

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

বিস্মিল্লাহির রাহমানির রাহীম



উদ্দামা

একাডেমিক এন্ড এডমিশন কেয়ার

Class: 9 Math (Chapter: 3.1, 3.2)

ALGEBRAIC EXPRESSIONS

LECTURE M-06

EXERCISE 3.1....

9) If $a + b + c = 6$ and $a^2 + b^2 + c^2 = 14$, what is the value of $(a - b)^2 + (b - c)^2 + (c - a)^2$?

$$\begin{aligned} (a-b)^2 + (b-c)^2 + (c-a)^2 &= a^2 - 2ab + b^2 + b^2 - 2bc + c^2 + c^2 - 2ca + a^2 \\ &= 2(a^2 + b^2 + c^2) - 2(ab + bc + ca) \\ &= 2(a^2 + b^2 + c^2) - \left\{ (a+b+c)^2 - (a^2 + b^2 + c^2) \right\} \\ &= 3(a^2 + b^2 + c^2) - (a+b+c)^2 \\ &= 3 \cdot 14 - 6^2 = 42 - 36 = 6 \end{aligned}$$

EXERCISE 3.1....

15) If $a^4 + a^2b^2 + b^4 = 8$ and $a^2 + ab + b^2 = 4$, what is the value of a) $a^2 + b^2$ b) ab ?

Question

$$\begin{aligned} & (a^2 + ab + b^2) = 4 \\ \Rightarrow (a^2 + b^2) &= (4 - ab) \\ \Rightarrow a^4 + 2a^2b^2 + b^4 &= 16 - 8ab + a^2b^2 \\ \Rightarrow a^4 + a^2b^2 + b^4 &= 16 - 8ab \\ \Rightarrow 8 &= 16 - 8ab \\ \Rightarrow 8ab &= 16 - 8 = 8 \\ \Rightarrow ab &= 1 \quad (\text{Ans}) \end{aligned}$$

$$\begin{aligned} & a^2 + ab + b^2 = 4 \\ \Rightarrow a^2 + b^2 &= 4 - ab \\ \Rightarrow a^2 + b^2 &= 4 - 1 \\ \Rightarrow a^2 + b^2 &= 3 \end{aligned}$$

(Ans).

Poll Question: 01

If $a + b = \sqrt{16}$ and $\underline{ab} = 1$, what is the value of $\underline{(a - b)^2}$?

- ☒ a) 12 b) 14 c) 18 d) 20

$$\begin{aligned}(a - b)^2 &= (a + b)^2 - 4ab \\&= (\sqrt{16})^2 - 4 \cdot 1 \\&= 16 - 4 = 12\end{aligned}$$

FORMULAE OF CUBES :-

 **Formula 6:** $(a + b)^3 = \underbrace{a^3}_{\text{a}^3} + \overbrace{3a^2b}^{3ab(a+b)} + \overbrace{3ab^2}^{3ab(a+b)} + \underbrace{b^3}_{\text{b}^3} = \underline{a^3 + b^3 + 3ab(a + b)}$
 $(a+b)^3 = (a+b)(a+b)(a+b) = (a+b)^2(a+b) = (a^2 + 2ab + b^2)(a+b) = \underline{a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3}$

 **Corollary 9:** $\underline{a^3 + b^3} = (a + b)^3 - 3ab(a + b)$

$(a+b)^3 = \underline{a^3 + b^3 + 3ab(a+b)} \Rightarrow \underline{a^3 + b^3} = (a+b)^3 - 3ab(a+b)$

 **Formula 7:** $(a - b)^3 = \underbrace{a^3}_{\text{a}^3} - \overbrace{3a^2b}^{+3ab(a-b)} + \overbrace{3ab^2}^{-3ab(a-b)} - \underbrace{b^3}_{\text{b}^3} = \underline{a^3 - b^3 - 3ab(a - b)}$
 $(a-b)^2(a-b) = \dots \dots \dots + 3ab(a-b) - 3ab(a-b)$

 **Corollary 10:** $\underline{a^3 - b^3} = (a - b)^3 + 3ab(a - b)$

$(a-b)^3 = \underline{a^3 - b^3 - 3ab(a-b)} \Rightarrow \underline{a^3 - b^3} = (a-b)^3 + 3ab(a-b)$

FORMULAE OF CUBES:-

$$\underline{a^n + b^n} \text{ or, } \underline{a^n - b^n}$$

➡ **Formula 8 :** $\underline{a^3 + b^3} = (\underline{a + b})(a^2 - ab + b^2)$

$$\begin{aligned} \underline{a^3 + b^3} &= (\underline{a + b})^3 - 3ab(a + b) = (a + b) \{ (\underline{a + b})^2 - 3ab \} = (\underline{a + b}) \{ \underline{a^2 + 2ab + b^2} - \underline{3ab} \} \\ &= (a + b) (a^2 - ab + b^2) \end{aligned}$$

