



CLASS 12 ACADEMIC PROGRAM-2020

# HIGHER MATH 2<sup>ND</sup> PAPER

Lecture : HM-10

Chapter 6 : Conic



$$x = \sqrt{\frac{c^2}{c} + c} - \frac{b}{2}$$



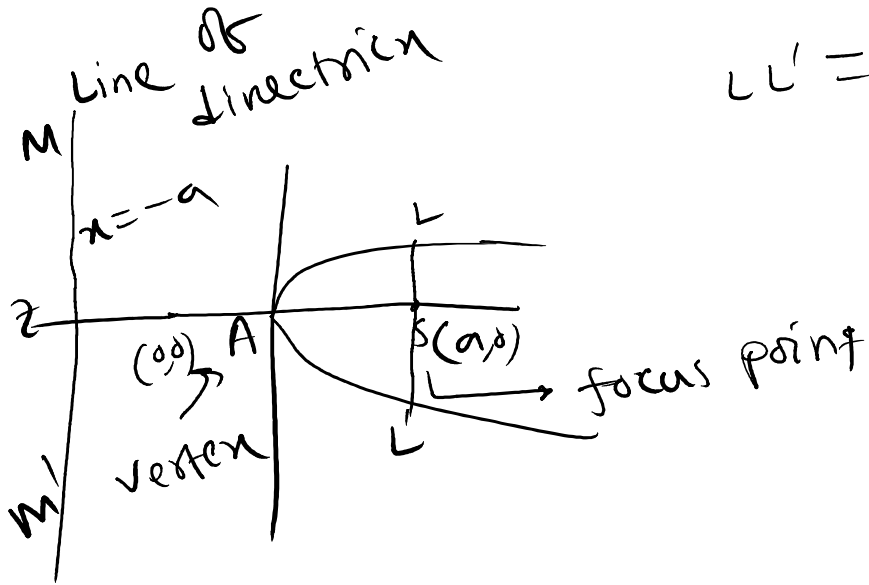
উদ্ভাস

একাডেমিক এন্ড এডভিশন কেয়ার



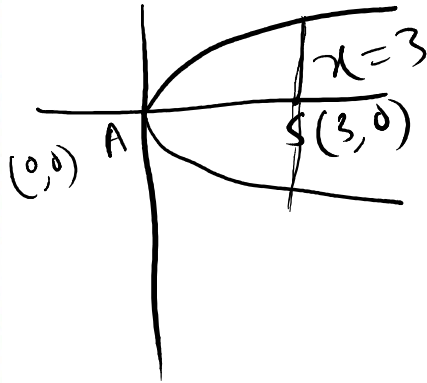
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$$y^2 = 4ax$$



## Mathematical Problem

1. Find vertex and equation of latus rectum of the parabola  $y^2 = 12x$



$$y^2 = 12x \\ = 4 \times 3 \times x$$

$$y^2 = 4ax$$

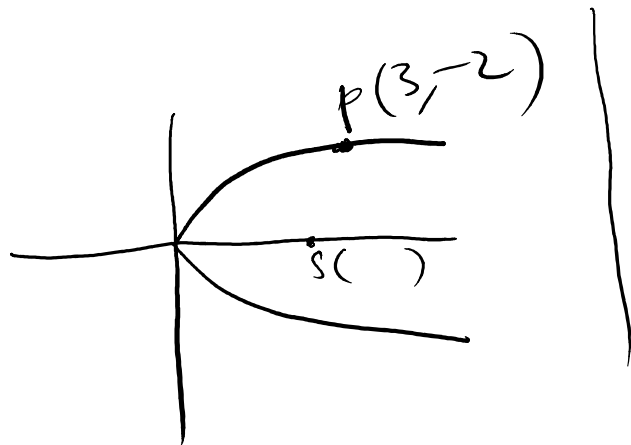
$$A(0,0)$$

$$x = 3$$

$$x - 3 = 0$$

## Mathematical Problem

2.  $y^2 = 4px$  parabola goes through  $(3, -2)$  point. Find the length of its latus rectum and focus.



