CLASS 8 ACADEMIC PROGRAM-2020

MATH

Lecture : M-18

Chapter 9 : Pythagoras Theorem

Creative Question

- In a hostel, 65% of the students like fish, 55% of the students like meat and 40% of the students like both.

(a) Express the stated information by Venn Diagram with short explanation.
(b) Find out the number of students who dislike both dishes.
(C) Find out the intersection set of the sets of factors of those students, who like only one dish.

ⓐ U = Total student

F = Those who like fish

M = " " " Meat

$F \cap M$ = Those who like both Fish & Meat

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(b) who dislikes both dishes.

40% → like both dishes

Those who likes only fish → $(65-40) = 25\%$

" " " only Meat → $(55-40)\% = 15\%$

Students who dislike both dishes,

$100 - (40+25+15) = 20\%$ Ans.

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(e)

Only fish → $\underline{25\%}$

Only Meat → $\underline{15\%}$

$P \cap Q$

$= \{1, 5, 25\} \cap \{1, 3, 5, 15\}$

$= \underline{\{1, 5\}}$ Ans

Factors of 25,

$1 \times 25 = 25$

$5 \times 5 = 25$

$\underline{P = \{1, 5, 25\}}$

$\underline{\text{Factors of } 15},$

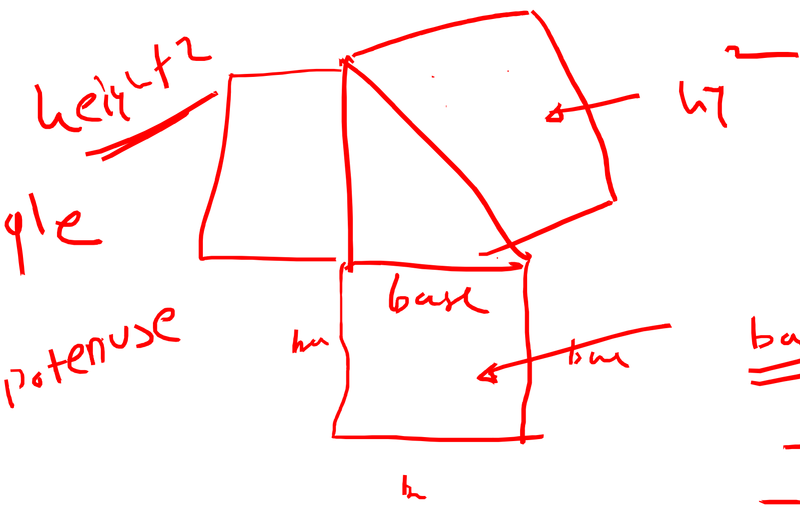
$1 \times 15 = 15$

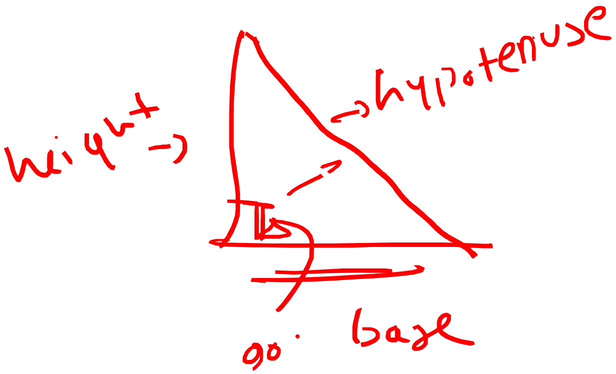
$3 \times 5 = 15$

$Q = \{1, 3, 5, 15\}$

What will we learn from chapter-9?

