بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

Class-8: Math(Chapter-10)

Circle

Lecture-13

Previous Homework

Here $9x - 7y = 13$ and $5x - 3y = 9$ two simple simultaneous equation.

(A) Which equation satisfied this point $(0, -3)$? 2

(B) Solve the equations by using the method of elimination .

4

(C) Draw the graph for the equations and find the abscissa and the ordinate of the point of intersection 4

Solution

(A) Given equations ,

$9x - 7y = 13 \dots\dots\dots\dots (i)$

$5x - 3y = 9 \dots\dots\dots\dots\dots (ii)$

L.H.S = R.H.S

Putting the value of x=0 and y=-3 in L.H.S. in equation (i) and we get,

$$9 \times 0 - 7(-3) = 21 \neq \text{R.H.S.}$$

Again,

Putting the value of x=0 and y=-3 in L.H.S. in equation (ii) and we get,

$5 \times 0 - 3(-3) = 9 =$ R.H.S.

Previous Homework

(B) Given, $9x - 7y = 13 \ldots \ldots \ldots \ldots (i)$

$5x - 3y = 9 \ldots \ldots \ldots \ldots \ldots (ii)$

Multiply equation (i) by 3 and equation (ii) by 7,

$27x - 21y = 39 \ldots \ldots \ldots \ldots (iii)$

$35x - 21y = 63 \ldots \ldots \ldots \ldots (iv)$

Now, subtracting (iv) and (iii), we get

$$8x = 24$$

$$\therefore x = 3$$

putting the value o x in equation (i) and we get,

$9 \times 3 - 7y = 13$

Or, $27 - 13 = 7y$

Or, $7y = 14$

$\therefore y = 2$

Required solution is $\boxed{(x, y) = (3,2)}$ → Ans

Previous Homework

(C) $9x - 7y = 13$ ... (1)

Or, $9x - 13 = 7y$

Or, $y = \frac{9x-13}{7}$ ... ... (i)

$y = \frac{9x-13}{7}$

We construct the table below by finding values of y for different values of x:

x	-4	3	10
y	-7	2	11

Table-1

$(-4,-7)$

$(3,2)$

$(10,11)$

Again, $5x - 3y = 9$ ... (11)

Or, $3y = 5x - 9$

Or, $y = \frac{5x-9}{3}$ ... ... (ii)

$y = \frac{5x-9}{3}$

We construct the table below by finding values of y for different values of x:

x	-3	3	6
y	-8	2	7

Table-2

$(-3,-8)$

$(3,2)$

$(6,7)$

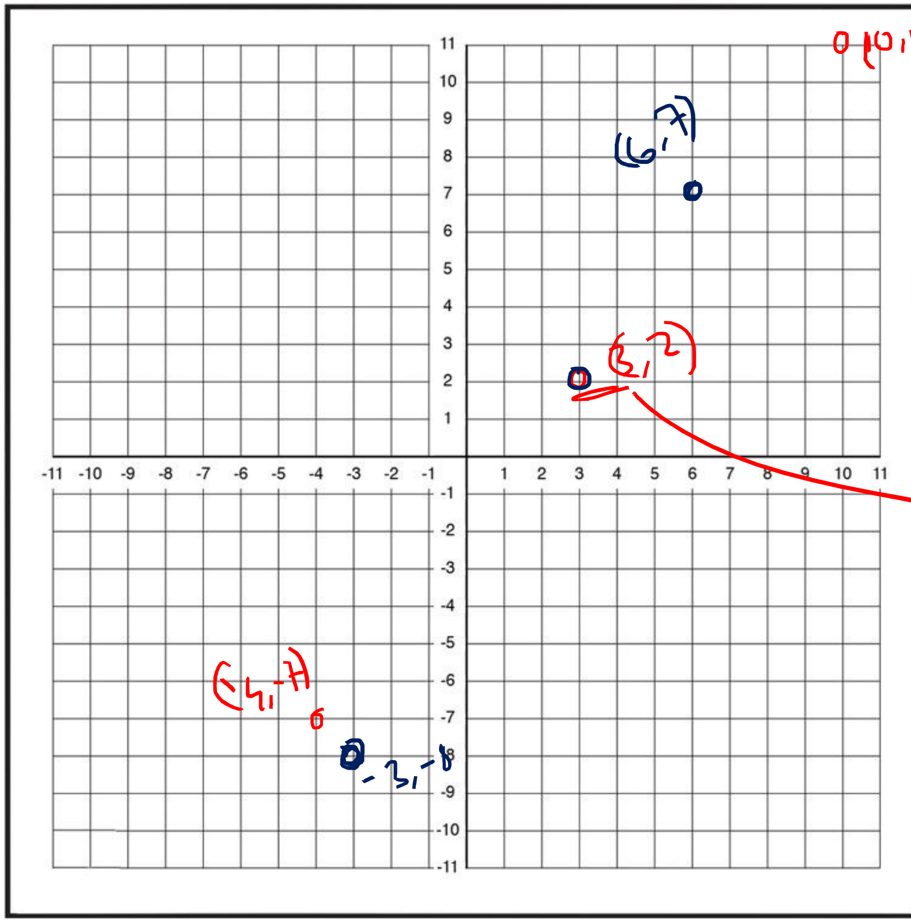
Previous Homework

eqn ①

(-4, -7)

(3, 2)

(10, 11)



eqn ②

(-3, -8)

(3, 2)

(6, 7)

3, 2

Ans

From previous lesson

Poll Question 01

Father's age is fourth times of his son age . 4 years before , summation of their age was 52 years . How old is the father now?

(a) 32 (b) 38 (c) 48 (d) 52

$5x = 60$

$x = 12$

12×4

Son's present age x

Father's " " $4x$

$(4x-4)+(x-4) = 52$

$5x - 8 = 52$

$\therefore 5x = 52+8$

From previous lesson

Poll Question 02

5×5

Tuhin is 5 years old. If rifat's age is fifth times greater than tuhin's age , how old is rifat ?

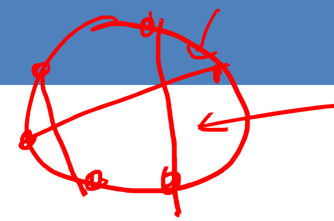
(a) 10 (b) 25 yrs (c) 30 (d) 20

What will we learn from chapter-10 ?

Information and theorem about circle

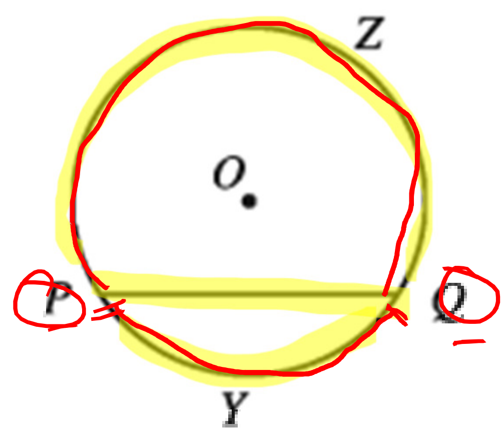
Mathematical problems about circle

Chapter-10.1



10.2 Chord and Arc of a circle

In the adjacent figure, a circle is drawn with the centre at O. Taking any two points P, Q on the circle, draw their joining line segment PQ. The line segment PQ is called a chord of the circle . The chord divides a circle into two parts.



Taking two points Y, Z on two sides of the chord and then we get two parts is PYQ and PZQ. Each part of the circle divided by the chord is called an arc of circle or in brief an arc. In the picture, two arcs, arc PYQ and arc PZQ are produced by the chord PQ.

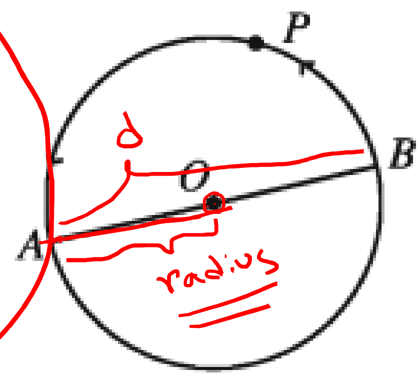
The joining line segment at any two points of a circle is the chord of the circle. Each chord divides a circle into two arcs.

Chapter-10.1

10.3 Diameter and Circumference

$\boxed{d = 2r}$

In the adjacent figure, a such chord AB of a circle is drawn which passes through the centre at O. In that case we call the chord a diameter of the circle. The length of a diameter is also called diameter. The arcs made by the diameter AB are equal; they are known as semi-circle. Any chord that passes through the centre is a diameter. The diameter is the largest chord of the circle. Half of the diameter is the radius of the circle. Obviously, diameter is twice the radius.