Given eq<sup>n</sup>  
 $y^2 = 4px \dots \textcircled{1}$

$\downarrow (3, -2)$

$$(-2)^2 = 4 \times p \times 3$$

$$4 = 12p$$

$$\therefore p = \frac{4}{12}$$

$$= \frac{1}{3}$$

$$y^2 = 4 \times \left(\frac{1}{3}\right) \times x \quad \Bigg| \quad y^2 = 4ax$$

length of the latus rectum =  $|4a|$

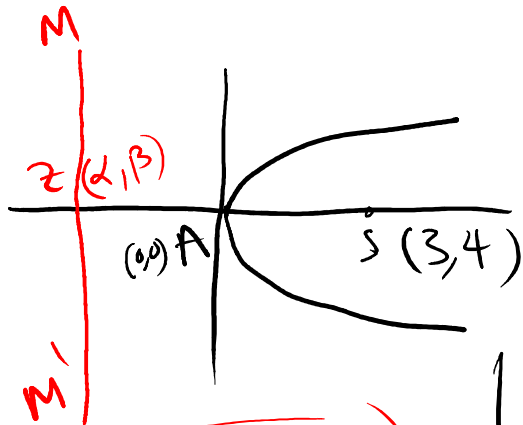
$$= \left| 4 \times \frac{1}{3} \right|$$

$$= \frac{4}{3}$$

focus,  $S(a, 0) = S\left(\frac{1}{3}, 0\right)$

## Mathematical Problem

3. Focus and vertex of a parabola is (3, 4) and (0, 0). Find its directrix.  $\checkmark \cdot \checkmark \cdot \checkmark$



Find  $(\alpha, \beta)$

$$\therefore \frac{\alpha + 3}{2} = 0$$

$$\alpha = -3$$

$$\frac{\beta + 4}{2} = 0$$

$$\beta = -4$$

Eqn of Axis (AS line)

$$A(0,0), S(3,4) \rightarrow \frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\frac{y - 0}{0 - 4} = \frac{x - 0}{0 - 3}$$

$$\frac{y}{-4} = \frac{x}{-3}$$

$$4x = 3y$$

$$4x - 3y = 0$$

Line of directrix

$$\rightarrow 3x + 4y + k = 0 \quad \text{--- (1)}$$

$\curvearrowright Z(-3, -4)$

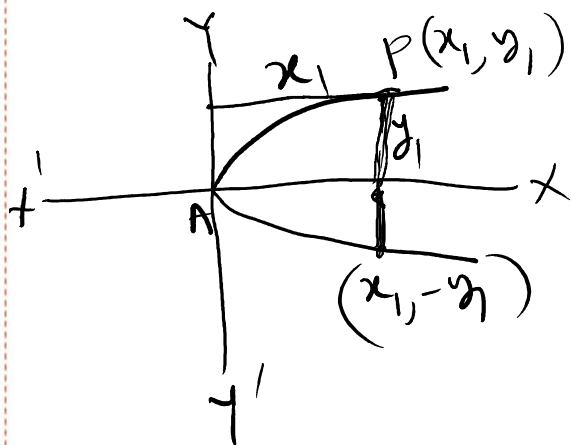
$$3 \times (-3) + 4 \times (-4) + k = 0$$

$$\therefore k = 25$$

$$\therefore 3x + 4y + 25 = 0$$

## Mathematical Problem

4. P is a point on the parabola  $y^2 = 12x$ . Distance of P from x-axis is double of the distance from y-axis. Find the co ordinate of P.



