بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

উদ্ভাস উন্মেষ
শিক্ষা পরিবার

Physics
1st Paper

Chapter-02: Vactor

Lecture: P-02

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Topics

- ❖ Quantity
- ❖ Representation of Vectors
- ❖ Different types of Vector
- ❖ Resultant of Vector
    - Law of Triangle
    - Law of Polygon
    - Law of Parallelogram
- ❖ Magnitude and direction of the resultant by the Parallelogram Law
- ❖ Mathematical Examples
- ❖ Some Properties of Vector Addition

Quantity

Physical characteristics of matter that can be measured are called quantities.

We can classify physical quantities into two categories-

magnitude

Scalar Quantity

magnitude
direction

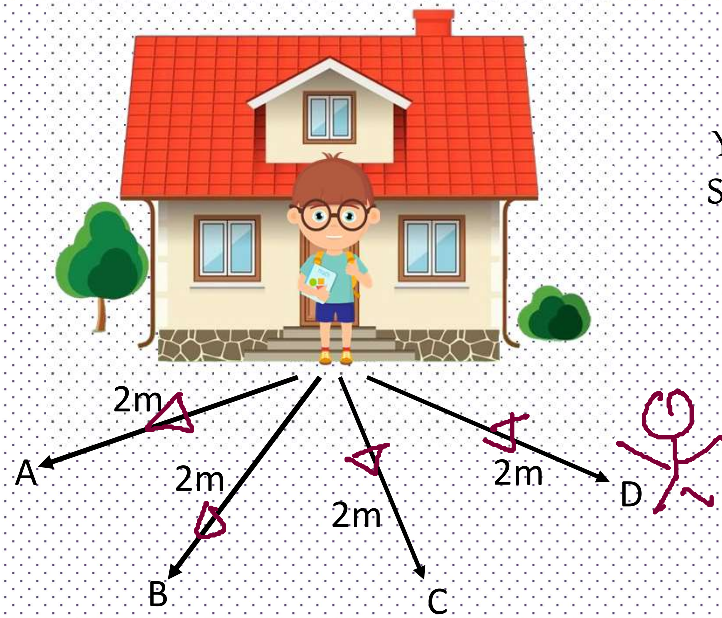
Vector Quantity

Scalar and Vector Quantity

What is the mass of this brick?

-500 gm

This is enough to express the mass. So, mass is scalar quantity.



What is your displacement in this case?

- 2m

You may be at any point of A, B, C, D. So, the information here is incomplete. To know the exact position you have to know the **Direction** of displacement too.

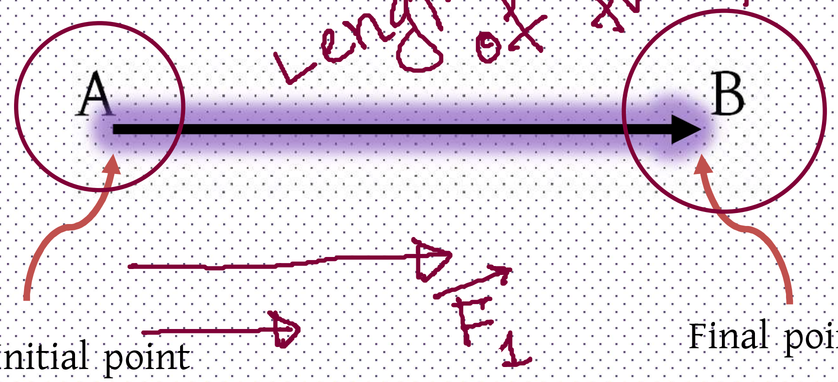
Therefore, displacement is a vector quantity.

Examples of Vector Quantity

- Displacement
- Velocity
- Momentum
- Acceleration
- Force

Representation of Vectors

Length of the line



$\vec{AB}$

$\vec{F_1}$

$\vec{F_2}$

$|\vec{AB}|$

Length of line → Magnitude of Vector

Direction of arrow → Direction of Vector

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AB or $\vec{AB}$

Bold

First Initial then Final Point

Magnitude of vector

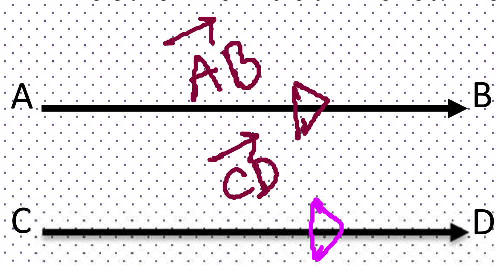
AB or $|\vec{AB}|$

Not Bold

Different Types of Vector

Equal vector

- Magnitude and length are equal
- Direction of both is same



equal length

$$\overrightarrow{AB} = \overrightarrow{CD}$$

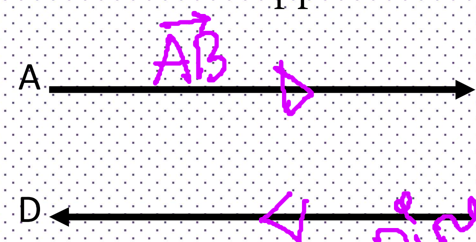
A vector can be moved to any place on the surface keeping the Direction and Magnitude constant.

