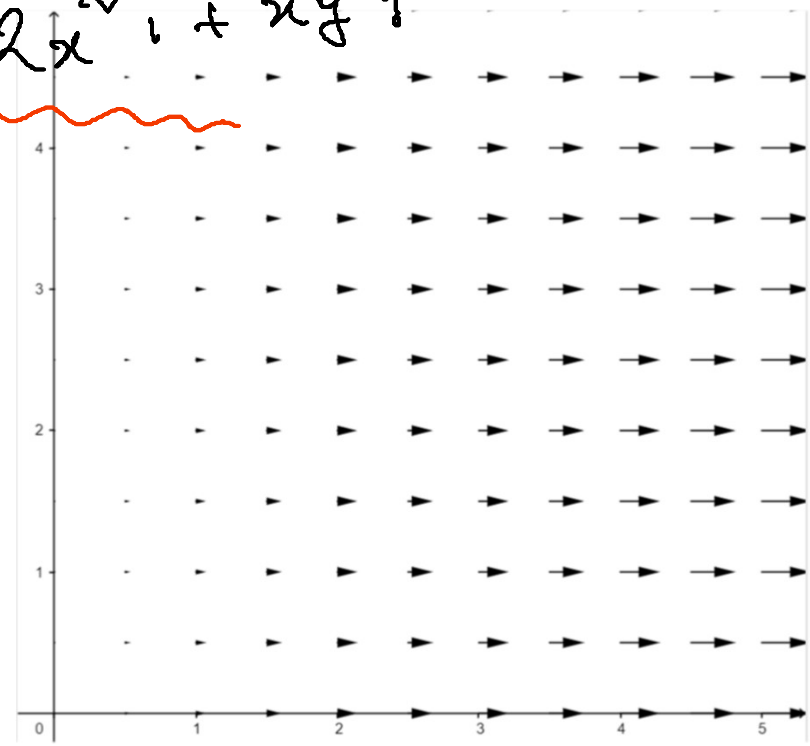
$$= (2xy + yz)\hat{i} + (x^2 + xz)\hat{j} + xy\hat{k}$$

$\vec{\nabla}\cdot$(vector field) = Divergence

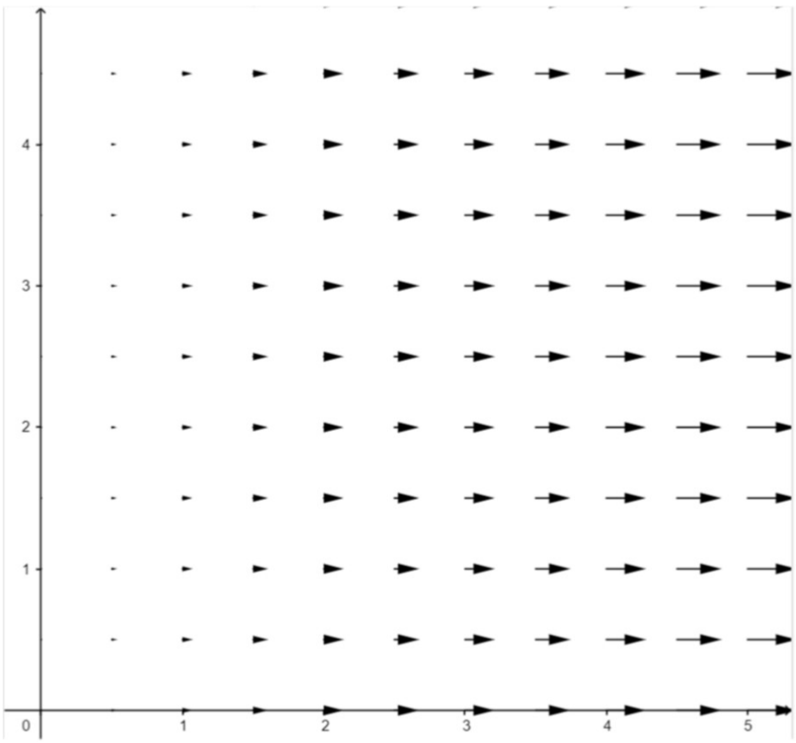
Divergence(ডাইভারজেন্স)

velocity of water

$\vec{V} = 2x^2\hat{i} + xy\hat{j}$



Divergence(ডাইভারজেন্স)



$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{\nabla}\cdot\vec{V}$

Practice Problem

$\vec{A}\cdot\vec{B}$

Find the divergence of velocity field, $\vec{v} = x^2y\hat{i} + yz\hat{j}$

$$\vec{\nabla}\cdot\vec{V} = \left(\boxed{\frac{\partial}{\partial x}\hat{i}} + \boxed{\frac{\partial}{\partial y}\hat{j}} + \boxed{\frac{\partial}{\partial z}\hat{k}}\right)\cdot\left(\boxed{x^2y\hat{i}} + \boxed{yz\hat{j}} + \boxed{0\hat{k}}\right)$$

$$= \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(0)$$

$$= y\frac{\partial}{\partial x}(x^2) + z\frac{\partial}{\partial y}(y) + 0$$

$$= y \times 2x + z \times 1$$

$$= 2xy + z \text{ (Scaler quantity)}$$

vector field

Poll Question - 02

For which of the following divergence can not be determined?