ATQ,

$$y_1 = \pm 2x_1$$

Given eq<sup>n</sup> of parabola,

$$y^2 = 12x \quad \text{--- (1)}$$

$$\hookrightarrow P(x_1, y_1)$$

$$y_1^2 = 12x_1$$

$$(\pm 2x_1)^2 = 12x_1$$

$$4x_1^2 = 12x_1$$

$$x_1^2 = 3x_1$$

$$\therefore x_1 = 3$$

$$\therefore y = \pm 2x(3) \\ = \pm 6$$

$$(3, 6), (3, -6)$$

## Mathematical Problem

5. Find a parabola with focus at  $(0, -4)$  and directrix  $y - 4 = 0$ .

$$SP = PM$$

ATQ,

$$SP = PM$$

$$\sqrt{(x-0)^2 + (y+4)^2} = \frac{|y-4|}{\sqrt{0^2+1^2}}$$

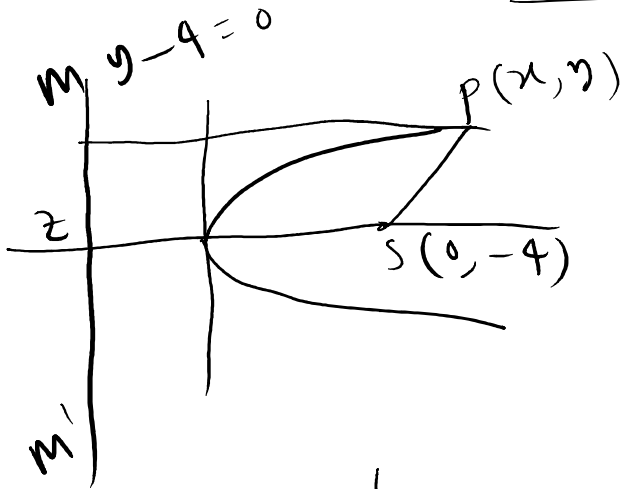
$$x^2 + (y+4)^2 = (y-4)^2$$

$$x^2 = \frac{(y-4)^2}{a} - \frac{(y+4)^2}{b}$$

$$= (y-4 + y+4)(y-4 - y-4)$$

$$= 2y \times (-8)$$

$$x^2 = -16y$$



$$ax + by + c = 0$$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

## Mathematical Problem

6. On a parabola  $y^2 = 16x$ , a point has a focal length 6. Find that point

H.W



## Mathematical Problem

7. P is a point on the parabola  $y^2 = 9x$  and its y-co ordinate is 12. Find the focal length of P.

$P(\alpha, \beta) \equiv P(\alpha, 12) \equiv P(16, 12)$   
 $S\left(\frac{9}{4}, 0\right)$