10 unit

Opposite vector

- Magnitude is equal
- Direction opposite

equal length



Direction opposite

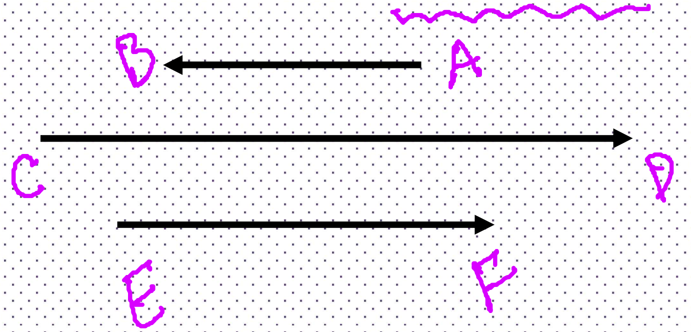
$$\overrightarrow{AB} = -\overrightarrow{CD}$$

Different Types of Vector

Collinear Vectors

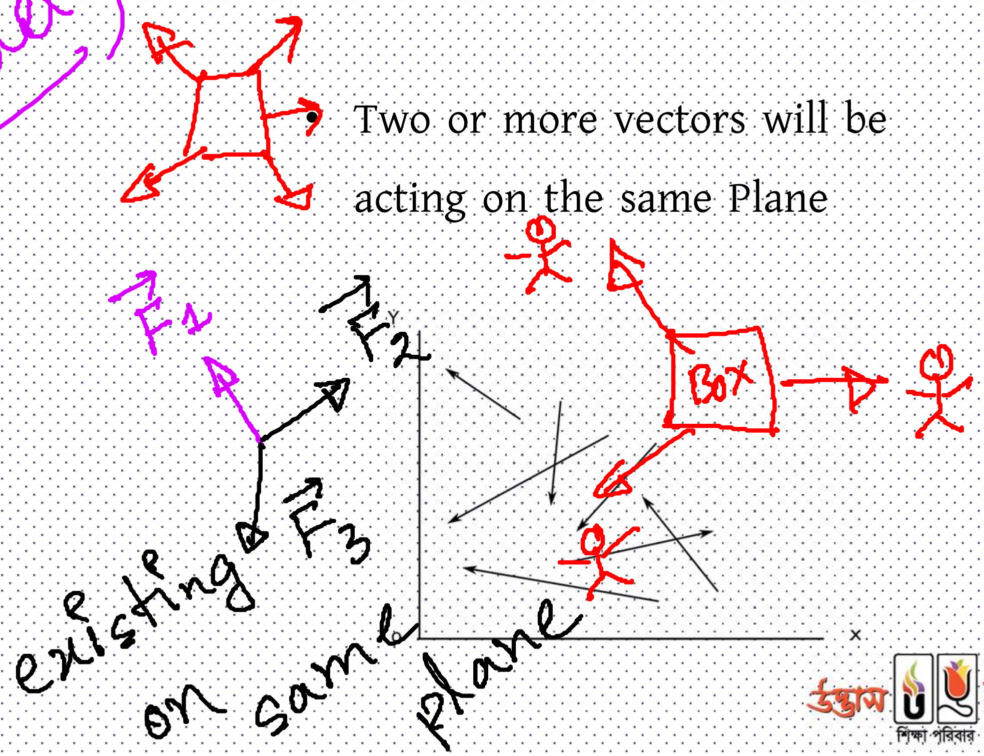
(parallel)

- Two vectors directed along the same line
- Magnitude can be Equal/Not-equal
- Direction can be same/opposite



Coplanar Vectors

- Two or more vectors will be acting on the same Plane



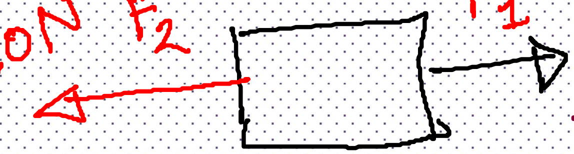
Different Types of Vector

Null Vector

- Magnitude Zero
- No particular Direction
- Used to denote the resultant of subtraction of two equal vectors
- Indicates the vector product of two parallel vectors
- Denoted by $\vec{0}$

Example:

10N $\vec{F_2}$ $\vec{F_1}$



10N

$\vec{AB} = \vec{CD}$

null vector 0N

A $\longrightarrow$ B

C $\longrightarrow$ D

$\vec{AB} - \vec{CD} = \vec{0}$

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Different Types of Vector

Direction same

$\vec{F_1}$ $|\vec{F_1}| = 10$

Reciprocal Vectors

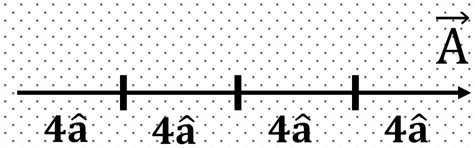
- Two vectors are Parallel (acting on same)
- Magnitude is reciprocal to one another

$\vec{F_2}$ $|\vec{F_2}| = \frac{1}{|\vec{F_1}|} = \frac{1}{10}$

Unit Vector

- Magnitude 1 unit
- When a vector is divided by its magnitude, unit vector is found along the direction of the vector
- ^ Sign is put on the small letter to denote unit vector

If $\vec{A}$ is a vector with magnitude 4 unit and $\hat{\mathbf{a}}$ is the unit vector along $\vec{A}$,



$$\hat{\mathbf{a}} = \frac{\vec{\mathbf{A}}}{\mathbf{A}}$$

$\vec{F_1}$ (parallel)

$\hat{F} = 1$ unit

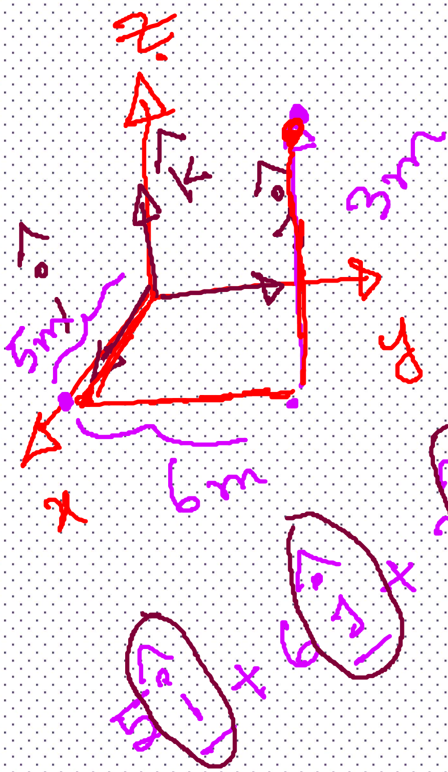
$\hat{F}$ is the unit vector of $\vec{F_1}$

$\vec{F}$

$\hat{F}$ (1 unit)