Right angled Triangle





$$\text{hypotenuse}^2 = \text{base}^2 + \text{height}^2$$

① Right angled Triangle

base² ② Pythagoras Theorem

→ Right angled - -

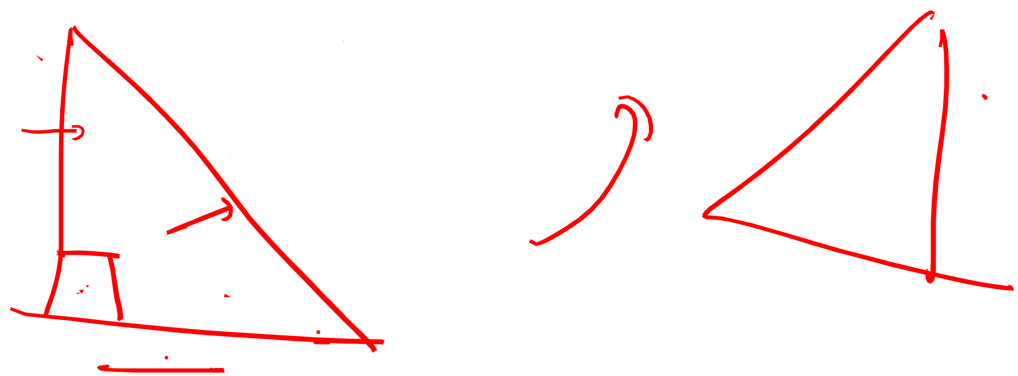
→ Similar —

→ AI —

③ Converse of Pythagoras theorem

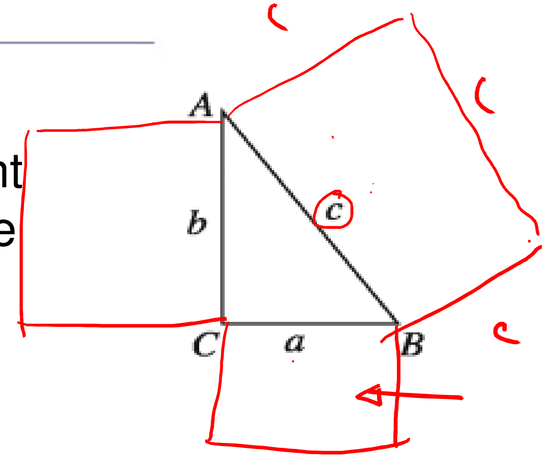
Pythagoras theorem

In 6th Century B.C Greek philosopher Pythagoras discovered a special property of right-angled triangle. This property of right-angled triangle is known as Pythagorean property. It is believed that before the birth of Pythagoras, in Egyptian and Greek era, this special property of right-angled triangle was in use. In this chapter, we shall discuss this property of right-angled triangle. We know that the sides of a right-angled triangle have got special names-the side opposite to right angle as hypotenuse and the sides containing the right angle as base and height. In this chapter, relation among these three sides will be discussed.



Pythagoras theorem

In the figure ABC a right-angled triangle with $\angle ACB$ as a right angle. Therefore, AB is the hypotenuse of the triangle. In the figure, we denote the sides by a, b, c.



Observe, $3^2 + 4^2 = 5^2$ i. e. the sum of the squares of two sides is equal to the square of the measurement of the hypotenuse. Therefore, for a right-angled triangle with sides a, b and c, $\boxed{c^2 = a^2 + b^2}$. This is the key point of Pythagoras theorem. This theorem has been proved in various methods. A few simple proofs of this theorem are given below.

$$\boxed{c^2 = a^2 + b^2}$$

Poll Question-01

For Which of the following measurements, is it possible to draw a right angled triangle?

(a) 3, 4, 5

(b) ~~4, 4, 5~~

(c) ~~6, 7, 8~~

(d) ~~1, 6, 7~~

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$

$25 = 25$

$7^2 = 6^2 + 1^2$

$49 \cancel{=} 36 + 1$

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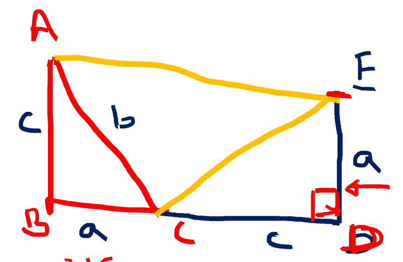
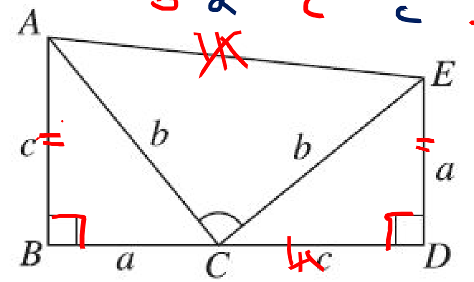
Pythagoras theorem

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the two other sides.