$$SP = \sqrt{\left(16 - \frac{9}{4}\right)^2 + (12 - 0)^2}$$

$$= \frac{73}{4} \quad \text{b}$$

$y^2 = 9x$        $y^2 = 4ax$

$= 4 \times \boxed{\frac{9}{4}} \times x$

$S \equiv (a, 0)$

---

Given eqn

$y^2 = 9x$

$\rightarrow$   $P(\alpha, 12)$

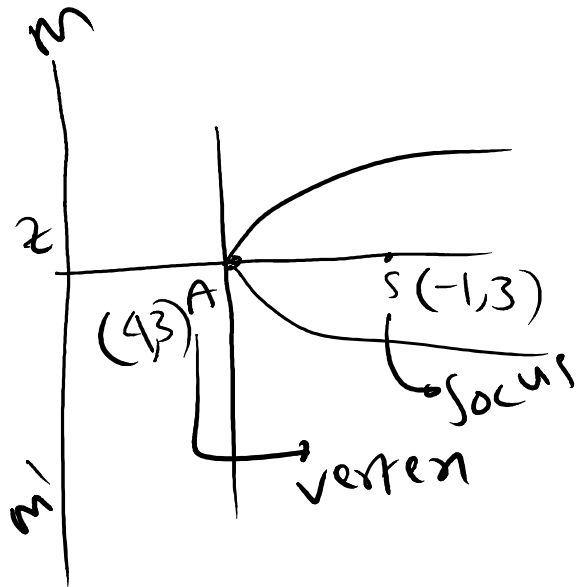
$(12)^2 = 9 \times \alpha$

$144 = 9\alpha$

$\alpha = 16$

## Mathematical Problem

8. Determine a parabola which has focus at  $(-1, 3)$  and vertex at  $(4, 3)$ .



→ (3) Math  
+ (5) line of direction

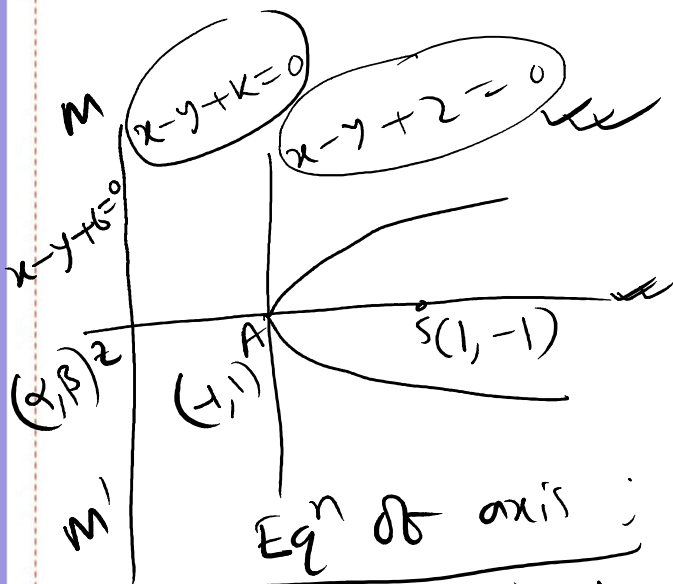
focus point  
+ directrix  
 $SP = PM$

buet

# Mathematical Problem

focus + directrix  $SP=PM$

9. Determine the equation of a parabola which has focus at  $(1, -1)$  and the line  $x - y + 2 = 0$  is perpendicular to axis at vertex.  $\checkmark \checkmark \checkmark \checkmark \checkmark$



Eq<sup>n</sup> of axis :

$$x - y + 2 = 0$$

$$\perp \text{ is } x + y + k = 0$$

↑ S(1, -1)

$$1 - 1 + k = 0$$

$$\therefore k = 0$$

$$\therefore x + y = 0 \rightarrow \text{Axis}$$

$$x - y + 2 = 0 \rightarrow A$$

$$x + y = 0 \rightarrow (ii)$$

$$2x + 2 = 0$$

$$2x = -2$$

$$x = -1$$

$$-1 + y = 0$$

$$y = 1$$

Finally

$$x - y + k = 0$$

$$\sim 2(-3, 3)$$

$$-3 - 3 + k = 0$$

$$k = 6$$

SP = PM

$$S(1, -1), 2(\alpha, \beta)$$

$$A(-1, 1)$$

$$\therefore \frac{1 + \alpha}{2} = -1$$

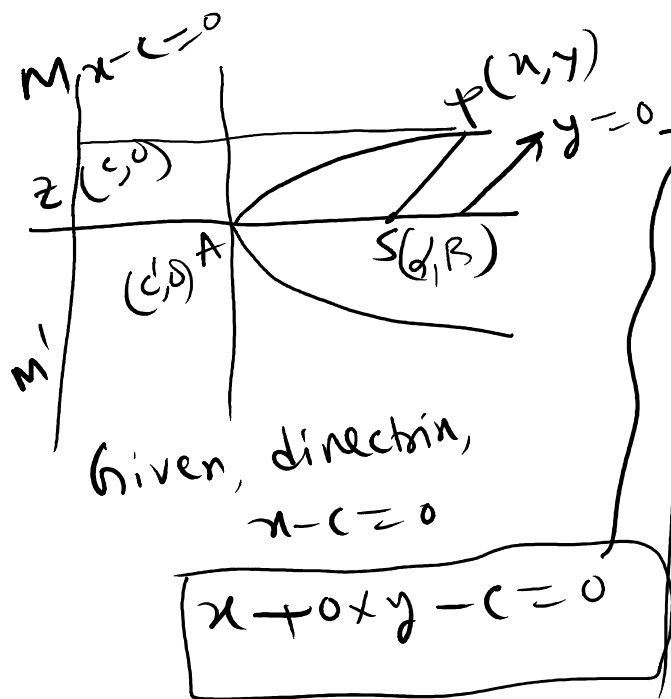
$$\alpha = -3$$

$$\frac{\beta - 1}{2} = 1$$

$$\beta = 3$$

## Mathematical Problem

10. A parabola has directrix  $x - c = 0$  and ~~focus~~ <sup>vertex</sup> at  $(c', 0)$ . Show that the equation of that parabola is  $y^2 = 4(c' - c)(x - c')$ .



Line of axis

$$x \cdot 0 - y + k = 0$$

$$c' \cdot 0 - 0 + k = 0$$

$$\therefore k = 0$$

$$\therefore x \cdot 0 - y + 0 = 0$$

$$\therefore y = 0$$

Find z:

$$x - c = 0 \rightarrow x = c$$

$$y = 0 \rightarrow y = 0$$

$$z(c, 0)$$

$$S(\alpha, \beta), z(c, 0)$$

$$A(c', 0)$$

$$\therefore \frac{\alpha + c}{2} = c' \quad \left| \quad \frac{\beta + 0}{2} = 0 \right.$$

$$\alpha = 2c' - c \quad \left| \quad \beta = 0 \right.$$

directrix  $x-c=0$   
focus  $(2c'-c, 0)$   
P(x,y)

