بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

Binomial theorem

Lecture: HM-07

Binomial Expression

Binomial Expression:

A polynomial equation with two terms usually joined by a plus or minus sign is called a binomial expression:

Example: $x + y, 3x + 2, ax + \frac{c}{dy}$

In this case, multiplication/division of two terms is not a binomial

Example: $\frac{x}{y}, xy, ax^3$

Binomial Expression

♦ **Discussion about terms in binomial expansion:**

- In the expansion of $(a + x)^n$
  If n is non-negative whole number then number of terms will be $(n + 1)$
  If n is negative or fractional then term number of terms will be ∞ [Here considering the conditions $|x| < a$ or $|x| > a$ the expansion is to be done]
- $(a + b \ldots \ldots \ldots \text{upto } r\ number)^n$ then number of terms ${}^{n+r-1}C_{r-1}$
- In expansion of $(a + x)^n$, $(r + 1)$th term, $T_{r+1} = {}^nC_r\, a^{n-r}.\, x^r$
- **Middle term:** In case of $(a + x)^n$, if n is even, middle term is $\left(\frac{n}{2} + 1\right)$ th term and if n is odd, middle terms are $\left(\frac{n+1}{2}\right)$ th and $\left(\frac{n+1}{2} + 1\right)$ th
- If ${}^nC_x = {}^nC_y$ and $x \neq y$, then $n = x + y$.
- In expansion of $(a + x)^n$, ratio of $(r + 1)$ th term and r- th term, $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} . \frac{x}{a}$

Binomial Expression

- x+y
- $x - y$
- $ax + y$
- $cx - dy$

Binomial Expression

- $x^2 + y^2$
- $x - y^3$
- $ax^2 + y^3$
- $cx^4 - dy^2$

Combination

$$^{n}C_{r} = \frac{n!}{(n-r)!}$$

$$^{n}C_{x} = {}^{n}C_{y} \rightarrow n = x + y$$

Poll Question 01

- **Which one is not a binomial expression?**

(i) $a + bx$ (ii) $b + \frac{8}{3y}$ (iii) $ab + xy + 2$ (iv) $ax^2 + \frac{cy^2}{dz^{-3}}$

Binomial Expression

গাণিতিক আরোহ বিধিঃ

1. আরোহ বিধি ও আরোহ পদ্ধতি:

গাণিতিক আরোহ বিধি (Principle of Mathematical Induction) ঃ যদি $\mathbb{N}$ স্বার্ভাবিক সংখ্যার সেট এবং অপর একটি সেট $S \subset \mathbb{N}$ এরূপ হয় যে $1 \in S$ এবং (ii) $m \in S$ হলে, $m + 1 \in S$ (যেখানে $m \in \mathbb{N}$) । তাহলে, $S = \mathbb{N}$, এর একটি মৌলিক স্বীকার্য। এ স্বীকার্যকে গাণিতিক আরোহ বিধি বলা হয়।

গাণিতিক আরোহ পদ্ধতিঃ চলরাশি স্বাভাবিক সংখ্যা $n \in \mathbb{N}$ সম্বলিত কোন উক্তি যদি $n = 1$ এর জন্য সত্য হয় এবং উক্তিটি $n = m \in \mathbb{N}$ এর জন্য সত্য ধরে যদি তা $n = m + 1 \in \mathbb{N}$ এর জন্যও সত্য হয়, তবে উক্তিটি সকল $n \in \mathbb{N}$

If $\mathbb{N}$ is set of natural numbers ans ssn is such a set that, if (i) $1 \in S$ and (ii) $m \in S$ is true then $m + 1 \in S$, then $S = \mathbb{N}$ This is a fundamental postulate of $\mathbb{N}$. This relation is called principal of mathematical induction.

For $n \in \mathbb{N}$, If a statement is true for

(i) $n = 1$

(ii) $m = 1 \in \mathbb{N}$ after assuming that the statement is true $n = m \in \mathbb{N}$