(Proof with the help of two right angled triangles)





Pro position:-

AC = b
AB = C
BC = a

Prove that

$b^2 = a^2 + c^2$

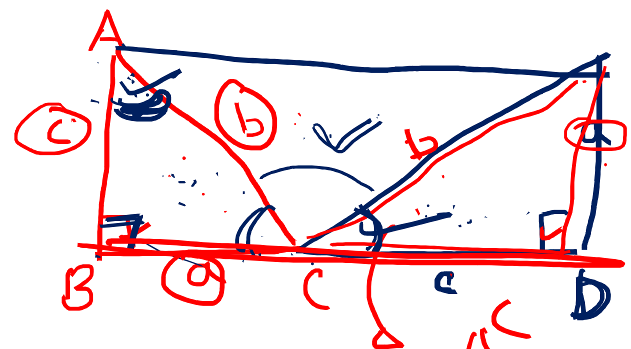
Construction:-

CD = c, DE = a

Join, C,E & A,E

Area of Tri = $\frac{1}{2}\times$ sum $\times$

Proof :-



$\triangle ABC$ & $\triangle DEC$

$\angle ABC = \angle CDE$ [Right angle]

$BC = DE$

$AB = CD$

$\triangle ABC \cong \triangle DEC$

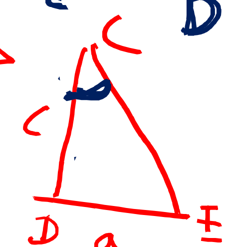
$\angle BAC = \angle ECD$

$\angle BAC + \angle ACB = 90^\circ$

$\angle ECD + \angle ACB = 90^\circ$

$\boxed{\angle ACE = 90^\circ}$

ABDE -> Trapezium



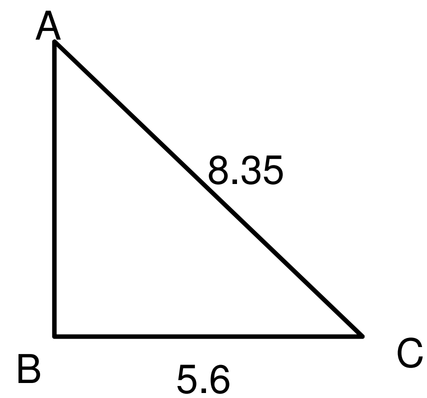
Area of ABDE = $\triangle ABC + \triangle ACE + \triangle CDE$

$\cancel{\frac{1}{2}}\times(c+a)\times(a+c) = \cancel{\frac{1}{2}}\times a\times c + \cancel{\frac{1}{2}}\,b\times b + \cancel{\frac{1}{2}}\,a\times c$

$\cancel{ca} + c^2 + \cancel{ca} + a^2 = \cancel{ca} + b^2 + \cancel{ca}$

$c^2 + a^2 = b^2 \rightarrow \boxed{b^2 = a^2 + c^2}$

Poll Question-02



In Figure AB=?

(a) 5.2

(b) 6.2

(a) 7.4

(d) 8

$$AC^2 = AB^2 + BC^2$$

$$AB = \sqrt{AC^2 - BC^2}$$

$$= \sqrt{8.35^2 - 5.6^2}$$

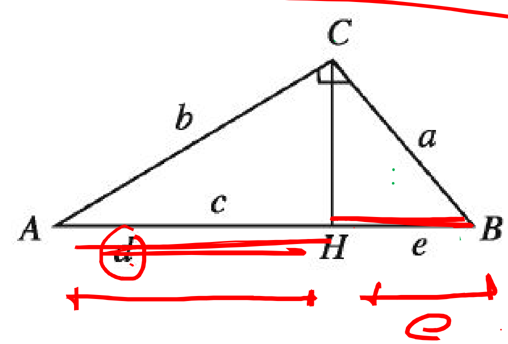
$$= 6.2$$

Pythagoras theorem

Step ① Angles are equal

Step ② Ratio of sides are equal

Alternative Proof of Pythagoras theorem
(By using similar triangles)