Different Types of Vector

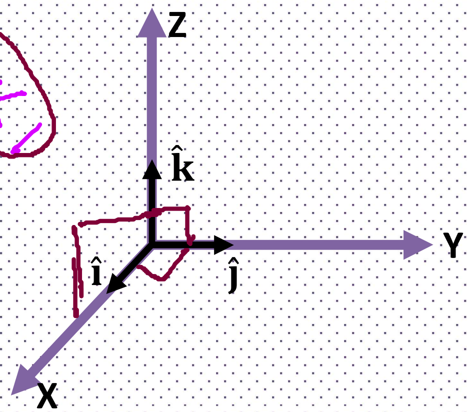
Rectangular Unit vector



- Three unit vectors along the three axes of 3-D co-ordinate system
- $\hat{\mathbf{i}}$ along X-axis, $\hat{\mathbf{j}}$ along Y-axis, $\hat{\mathbf{k}}$ along Z-axis

$\vec{S} = 5\hat{i}$ m

$\vec{F} = 10\hat{k}$ N

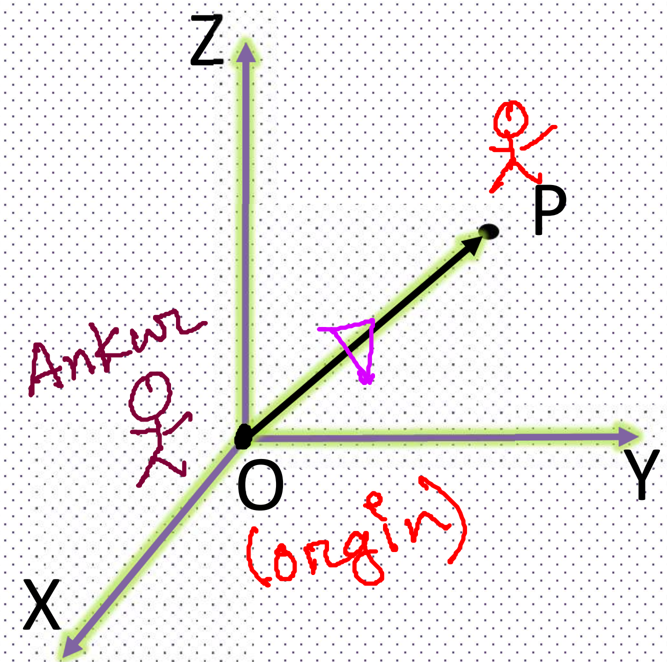


Example –
If $\vec{A}$ is a vector of Magnitude 5 along X axis,
$\vec{A} = 5\,\hat{\imath}$

Different Types of Vector

Position Vector

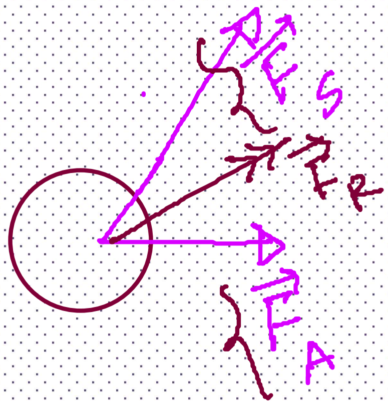
- Vector used in 3-D co-ordinate system to know the position of a point w.r.t origin
- Also called radius vector

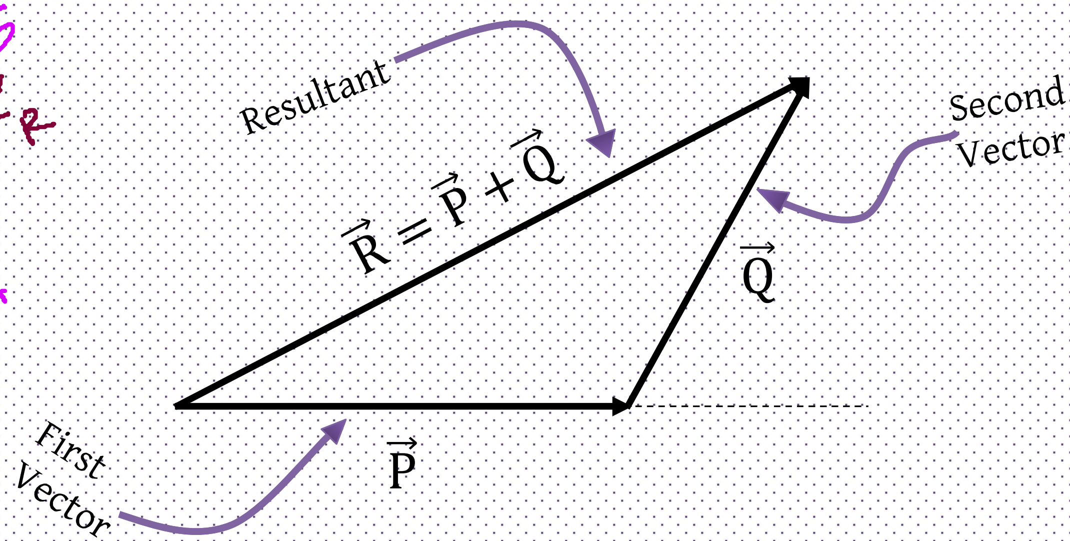


$\overrightarrow{\mathbf{OP}}$ - Position Vector

Resultant of Vectors : General Law

Statement: Of the two vectors, the final point of first vector and the initial point of second vector are placed on the same point, then the straight line connecting the initial point of the first vector and the final point of second vector will express the resultant. Magnitude is the length of the line and direction is from the initial point of the first vector to the final point of second vector.