ATQ,  
SP=PM

$$\sqrt{\{x-(2c'-c)\}^2 + (y-0)^2} = \frac{|x-c|}{\sqrt{1+0}}$$

$$(x-2c'+c)^2 + y^2 = (x-c)^2$$

$$y^2 = \underline{(x-c)^2} - \underline{(x-2c'+c)^2}$$

$$= (x-c + x-2c'+c) (x-c - x+2c'-c)$$

$$= (2x-2c') (2c'-2c)$$

$$= 2(x-c') 2(c'-c)$$

$$= 4(c'-c)(x-c')$$

## Mathematical Problem

11. Find a parabola which goes through the point  $(2, 5)$  has a vertex at  $(0, 2)$  and axis is parallel to y-axis.

Let the eq<sup>n</sup> of parabola, whose axis is parallel to y axis

$$x^2 = 4ay, \text{ where vertex } (0, 0)$$

vertex  $(0, 0)$  is now ~~replaced~~ <sup>transferred</sup> by  $(0, 2)$

$$\therefore (x-0)^2 = 4a(y-2) \dots (1)$$

$\hookrightarrow (2, 5)$  point passing

$$2^2 = 4a(5-2)$$

$$4 = 4a \times 3$$

$$a = \frac{1}{3}$$

$$x^2 = 4 \times \frac{1}{3} (y-2)$$

Ans

## Mathematical Problem

12. Find the equation of a parabola which has axis parallel to x-axis, vertex on y-axis and the parabola goes through  $(0, 2)$  and  $(1, 0)$ .

$$y^2 = 4ax \rightarrow \text{vertex } (0, 0)$$

(i)  $(0, 0)$  transferred by  $(0, \beta)$

$$(y - \beta)^2 = 4a(x - 0) \\ = 4ax \quad \text{--- (1)}$$

$(0, 2)$  point passing

$$(2 - \beta)^2 = 4a \times 0$$

$$(2 - \beta)^2 = 0$$

$$2 - \beta = 0$$

$$\therefore \beta = 2$$

$$(y - 2)^2 = 4ax$$

--- (1)

$(1, 0)$  point passing

$$(0 - 2)^2 = 4a \times 1$$

$$4 = 4a$$

$$a = 1$$

Finally

$$(y - 2)^2 = 4 \times 1 \times x$$

## Mathematical Problem

13. A parabola has axis parallel to x-axis and goes through  $(-2, 1)$ ,  $(1, 2)$  and  $(-1, 3)$ . Find its equation.

$$x = ay^2 + by + c \quad \text{--- (i)}$$

$(-2, 1) \rightarrow$  (i)

$$-2 = a + b + c \quad \text{--- (ii)}$$

$(1, 2) \rightarrow$  (i)

$$1 = 4a + 2b + c \quad \text{--- (iii)}$$

$(-1, 3) \rightarrow$  (i)

$$-1 = 9a + 3b + c \quad \text{--- (iv)}$$

(ii), (iii), (iv)