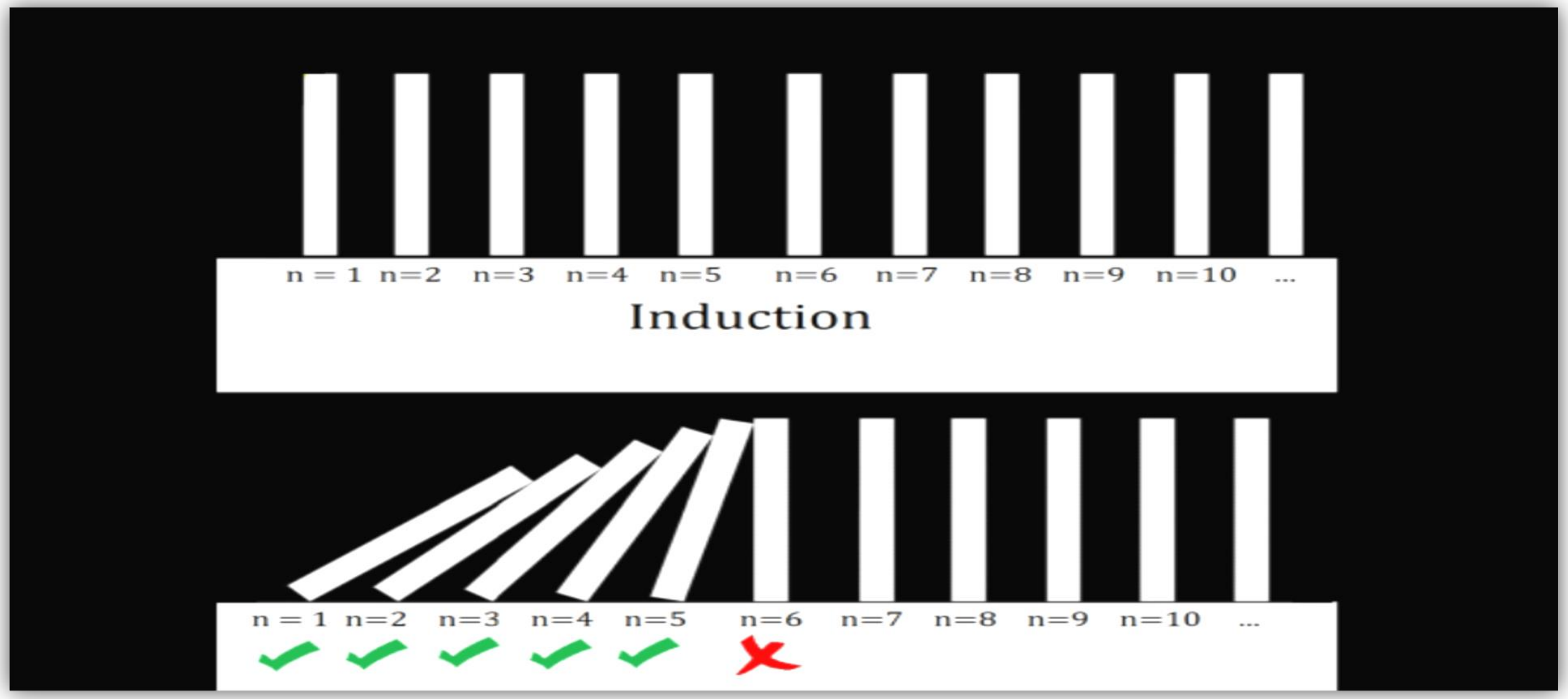
Then the statement is true for all $n \in \mathbb{N}$

Binomial Expression

Principle of Mathematical Induction

Binomial Expression

Principle of Mathematical Induction:



Binomial Expression

<u>Binomial Theorem:</u>

$$(a + x)^n = a^n + {}^nC_1\, a^{n-1}\, x + {}^nC_2\, a^{n-2}\, x^2 + {}^nC_3\, a^{n-3}\, x^3 + \ldots\ \ldots\ \ldots + {}^nC_r\, a^{n-r}\, x^r + \ldots\ \ldots\ \ldots + x^n$$

Isaac Newton discovered this theorem about 1665 and later stated, in 1676, with proof, the general form of the theorem (for any real number n), and a proof by john Colson was published in 1736

Sir Isaac Newton

John Colson

Binomial Expression

Example:

Binomial Expression

Binomial Theorem:

Proof 1:

proof by induction:

Binomial Expression

Binomial Theorem:

Proof 1:

Binomial Expression

Binomial Theorem:

Proof 2:

Suppose, we want to determine $(a+x)^2$. Now, $(a+x)^2$ is equivalen to $(a+x)(a+x)$. This multiplication can be done in follow way:

Probable term from first (a+x)	Probable term from first (a+x)	Multiplication	Number of ways to select
a	a	a^2	1
a	x	ax	1
x	a	ax	1
x	x	x^2	1
		$a^2+2ax+x^2$	

$2ax$

Binomial Expression

Binomial Theorem:

Proof 2:

After expanding $(a+x)^3$ if we want a^3, we will need three a's. From three $(a+x)$ we can select thrice a's in ${}^3C_3$ or 1 way.