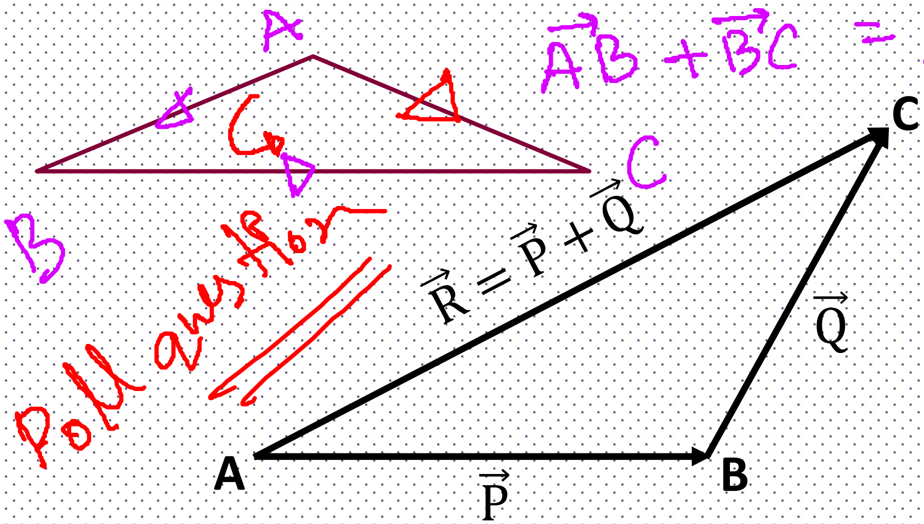




Resultant of Vectors : Triangle Law

Statement: If two similar vectors acting at a point can be represented by two consecutive sides of a triangle taken in order, then the third side will give the resultant vector in the reverse order.

$\vec{AB} + \vec{BC} = \vec{AC}$



$$\vec{P} + \vec{Q} = \vec{R}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$$

If three vectors acting simultaneously at a point are represented by three sides of a triangle taken in order, then the resultant will be Zero.

POLL QUESTION

$\vec{A} = 5\ \hat{\imath}$ and $\vec{B} = 0.2\ \hat{\imath}$ -Which of the following is true for these two?

$\vec{A} = 5\hat{i}$ (x axis)

$\vec{B} = 0.2\hat{i}$ (x axis)

(a) Colinear Parallel

(b) Non-zero

(c) Reciprocal

(d) All

$|\vec{A}| = 5$

$|\vec{B}| = 0.2 = \frac{1}{5}$

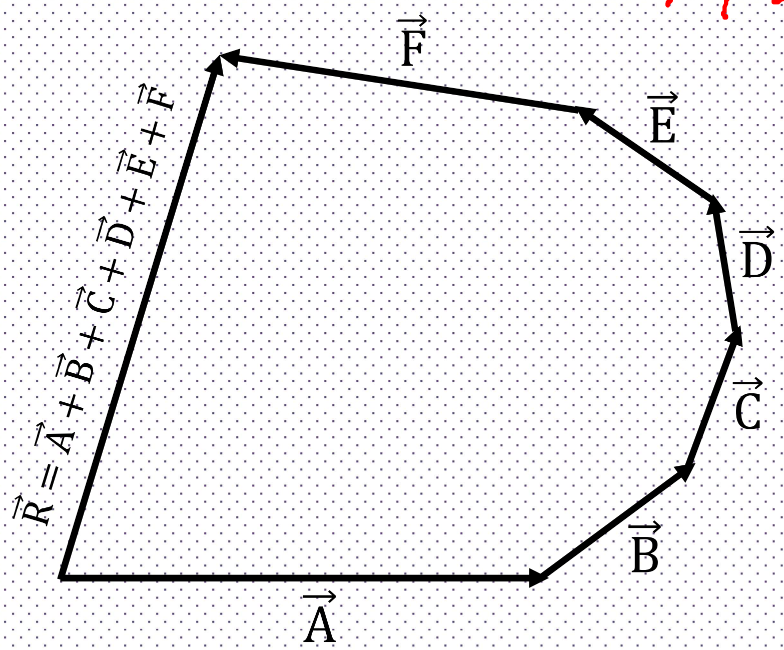
$= \frac{1}{|\vec{A}|}$

Determination of Resultant : Law of Polygon

Statement: If a vector polygon be drawn, placing the tail-end of each succeeding vector at the head or arrow-end of the preceding one, their resultant is drawn from the tail-end of the first to the head of the last.

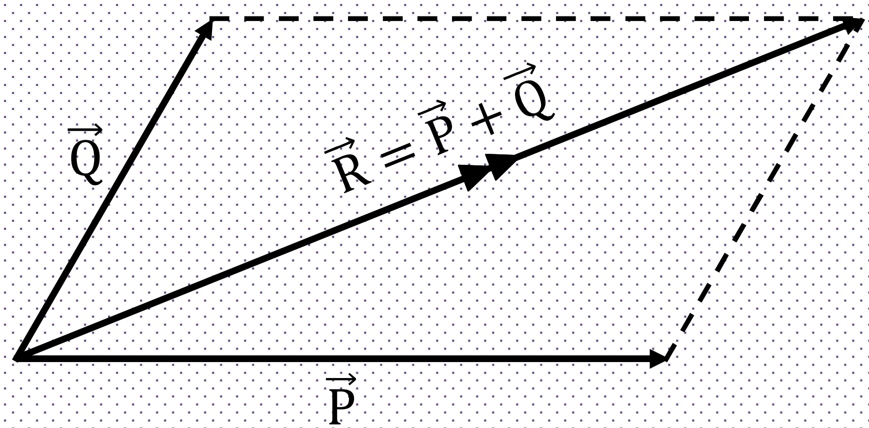
$\vec{A}, \vec{B}, \vec{C}, \vec{D}$

$\vec{P}, \vec{Q}$



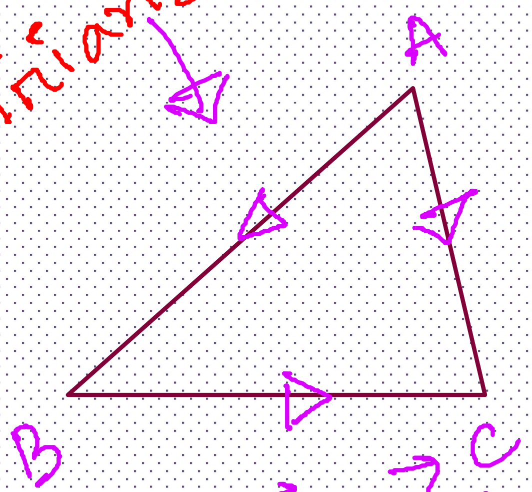
Determination of Resultant : Parallelogram Law

Statement: If two similar vectors acting simultaneously at a point can be represented both in magnitude and direction by two adjacent sides of a parallelogram, then the diagonal from the point of intersection of these sides gives the resultant vector both in magnitude and direction.