The complete length of the circle is called its circumference. That means, starting from a point P, the distance covered along the circle until you reach the point P, is the circumference.

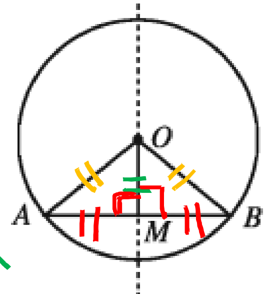
The circle is not a straight line, so its circumference can not be measured with a ruler.

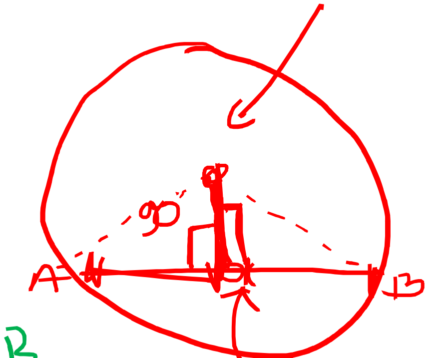
Center

Theorem 1

The line segment joining the centre of a circle to the midpoint of a chord other than diameter, is perpendicular to the chord.

Proposition: Let AB be a chord other than diameter of a circle with centre O and O is joined to the midpoint M of AB. It is to be proved that OM is perpendicular to AB.





$\angle OMA$

$= \angle OMB$

Construction: Join O,A and O,B.

Proof:

Steps	Justification
(1) In ΔOAM and ΔOBM,	
$AM = BM$	[M is the midpoint of AB]
$OA = OB$	[radius of same circle]
and $OM = OM$	[common side]
Therefore, $\Delta OAM \cong \Delta OBM$	[SSS theorem]
$\therefore \angle OMA = \angle OMB$	
(2) Since the two angles are equal and make a straight angle	
$\angle OMA = \angle OMB = 1$ right angle.	
Therefore, $OM \perp AB$. (Proved)	

90°

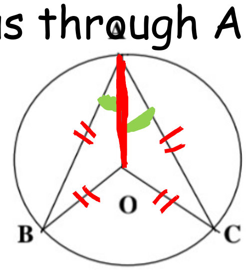
180°

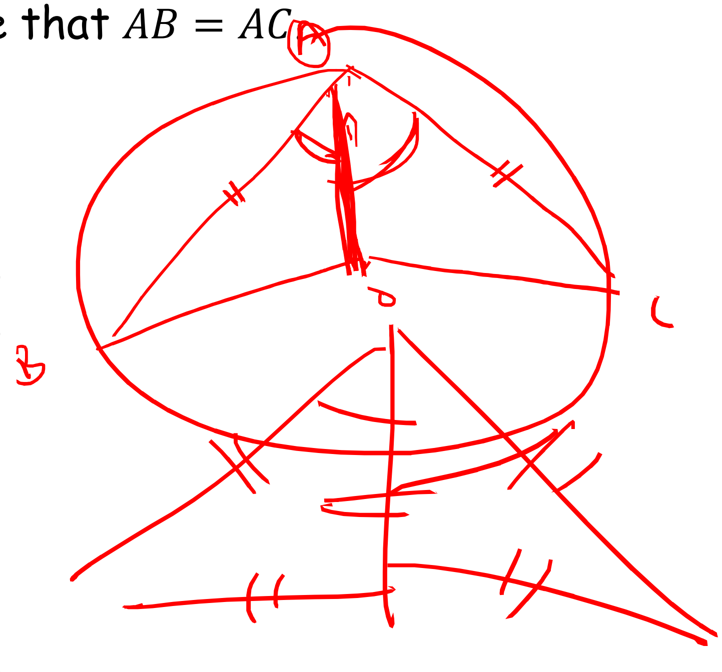
90°

Exersice-10.1

3. Two chords AB and AC of a circle make equal angles with the radius through A . Prove that AB= AC .

Solution:
Proposition: Let, O be the center of the circle and Two chords AB and AC of a circle make equal angles with the radius through A .We have to Prove that $AB = AC$.
Proof:





Steps	Justification
(1) In ΔAOB and ΔAOC,	
$BO = CO$	[radius of same circle]
$\angle BAO = \angle CAO$	
and $OA = OA$	[common side]
$\therefore \Delta AOB \cong \Delta AOC$	[R.H.S. theorem] SAS
$\therefore AB = AC$ (proved)	

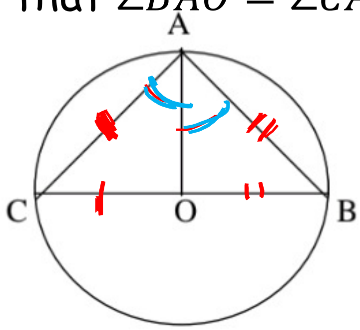
Exercise-10.1

4.In the figure , O is the center of the circle and the chord AB=chord AC . Prove that $\angle BAO = \angle CAO$

Solution:

Proposition: In the figure , O is the center of the circle and the chord AB=chord AC . We have to Prove that $\angle BAO = \angle CAO$

Proof:



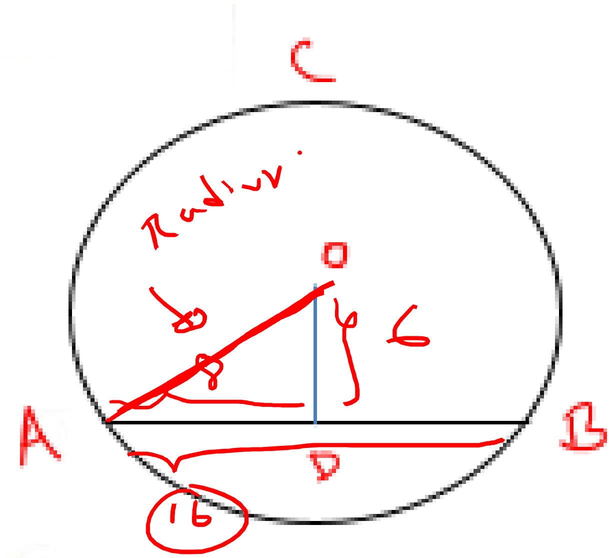
$\angle BAO = \angle CAO$

Steps	Justiication
(1) In ΔAOB and ΔAOC	
$AB = AC$	[supposition]
$OB = OC$	[radius of same circle]
and $OA = OA$	[common side]
$\therefore \Delta AOB \cong \Delta AOC$	[~~RHS~~ theorem] SAS
$\therefore \angle BAO = \angle CAO$ (Proved)	

O be the center of ABC circle where OD⊥AB . If AB=16cm and OD=6 cm , what is the value of the radius ?[Cm.B.-15]

(A)10cm (B)14cm

(C)17cm (D)22cm



$h^2 = b^2 + p^2$

AB : 16 cm

$r = \sqrt{8^2+6^2}$

$= \sqrt{64+36}$

$= \sqrt{100}$

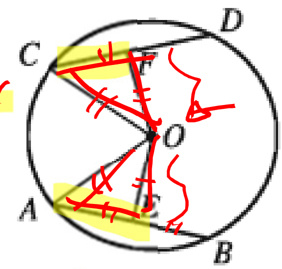
$= 10$

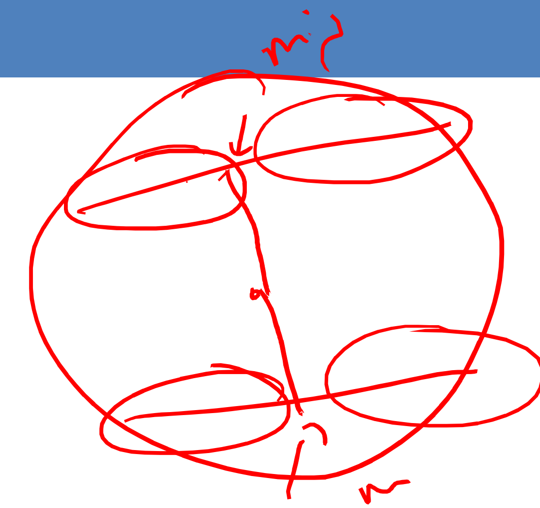
Theorem 2

Equal chords of a circle are equidistant from the centre.