Proposition :-

ABC

$\angle C = 90^\circ$

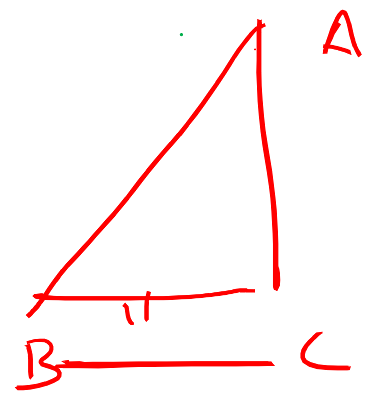
$AB = c$

$AC = b$

$BC = a$

$$\boxed{c^2 = a^2 + b^2}$$

Construction :-



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Proof :-

ΔABC & ΔBCH

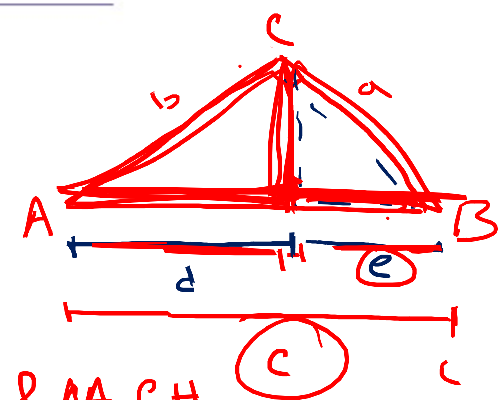
$\angle ACB = \angle BHC$ = right angle

$\angle CBH = \angle ABC = \angle B$ is common angle.

ΔABC & ΔBCH are similar.

$$\frac{BC}{AB} = \frac{BH}{BC}$$

$$\frac{a}{c} = \frac{e}{a} \rightarrow a^2 = ce \text{ --- (i)}$$



ΔABC & ΔACH Similar,

$$\frac{AC}{AB} = \frac{AH}{AC}$$

$$\frac{b}{c} = \frac{d}{b}$$

$$b^2 = cd \text{ --- (ii)}$$

(i) + (ii)

$$a^2 + b^2 = ce + cd$$

$$a^2 + b^2 = c(e+d)$$

$$a^2 + b^2 = c \cdot c$$

$$\boxed{a^2 + b^2 = c^2}$$

Poll Question-03

The length of a rectangle 8 cm and breadth 6 cm , what is length of the diagonal (in cm) ?

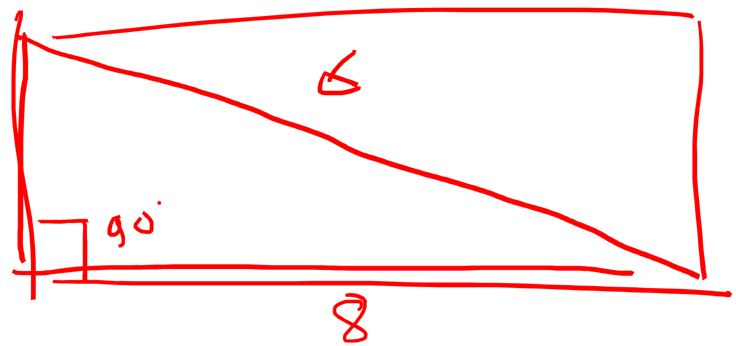
(a) 56

(b) ~~48~~

(c) 28

(d) 10

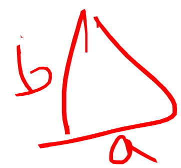
C



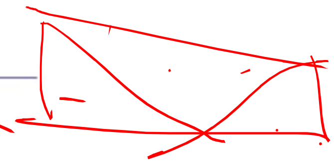
$$\sqrt{8^2+6^2} = \sqrt{64+36}$$
$$= \sqrt{100} = 10$$

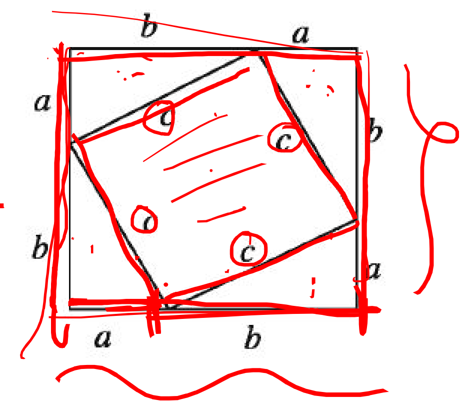
Pythagoras theorem

Alternative Proof of Pythagoras theorem
(Algebraic proof)