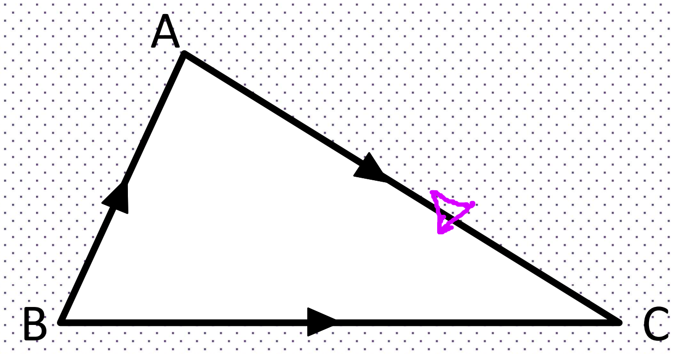


POLL QUESTION

Triangle law



$\vec{AB} + \vec{BC} = \vec{AC}$



$\vec{AB} + \vec{BC} = \vec{AC}$

$= -\vec{CA}$

Which one is True?

(a) $\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$

(b) $\overrightarrow{BA} + \overrightarrow{BC} = \overrightarrow{AC}$

(c) $\overrightarrow{CB} + \overrightarrow{AB} = \overrightarrow{CA}$

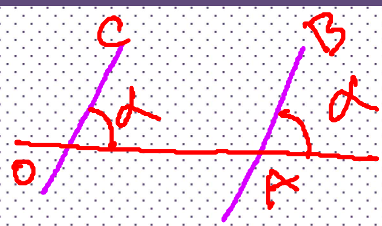
(d) All

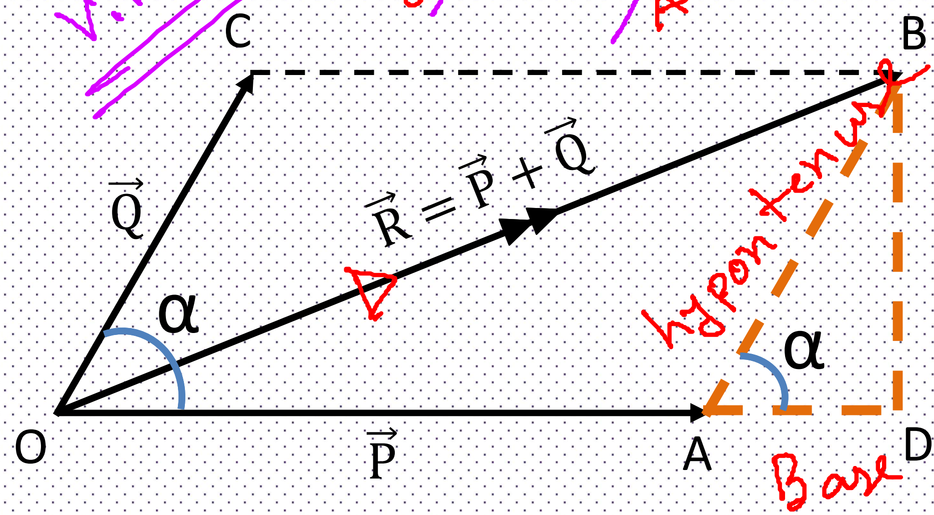
$\vec{AC} = -\vec{CA}$

opposite vector

Magnitude of Resultant from Parallelogram Law

v.v.imp





R = ?

θ = ?

$|\vec{P}| = P = OA = BC$ $OA \parallel BC$

$|\vec{Q}| = Q = OC = AB$ $OC \parallel AB$

$|\vec{R}| = R = OB$

ADB

In ΔADB, (Right-angle triangle

$$\sin\alpha = \frac{BD}{AB} = \frac{BD}{Q} \qquad [AB = OC = Q]$$

$$\Rightarrow BD = Q\sin\alpha$$

Again, $$\cos\alpha = \frac{AD}{AB} = \frac{AD}{Q}$$

$$\Rightarrow AD = Q\cos\alpha$$

$$\sin\alpha = \frac{\text{Perpendicular (BD)}}{\text{hypotenuse (AB)}}$$

Magnitude of Resultant from Parallelogram Law

√ $P = OA$

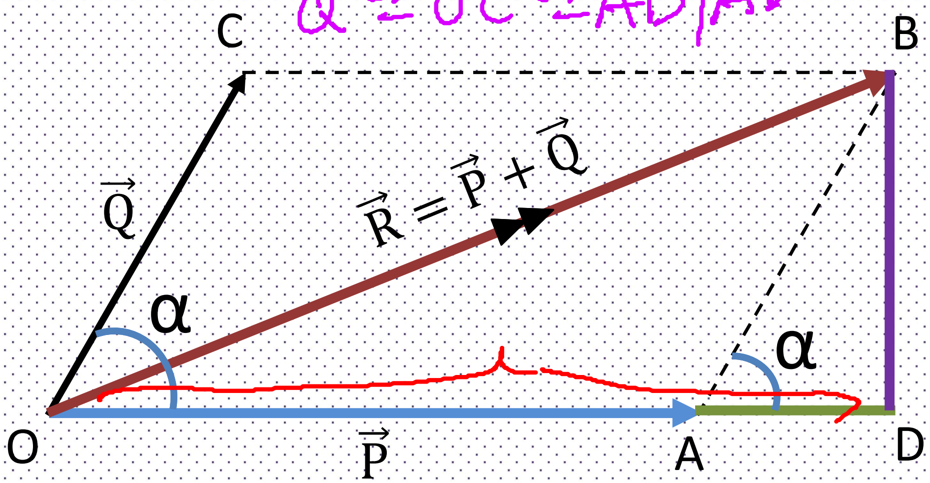
$Q = OC = AB$

$BD = Q\sin\alpha$

$AD = Q\cos\alpha$

OBD

OBD (Right-angle)



$$R^2 = OB^2 = OD^2 + BD^2 \quad \text{— (1)}$$

$$\Rightarrow R^2 = (OA + AD)^2 + BD^2 \quad \text{— (2)}$$

$$\Rightarrow R^2 = OA^2 + AD^2 + BD^2 + 2.OA.AD \quad \text{— (3)}$$

$$\Rightarrow R^2 = P^2 + Q^2(\cos^2\alpha + \sin^2\alpha) + 2PQ\cos\alpha$$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$\Rightarrow \mathbf{R} = \sqrt{\mathbf{P^2 + Q^2 + 2PQ\cos\alpha}}$$