Proposition: Let AB and CD be two equal chords of a circle with the centre O. It is to be proved that the chords AB and CD are equidistant from the centre. Prove, $OE = OF$

Construction: Draw from O, the perpendiculars OE and OF to the chords AB and CD respectively. Join O,A and O,C.





Steps	Justification
(1) $OE \perp AB$ and $OF \perp CD$ Therefore, $AE=BE$ and $CF=DF$. $\therefore AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$	[Perpendicular from the centre bisects the chord]
(2) ~~But~~ $AB = DC$	[supposition]
$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$	[radius of same circle]
$\therefore AE = CF$.	[Step 2]
(3) Now between the right-angled ΔOAE and ΔOCF	[RHS theorem]

AE = BE
CF = DF

Chapter-10.2

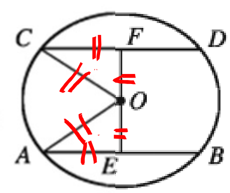
hypotenuse OA = hypotenuse OC and $AE = CF$. $\therefore \quad \Delta OAE \cong \Delta OCF$ $\therefore \quad OE = OF$. (4) But OE and OF are the distances from O to the chords AB and CD respectively. Therefore, the chords AB and CD are equidistant from the centre of the circle. (Proved)	

Chapter-10.2

Theorem 3

Chords equidistant from the centre of a circle are equal.

Proposition: Let AB and CD be two chords of a circle with centre O. OE and OF are the perpendiculars from O to the chords AB and CD respectively. Then OE and OF represent the distances from centre to the chords AB and CD respectively.



If $OE = OF$, it is to be proved that $AB = CD$.

Construction : Join O,A and O,C.

Proof:

Steps	Justification
(1) Since $OE \perp AB$ and $OF \perp CD$. Therefore, $\angle OEA = \angle OFC =$ 1 right angle	[right angles]
(2) Now, between the right-angled ΔOAE and ΔOCF hypotenuse $OA =$ hypotenuse OC and $OE = OF$ $\therefore \quad \Delta OAE \cong \Delta OCF$ $\therefore \quad AE = CF$.	[radius of same circle] [supposition] [RHS theorem]
(3) $AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$	[Perpendicular from the centre bisects the chord]
(4) Therefore, $\frac{1}{2}AB = \frac{1}{2}CD$ i.e., $AB = CD$	

ΔOCF

ΔOAE

$AE = \frac{1}{2}AB$; $CF = \frac{1}{2}CD$.

$\frac{1}{2}AB = \frac{1}{2}CD$

$AB = CD$

Exercise-10.2

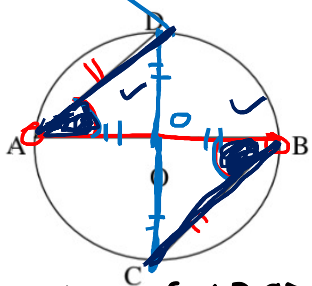
3. Show that equal chords drawn ~~drawn~~ from the end points on opposite sides o a diameter are parallel.

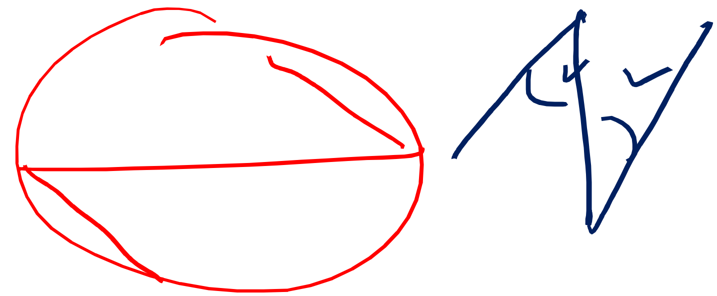
Solution:

OA = OB [radius]

OC = OD ["]

AD = BC





Proposition: let, O be the center of ABCD circle . Draw chord AD of point A and chord BC of point B from Diameter AB . We have to show that AD ∥ BC .