After expanding $(a+x)^3$ if we want a^2x, we will need two a's. From three $(a+x)$ we can select two a's in ${}^3C_2$ or 3 ways.

After expanding $(a+x)^3$ if we want ax^2, we will need one a. From three $(a+x)$ we can select one a in ${}^3C_1$ or 3 ways.

After expanding $(a+x)^3$ if we want x^3, we will need 0 a. From three $(a+x)$ we can select 0 a in ${}^3C_0$ or 1 ways.

Binomial Expression

Binomial Theorem:

Proof 2:

Therefore,

$$(a+x)^3 = a^3 + {}^3C_2 a^2 x + {}^3C_1 a\,x^2 + \mathrm{x}^3,$$

Or, $(a+x)^3 = a^3 + {}^3C_1 a^2 x + {}^3C_2 a\,x^2 + \mathrm{x}^3$ (As ${}^3C_1 = {}^3C_2$)

$$(a+x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$$

Similarly,

$$(a+x)^n = an + {}^nC_1 a^{n-1} x + {}^nC_2 a^{n-2} x^2 + \ \ldots\ldots + x^n$$

Poll Question 02

- **How many terms there will be in expansion of** $(a+2x)^{21}$ **?**

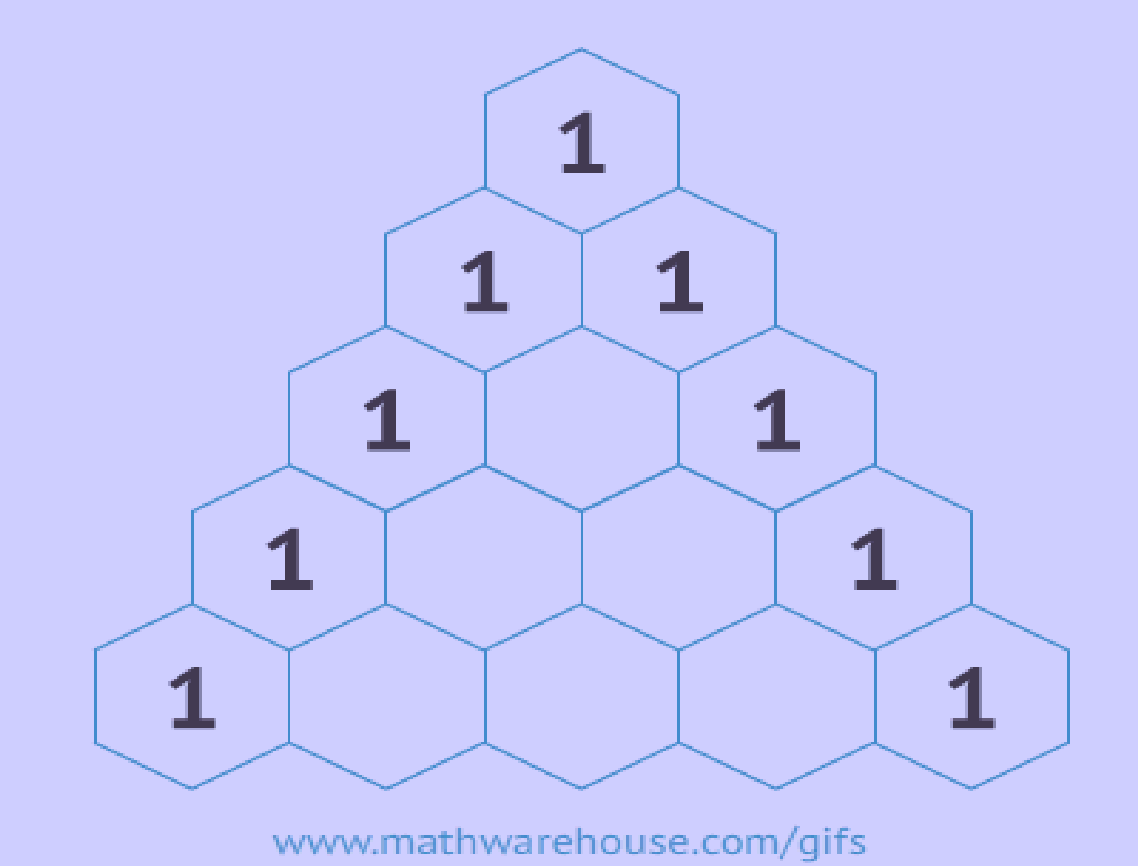
(i) 20 (ii) 21 (iii) 22 (iv) 23

Binomial Expression

Pascal's Triangle:

Binomial Expression

Pascal's Triangle:



Binomial Expression

Pascal's Triangle:

Use:

Binomial Expression

Pascal's Triangle:

Use:

$(a + x)^2 = a^2 + \bigcirc ab + b^2$

$(a + x)^3 = a^3 + \bigcirc a^2b + \bigcirc ab^2 + b^3$

$(a + x)^4 = a^4 + \bigcirc a^3b + \bigcirc a^2b^2 + \bigcirc ab^3 + b^4$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

Binomial Expression

(r+1) th term

$$(a + x)^n = a^n + {}^nC_1\, a^{n-1}\, x + {}^nC_2\, a^{n-2}\, x^2 + {}^nC_3\, a^{n-3}\, x^3 + \dots \dots$$

Here,

1^{st} term = a^n

2^{nd} term = ${}^nC_1\, a^{n-1}\, x$

3^{rd} term = ${}^nC_2\, a^{n-2}\, x^2$

4^{th} term = ${}^nC_3\, a^{n-3} x^3$

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(r+1)-th term = ${}^nC_r\, a^{n-r} x^r$

Binomial Expression

<u>(r+1)-th term</u>

$$T_{r+1} = {}^{n}C_{r}\, a^{n-r}\, x^{r}$$

Binomial Expression

Equidistant terms:

$(a + x)^n = a^n + {}^nC_1\, a^{n-1}\, x + {}^nC_2\, a^{n-2}\, x^2 + {}^nC_3\, a^{n-3}\, x^3 + \ldots \ldots \ldots + {}^nC_r\, a^{n-r}\, x^r + \ldots \ldots \ldots + x^n$

If a = 1

$(1 + x)^n = 1 + {}^nC_1\; x + {}^nC_2\; x^2 + {}^nC_3\; x^3 + \ldots \ldots \ldots + {}^nC_r\; x^r + \ldots \ldots + {}^nC_{n-2}x^{n-2} + {}^nC_{n-1}x^{n-1} + x^n$

By complementary combination,

$${}^nC_1 = {}^nC_{n-1}$$
$${}^nC_2 = {}^nC_{n-2}$$

.

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.

$${}^nC_r = {}^nC_{n-r}$$

Therefore, The binomial coefficients which are equidistant from the beginning and from the ending are equal

Binomial Expression

Middle term:

$$(a + x)^2 = a^2 + 2ab + b^2$$

$$(a + x)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + x)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Binomial Expression

Middle term:

$(a + x)^2 = a^2 + 2ab + b^2$ → 2

$(a + x)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ → 2 , 3

$(a + x)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ → 3

Binomial Expression

Middle term:

If n is even-

Binomial Expression

Middle term:

If n is even: $\frac{n}{2} + 1$ th term

If n is odd: $\frac{n+1}{2}$ and $\frac{n+1}{2} + 1$ th term

Binomial Expression

Ratio of consecutive terms:

$$\frac{T_{r+1}}{T_r} = \frac{{}^{n}C_{r}\; an^{-r}\; xr}{{}^{n}C_{r-1}a^{n-r+1}x^{r-1}}$$

$$= \frac{n-r+1}{r}\,\frac{x}{a}$$

Binomial Expression

Ratio of consecutive terms:

$$\frac{T_{r+1}}{T_r} = \frac{{}^nC_r\, an^{-r}\, xr}{{}^nC_{r-1}a^{n-r+1}x^{r-1}}$$

$$= \frac{n-r+1}{r}\,\frac{x}{a}$$

Binomial Expression

1. Expand: $(x+3y)^4$

Binomial Expression

2. Determine 7-th term from expansion of $(1 - \frac{1}{x})^{10}$

Binomial Expression

3. Find the term independent of x in $(2x^3 - \frac{1}{x})^{12}$

Binomial Expression

4. Find the term independent of x in $(x^2 - 2 + \frac{1}{x^2})^{12}$

Binomial Expression

5. Find the value of a, if coefficient of x^3 in expansion of $(a + 2x)^5$ is 320.

Binomial Expression

6. If coefficient of x^5 and x^{15} in expansion of $(2x^2 + \frac{k}{x^3})^{10}$ is equal, find the value of k.

Binomial Expression

7. Determine Middle term:

(i) $\left(\frac{x}{y}+\frac{y}{x}\right)^{21}$

Binomial Expression

8. Determine Middle term:

(ii) $\left(x^2 - 2 + \frac{1}{x^2}\right)^n$

Binomial Expression

9. In expansion of $(a+3x)^n$, if first three consecutive terms are b , $\frac{21}{2}bx$ and $\frac{189}{4}bx^2$, determine value of a , b and n.