$OD = OA + AD$

P, Q, α

R

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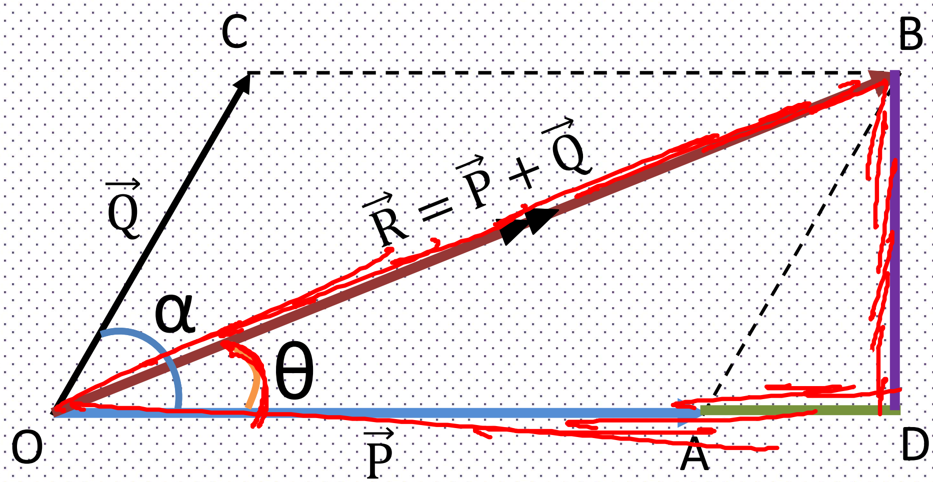
Direction Of Resultant from Parallelogram Law



If resultant makes an angle θ with $\vec{P}$,

OBD

$$\tan\theta = \frac{BD}{OD} = \frac{\text{Perpendicular}}{\text{Base}}$$

$$\Rightarrow \tan\theta = \frac{BD}{OA + AD}$$

$OD = OA + AD$

$$\Rightarrow \boldsymbol{\tan\theta = \frac{Q\ \sin\alpha}{P + Q\ \cos\alpha}}$$

$BD = Q\ \sin\alpha$

$AD = Q\ \cos\alpha$

$P = OA$

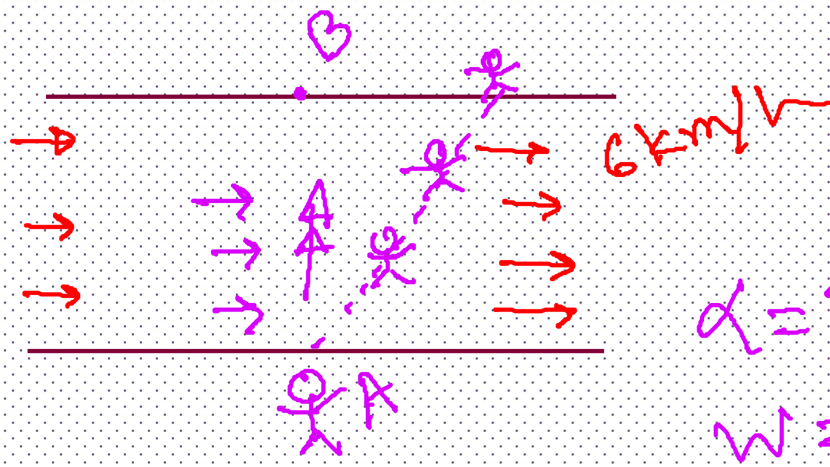
$$\theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P + Q\cos\alpha}\right)$$

Practice Problem

One day the velocity of tide on the river is 6 km/h and velocity of boat is 12 km/h. A boatman wants to cross the river straight and reach the opposite point of the river.

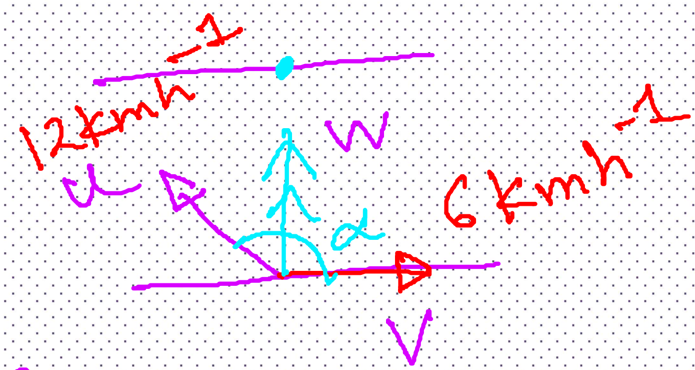
At which direction he should start?

What will be the resultant velocity?



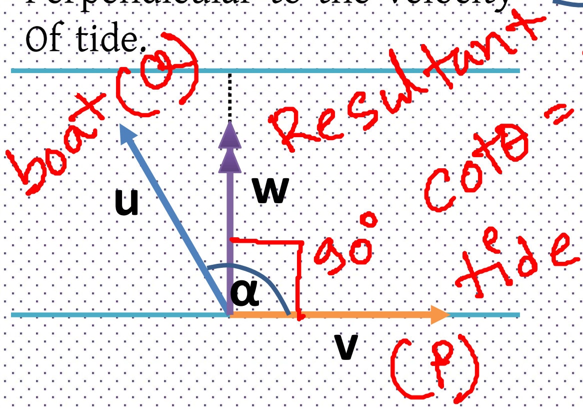
$\alpha = ?$

$w = ?$



Solution

The resultant velocity works Perpendicular to the velocity Of tide.