(a) Electromagnetic Force

(b) Velocity of air

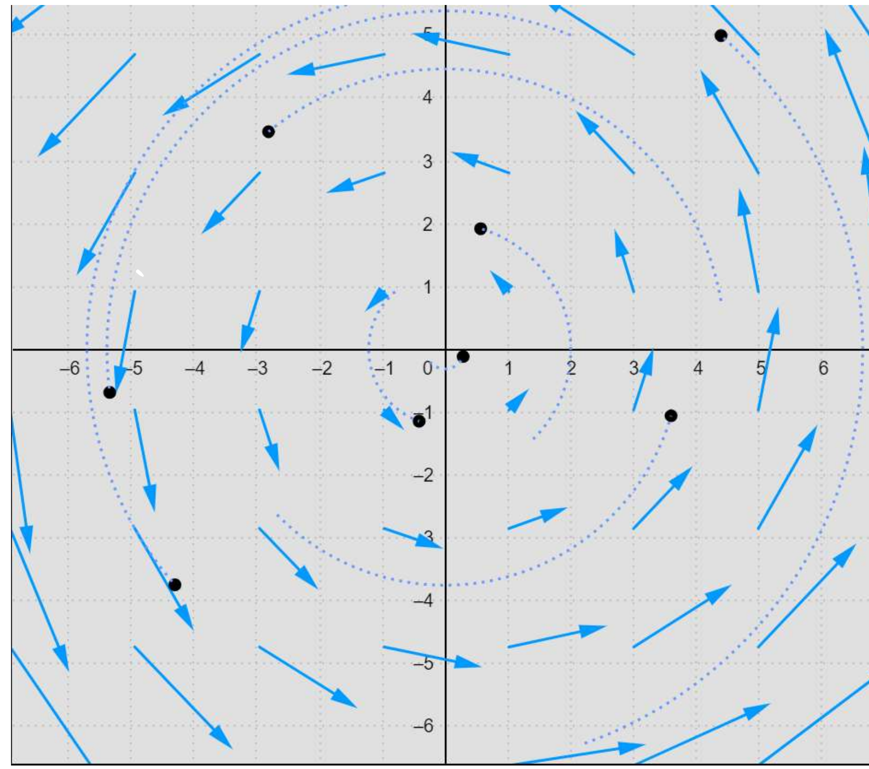
(c) Temperature

Scalar field

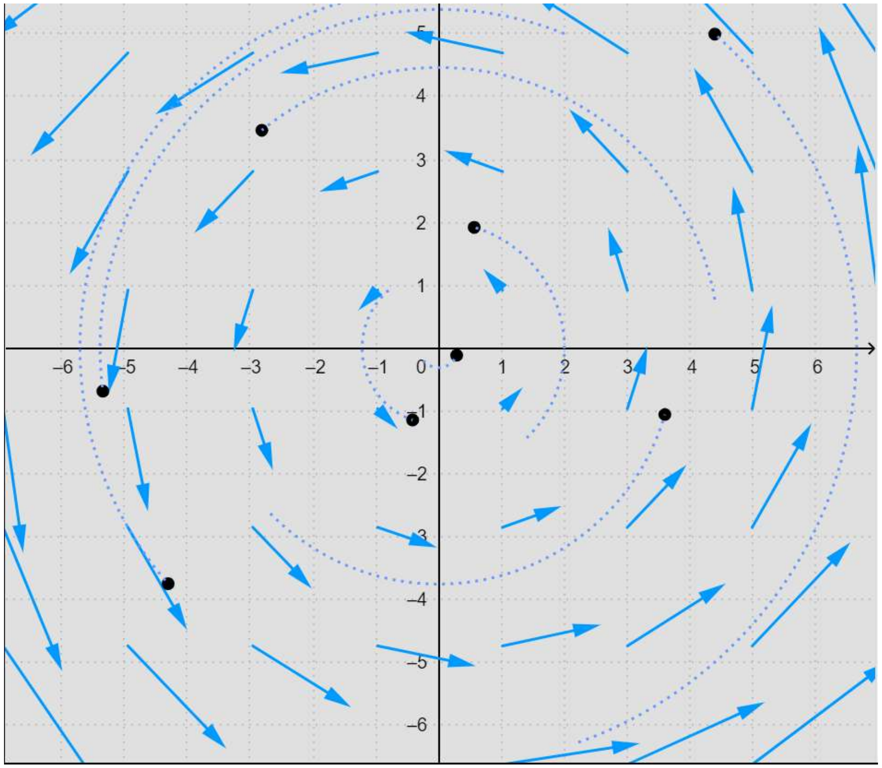
$\vec{\nabla}\cdot\vec{V}$ = divergence = scaler quantity

$\vec{\nabla}\times\vec{V}$ = curl = vector quantity

Curl(কার্ল)



Curl(কার্ল)



Practice Problem

Find the curl of velocity field, $\vec{v} = -y\hat{\imath} + x\hat{\jmath}$

$\vec{v} = -y\hat{i} + x\hat{j} + 0\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$$= \hat{i}\left\{\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(x)\right\} - \hat{j}\left\{\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(-y)\right\}$$

$$+ \hat{k}\left\{\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y)\right\}$$

0 0

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}\{1-(-1)\}$$

$$= 2\hat{k}$$

Curl

Resources

Derivative: https://www.geogebra.org/m/WbcrsCm7
https://www.geogebra.org/m/DSEBMEyM
Partial Derivative: https://www.geogebra.org/m/EWMQ8qnr
Gradient: https://www.geogebra.org/m/QhfcuhqA
Divergence & Curl: https://www.geogebra.org/m/GmJqrGsC#material/xacMPzSj

https://openstax.org/books/calculus-volume-3/pages/6-5-divergence-and-curl

না বুঝে মুখস্থ করার অভ্যাস
প্রতিভাকে ধ্বংস করে।