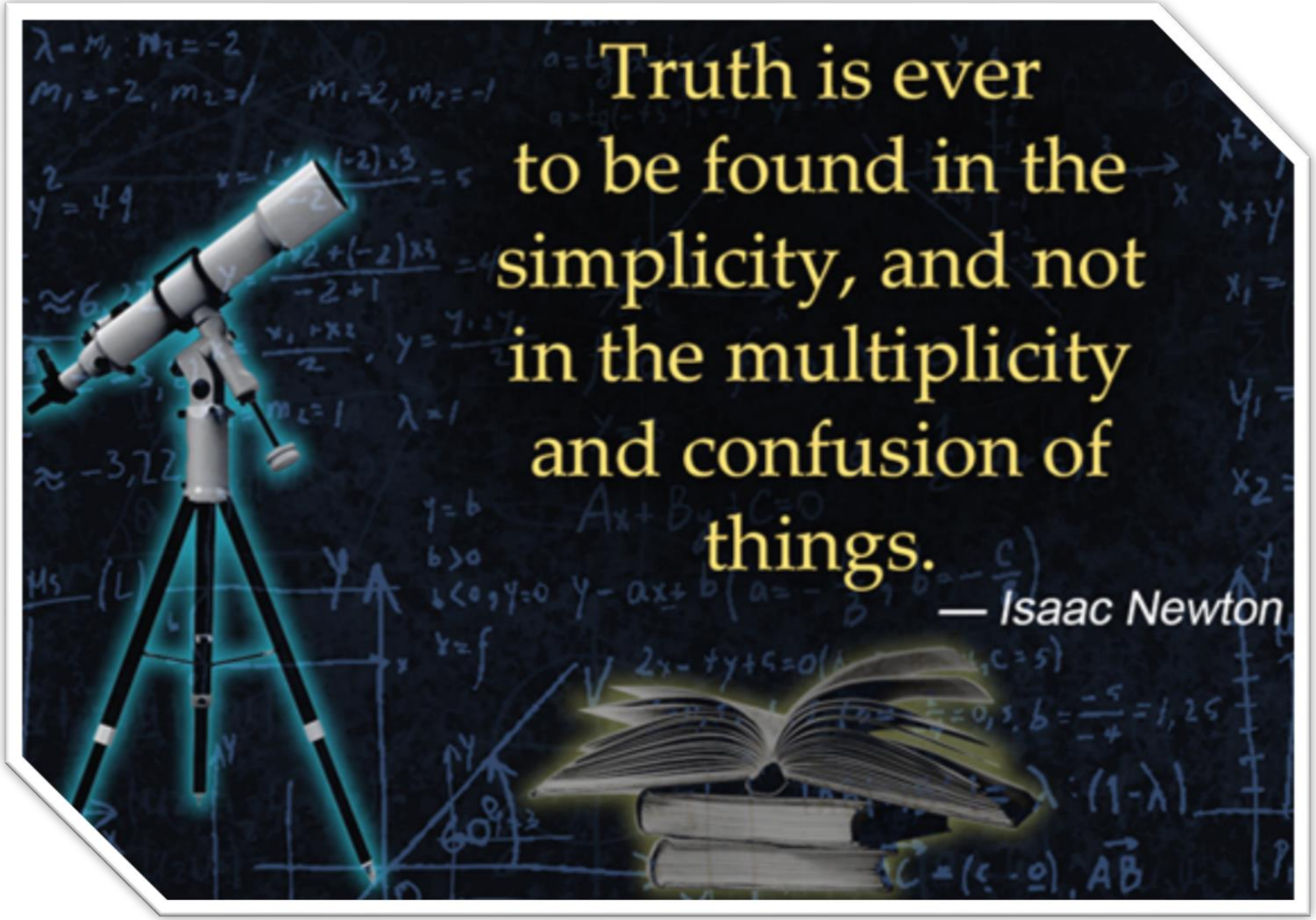
Binomial Expression

10. If n is a natural number, find $(n+1)$ th term from end in expansion of $\left(x^p + \frac{1}{x^p}\right)^n$

Binomial Expression

11. If in expansion of $(1+x)^n$ sum of odd terms and even terms are consecutively S_1 and S_2, show that $(1-x^2) = {S_1}^2 - {S_2}^2$; Where n is a natural number.

Binomial Expression



লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

উদ্ভাস-উন্মেষ শিক্ষা পরিবার

Thank You