$\cot\theta = \frac{1}{\tan\theta}$

u → Velocity of boat 12 kmh^{-1}

v → Velocity of tide 6 kmh^{-1}

w → Resultant velocity

$w = ?$ $\alpha = ?$

$$\tan 90^\circ = \frac{u \sin\alpha}{v + u\cos\alpha}$$

V remains alone (without sin/cos) in the denominator as the angle is measured with v.

$$\Rightarrow \cot 90^\circ = \frac{v + u\cos\alpha}{u \sin\alpha}$$

$$\Rightarrow 0 = \frac{v + u\cos\alpha}{u\sin\alpha}$$

$$\Rightarrow v + u\cos\alpha = 0$$

$$\Rightarrow \cos\alpha = -\frac{v}{u}$$

$$\Rightarrow \alpha = \cos^{-1}\left(-\frac{6}{12}\right)$$

$$\Rightarrow \alpha = 120^\circ$$

Resultant velocity,

$$w = \sqrt{u^2 + v^2 + 2uv\cos\alpha}$$

$$\Rightarrow w = \sqrt{144 + 36 + 2.12.6.\cos 120^\circ}$$

$$\Rightarrow w = 6\sqrt{3} \text{ km/h}$$

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$P \wedge R$

Some Special Cases

$\alpha = 0°$

Rmax

Both the vectors are in Same direction.
This will produce **Maximum** resultant.

$\vec{Q}$ $\vec{P}$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0°}$$
$$\Rightarrow R = \sqrt{P^2 + Q^2 + 2PQ}$$
$$\Rightarrow R = \sqrt{(P+Q)^2}$$
$$\Rightarrow \mathbf{R_{max} = P + Q}$$

10 cm

Q

Rmin P

$\alpha = 180°$

$\alpha = 180°$

The vectors are in Opposite direction.
This will produce **Minimum** resultant.

$\vec{Q}$ $\vec{P}$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180°}$$
$$\Rightarrow R = \sqrt{P^2 + Q^2 - 2PQ}$$
$$\Rightarrow R = \sqrt{(P \sim Q)^2}$$
$$\Rightarrow \boxed{\mathbf{R_{min} = P \sim Q}}$$

(-1)

Q P

$R = \sqrt{(P-Q)^2}$
$= P - Q$
$\Rightarrow R_{min}$

$R_{min} = P \sim Q$

Practice Problem

Maximum and minimum resultant of two vectors are 20 unit and 8 unit respectively.

What will be the resultant when they act at an angle of 60°?

$R_{max}[\alpha = 0] = P + Q = 20$

$R_{min}[\alpha = 180°] = P - Q = 8$

$R[\alpha = 60°] = ?$

Solution

$R_{max} = 20 = P + Q$
$R_{min} = 8 = P - Q$ [Assuming, P > Q]

Adding the equations, P = 14 unit
Subtracting the second one
from the first, Q = 6 unit

Threrefore, $R = \sqrt{14^2 + 6^2 + 2.14.6.\cos 60°}$

=> $R = 2\sqrt{79}$ unit

$2P = 28$
$P = 14$
$Q = 6$

$Q > P$
$P + Q = 14$
$Q - P = 6$
$Q = 14$
$P = 6$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos 60°}$

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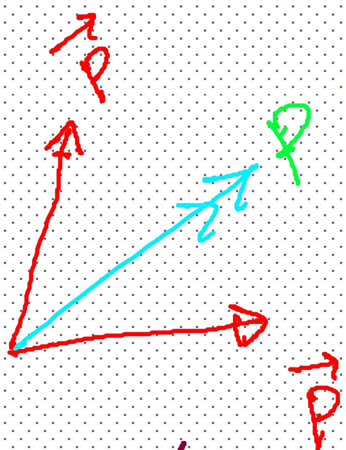
Poll Question

The resultant of two equal vectors is equal to each of the vectors.
What's the angle between them?

$\alpha = ?$



(a) 60°

(b) 45°

(c) 120° ✓

(d) 135°

$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$

$P^2 = P^2 + P^2 + 2P^2\cos\alpha$

$1 = 1 + 1 + 2\cos\alpha$

$\Rightarrow \cos\alpha = -\frac{1}{2}$

$\alpha = 120^\circ$

Subtraction of Vectors

FACT : There's nothing like addidion or subtraction of vector.
We just calculate Resultant.

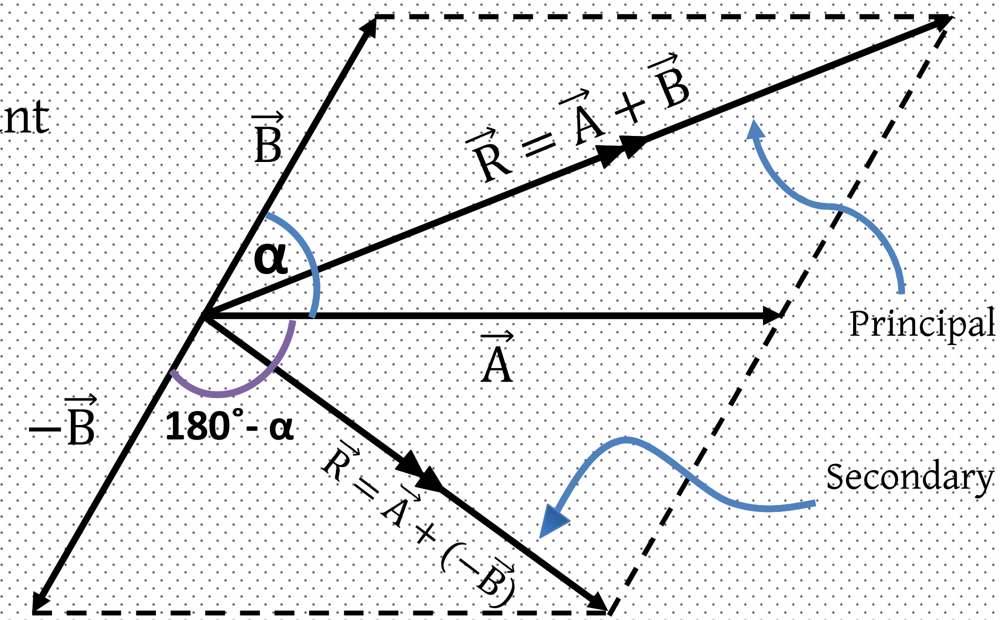
$\vec{A} + \vec{B}$

It indicates the resultant of $\vec{A}$ and of $\vec{B}$.