$\frac{1}{2} \times b \times a$





Proposition,

$c^2 = a^2 + b^2$

Construction:-

Side of large square $a+b$

Area $\rightarrow (a+b)^2$

Side of small square $\rightarrow c$

Area - - - - $\rightarrow c^2$

$(a+b)^2 = 4 \times$ Each triangle Area $+ c^2$

$a^2 + 2ab + b^2 = 4 \times \frac{1}{2} \times ab + c^2$

$a^2 + \cancel{2ab} + b^2 = \cancel{2ab} + c^2$

Proof

$\boxed{a^2 + b^2 = c^2}$

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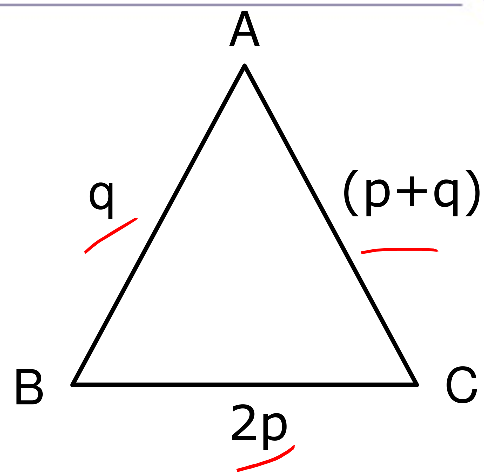
Poll Question-04



$q + p + q + 2p = 12$

$3p + 2q = 12$

If the perimeter of a of the triangle is 12 m , which one is right ?

(a) $p - q = 6$

(b) $3p + 2q = 12$

(c) $p - 2q = 6$

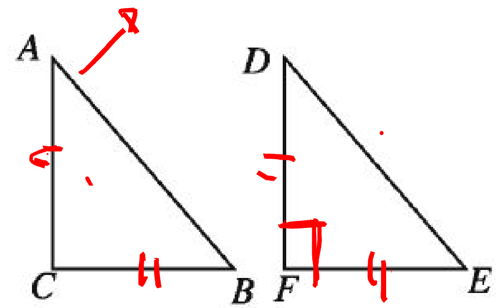
(d) $2p - q = 12$

Converse of

Pythagoras theorem

If the square of a side of any triangle is equal to the sum of the squares of other two sides, the angle between the latter two sides is a right angle.

$AB^2 = AC^2 + BC^2$



Proposition :- ΔABC, $AB^2 = AC^2 + BC^2$

prove that $\angle C = 90^\circ$

Construction :- ΔDFE, $FE = CB$, $DF = AC$ } $\angle F = 90^\circ$

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Proof:-

As $\Delta DFE \rightarrow$ Right anged

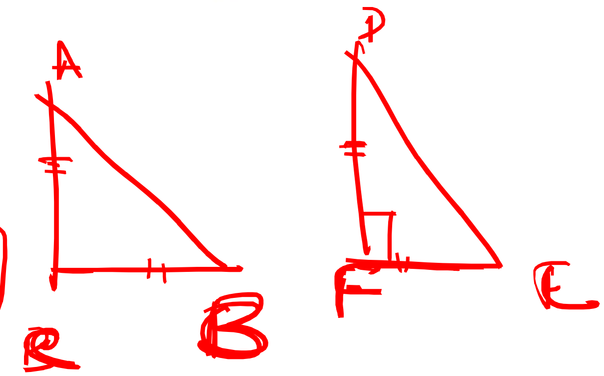
$DE^2 = DF^2 + EF^2$

$= AC^2 + BC^2$

$= AB^2$

$\therefore DE = AB$

$(\because AB^2 = AC^2 + BC^2)$



ΔABC, ΔDFE

$\Delta ABC \cong \Delta DFE$

$\angle F = \angle C = 90^\circ$

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ধ্বংস করে