$$a = -\frac{5}{2}$$

$$b = -\frac{21}{2}$$

$$c = -10$$

$$x = ay^2 + by + c \rightarrow \text{Axis parallel to x axis + three points}$$

$$y = ax^2 + bx + c \quad \text{--- (ii)}$$

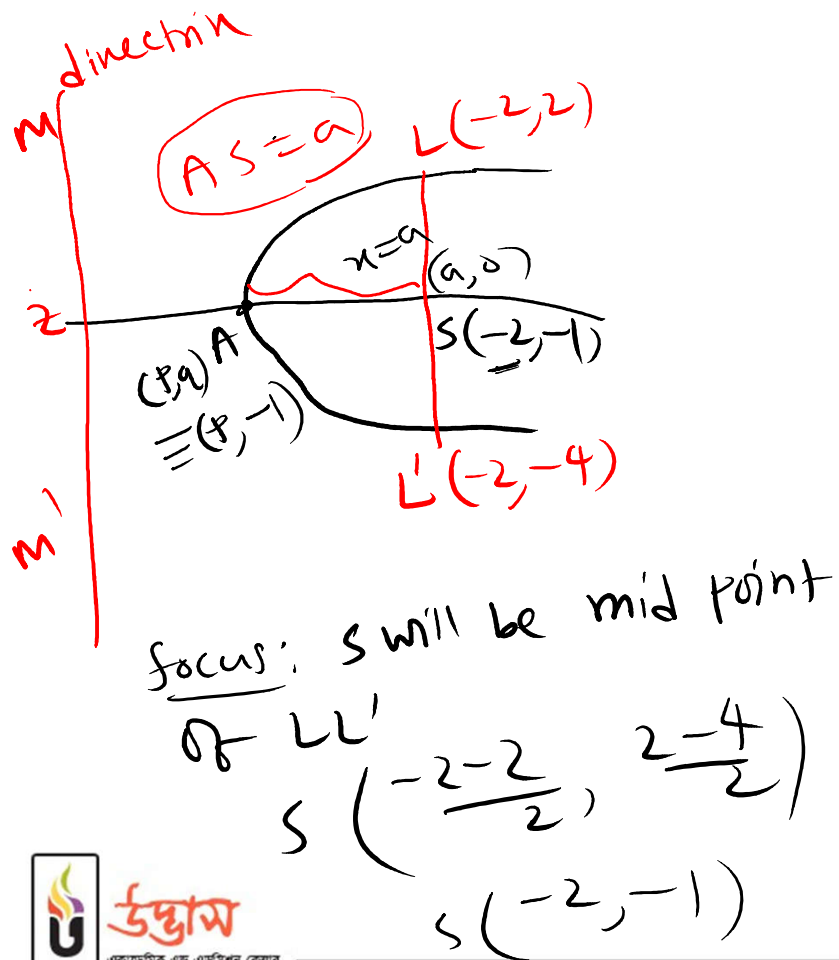
↳ Axis parallel to y axis + three passing points



# Mathematical Problem

focus + directrix  
SP = PM

14. If end points of latus rectum are  $(-2, 2)$  and  $(-2, -4)$  find the equation of the parabola.



Eqn of latus rectum:

$$x = -2$$

$$x + 2 = 0$$

$$x + 0 \times y + 2 = 0$$

$$-y - 1 = 0$$

$$y + 1 = 0$$

$$y = -1$$

Eqn of Axis

$$x \times 0 - y + k = 0$$

$$\rightarrow S(-2, -1)$$

$$-2 \times 0 - (-1) + k = 0$$

$$1 + k = 0$$

$$k = -1$$

$$LL' = \sqrt{(-2+2)^2 + (2-4)^2}$$

$$|4a| = 6$$

$$4a = \pm 6$$

$$a = \pm \frac{3}{2}$$

$A(P, -1)$   
 $S(-2, -1)$   
 $a = \pm 3/2$

$AS = a$

$$\sqrt{(P+2)^2 + (-1+1)^2} = \frac{3}{2}$$

$$\sqrt{(P+2)^2} = \frac{3}{2}$$

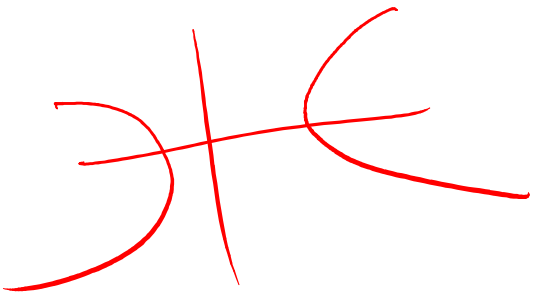
$$(P+2)^2 = \frac{9}{4}$$

$$P+2 = \pm \frac{3}{2}$$

$$\therefore P = -\frac{1}{2}, -\frac{7}{2}$$

vertices  $(-\frac{1}{2}, -1)$

$(-\frac{7}{2}, -1)$



Vertex + focus

↓  
line of direction

↓ + focus

$SP = PM$

parabola

## Mathematical Problem

15.  $y = 3x + 1$  is a tangent to the parabola  $y^2 = 4ax$ . Find the length of latus rectum.

$$y = mx + c$$

$$c = \frac{a}{m}$$

$$1 = \frac{a}{3}$$

$$\therefore a = 3$$

$$|4a|$$
$$= |4 \times 3|$$

$$= 12$$

Ans)

$$y^2 = 4ax$$
$$y = mx + c$$

$$c = \frac{a}{m}$$

না বুঝে  
মুখস্থ করার  
অভ্যাস প্রতিভাকে  
ধ্বংস করে

$$X = caP \frac{V^2}{2S}$$

$$X = caP \frac{V^2}{2S}$$

$$E = mc^2$$

$$x = \sqrt{\frac{a^2}{c^2} + c} - \frac{b}{2}$$



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