$\vec{A} - \vec{B}$

It indicates the resultant of $\vec{A}$ and the opposite vector of $\vec{B}$. In other words, resultant of $\vec{A}$ and -$\vec{B}$.

Or, $\vec{A} + (\overrightarrow{-B})$



Relative Velocity

Relative velocity is the velocity of any object with respect to an observer. To determine the relative velocity, the velocity of the observer is to be subtracted from the velocity of the object. (But WHY???)

Compare

\$500 Ankur

\$1000 student

\$1000 − \$500

\$500 − \$1000

= −\$500

Relative velocity of B w.r.t A,

$$\vec{v}_{\mathrm{BA}} = \vec{v}_B - \vec{v}_A$$

Student $\vec{v}_S$

Ankur $\vec{v}_A$

$\vec{v}_S - \vec{v}_A$

with respect to me

with respect to your velocity

Practice Problem

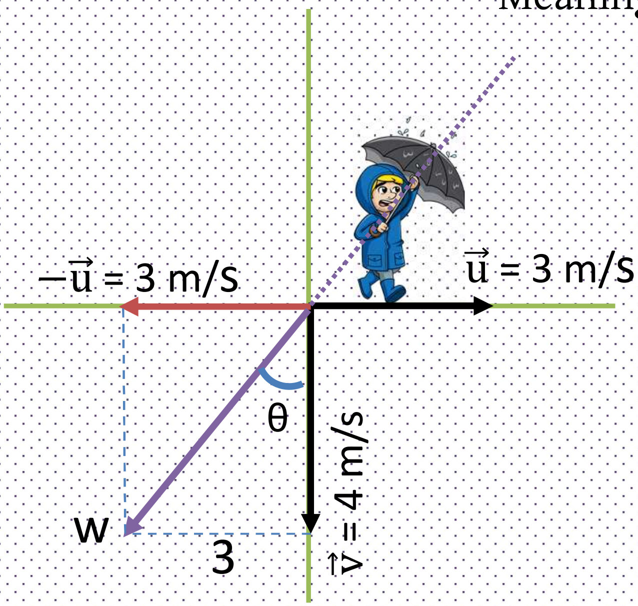
One day the velocity of rainfall was 4 m/s straight downwards.
A person was moving with 3 m/s speed. At what velocity rain hits the person?
Also find the direction at which the umbrella should be placed.

with respect to person
the velocity of the
Rain

$\vec{V}_R - \vec{V}_P$

Solution

Here, we've to determine the relative velocity of rain w.r.t the person.
Meaning, the resultant of rain and the opposite vector of the persons velocity.



$$w = \sqrt{3^2 + 4^2}$$

$$=> w = 5 \text{ m/s}$$

Rain hits the person
with this velocity

$$\tan\theta = \frac{|-\vec{u}|}{|\vec{v}|}$$

$$\Rightarrow \tan\theta = \frac{3}{4}$$

$$\Rightarrow \theta = 36.87°$$

Umbrella should be placed
at this angle
with the vertical axis.

Practice Problem

The driver of a Car, while going towards east at 30 km/h velocity, notices a Truck going towards North at $30\sqrt{3}$ km/h.

What is the actual velocity and direction of the truck?

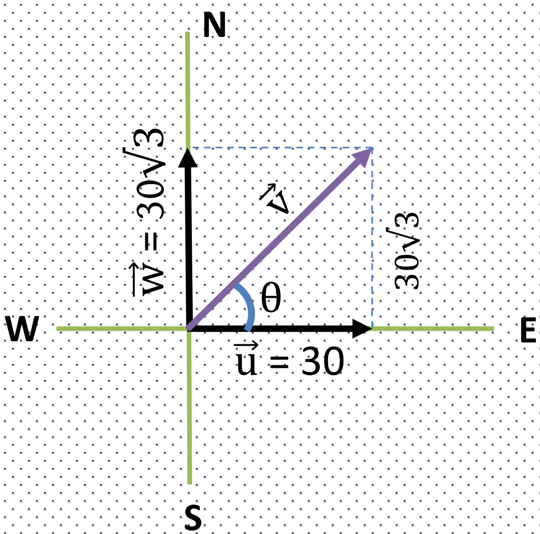
Solution

$$\vec{\mathbf{w}} = \vec{\mathbf{v}} - \vec{\mathbf{u}}$$

Velocity of car, u = 30 km/h
Velocity of Truck w.r.t. car, w = 30√3 km/h
Velocity of Truck v = ?

$$=> \vec{\mathbf{v}} = \vec{\mathbf{w}} + \vec{\mathbf{u}}$$

Therefore, the resultant of truck's relative velocity and the car's velocity is the truck's velocity.



$$v = \sqrt{30^2 + (30\sqrt{3})^2}$$

$$=> w = 60 \text{ km/h}$$

$$\tan\theta = \frac{|\vec{w}|}{|\vec{u}|}$$

$$\Rightarrow \tan\theta = \frac{30\sqrt{3}}{30}$$

$$\Rightarrow \theta = 60°$$

Velocity of the truck is 60 kmh^{-1} , Direction 60° E-N

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

উদ্ভ্রাস-উন্মেষ শিক্ষা পরিবার