Proof:

AOD ≅ BOC

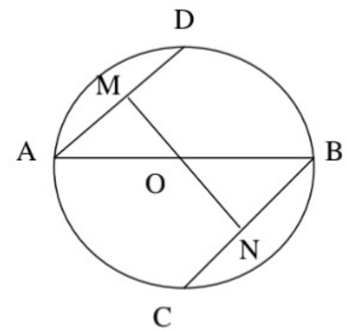
Steps	Justifications
(1) Here, $AD = BC$ [wrong] And AB is their transversal , $\therefore \angle BAD = \angle ABC$	[~~suppositions~~] [radius of same circle]
(2) Alternate angles are always equal . So, AC and BC are parallel. $\therefore AD \parallel BC$ (Proved)	

Exercise-10.2

41 show that, Parallel chords are drawn from the end points of a diameter are equal.

Solution:

Proposition: Let, O be the center of a circle where AB is the diameter . Draw chord AD of point A and chord BC of point B from Diameter AB and $AD \parallel BC$. We have to show that , $AD = BC$



Construct : Draw perpendicular lines OM and ON from center O on AD and BC respectively .

Proof:

Steps	Justifuactions
(1) Now , between the right angled ΔAOM and ΔBON , $AO = BO$ and AM = BN $\therefore \Delta AOM \cong \Delta BON$ $\therefore OM = ON$	[supposition]
(2) $\therefore AD = BC$(Proved)	[Chords equidistant from the center of a circle are equal]

Here ,OE=OF, If AB=6cm , CF=?

[J.B.-14]

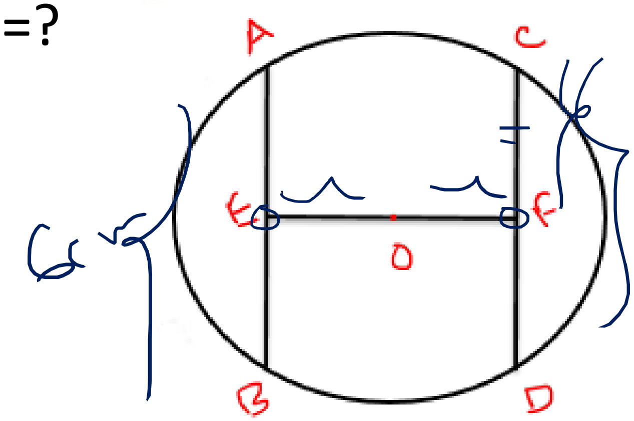
(A)2

(B)3

(C)4

(D)6

$6/2 = 3$



CD = 6cm

Chapter-10.3

Example 1. The diameter of a circle is 10 cm. What is the $\boxed{\text{circumference}}$ of the circle ? (use $\pi \approx 3{\cdot}14$)

Solution:

Diameter of the circle, $d = 10$ cm

Circumference of the circle $= \pi d$

$\approx 3.14 \times 10 \text{ cm} = 31{\cdot}4$ cm

Therefore, the circumference of the circle with ~~radius~~ dia 10 cm is 31·4 cm.

Area

$2\pi r$

radius

$d = 2r$

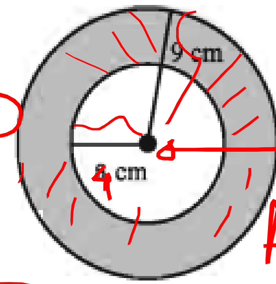
3.14

Circumference means the length of a circle.

Chapter-10.3

πr^2

Example 4. The adjoining figure shows two circles with the same centre. The radius of the larger circle is 9 cm and the radius of the smaller circle is 4 cm. What is the area of the shaded region between the two circles ?



Area of a circle $= \boxed{\pi r^2}$

Solution :

The radius of the larger circle $r = 9$ cm

So, the area of the larger circle $= \pi r^2$ sq. cm

$\approx 3.14 \times 9^2$ sq. cm $= 254.34$ sq. cm

The radius of the smaller circle $r = 4$ cm

$\therefore$ The area of the smaller circle $= \pi r^2$ sq. cm

$\approx 3.14 \times 4^2$ sq. cm $= 50.24$ sq. cm

$\therefore$ The area of the shaded region $= (254.34 - 50.24)$ sq. cm (approx.)

$= 204.10$ sq. cm (approx.) Ans

Creative question

Let, O be the center of a circle where AB is a diameter and AD is a chord . A.find the area of a fish where his diameter is 6.4 meter . 2

B. Prove that, $AB > CD$. 4

C. If E is the midpoint of CD, Prove that, $OE \perp CD$. 4

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

উদ্ভ্রাস-উন্মেষ শিক্ষা পরিবার