➡ **Formula 9 :** $\underline{a^3 - b^3} = (\underline{a - b})(a^2 + ab + b^2)$

$$\begin{aligned} \underline{a^3 - b^3} &= (\underline{a - b})^3 + 3ab(a - b) = (a - b) \{ (\underline{a - b})^2 + 3ab \} = (\underline{a - b}) (\underline{a^2 - 2ab + b^2} + \underline{3ab}) \\ &= (a - b) (a^2 + ab + b^2) \end{aligned}$$

Poll Question: 02

Which one is not correct ?

(a) $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$ C-10

☒ (c) $a^3 + b^3 = (a + b)(a^2 + ab \ominus b^2)$ x

(b) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ F-9

(d) $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ C-9

ACTIVITY : [Page no-51]

Find the cube with the help of the formulae :

a) $(3x+2y)^3$

$(\underline{a} + \underline{b})^3$

b) $3x - 4y$

c) 397^3

$(\underline{400} - \underline{3})^3$

ACTIVITY : [Page no-53]

1) If $x = -2$, what is the value of $27x^3 - 54x^2 + 36x - 8$?

ACTIVITY : [Page no-53]

2) If $a + b = 5$ and $ab = 6$, what is the value of $a^3 + b^3 + 4(a - b)^2$?

ACTIVITY : [Page no-53]

3) If $x = \sqrt{5} + \sqrt{3}$, what is the value of $x^3 + \frac{1}{x^3}$?

Poll Question: 03

If $x + \frac{1}{x} = 5$, then $x^3 + \frac{1}{x^3} = ?$

$x \rightarrow a$
 $\frac{1}{x} \rightarrow b$ } $a^3 + b^3$

(a) 21

(b) 23

☒ (c) 110

(d) 140

$$\begin{aligned} x^3 + \frac{1}{x^3} &= \left(x + \frac{1}{x}\right)^3 - 3 \cdot x \cdot \frac{1}{x} \cdot \left(x + \frac{1}{x}\right) \\ &= 5^3 - 3 \cdot 1 \cdot 5 \\ &= 125 - 15 = 110 \end{aligned}$$

$$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

EXERCISE 3.2....

c) $(m+n)^6 - (m-n)^6 - 12mn(m^2 - n^2)^2$

$$\neq \left\{ \underline{(m+n)^2}^3 - \underline{(m-n)^2}^3 - 3 \underline{(m+n)(m-n)}^2 \underline{4mn} \right\} \quad (a+b)^2 = a^2 + b^2$$

$$= \left\{ \underline{(m+n)^2}^3 - \underline{(m-n)^2}^3 - 3 \cdot \underline{(m+n)(m-n)}^2 \cdot \underline{(m+n)^2 - (m-n)^2} \right\}$$

$$= \left\{ \underline{(m+n)^2 - (m-n)^2}^3 \right\}$$

$$= \underline{(4mn)^3 = 64m^3n^3} \quad (\text{Ans.})$$

$$\underline{(a+b)^2 - (a-b)^2 = 4ab}$$

$$\underline{a^3 - b^3 - 3ab(a-b)} = (a-b)^3$$

Poll Question: 04

Simplify: $(2x + 3y - 4z)^3 + (2x - 3y + 4z)^3 + 12x \{4x^2 - (3y - 4z)^2\}$

(a) $64x$ (b) $128z^2$ (c) $64x^3$ (d) $64z^3$

$$\begin{aligned}
 & a^3 + b^3 + 3 \cdot 4x \{ (2x + 3y - 4z)(2x - 3y + 4z) \} \\
 &= a^3 + b^3 + 3 \left\{ \underbrace{(2x + 3y - 4z)}_a + \underbrace{(2x - 3y + 4z)}_b \right\} \underbrace{(2x + 3y - 4z)}_a \underbrace{(2x - 3y + 4z)}_b \\
 &= a^3 + b^3 + 3 \cdot ab(a + b) \\
 &= (a + b)^3 = \underbrace{(2x + 3y - 4z + 2x - 3y + 4z)}_{4x}^3 = \underline{64x^3}
 \end{aligned}$$

EXERCISE 3.2....

4) If $a^3 - b^3 = 513$ and $a - b = 3$, what is the value of ab ?

$$\begin{aligned}(a-b)^3 &= a^3 - b^3 - 3ab(a-b) \\ 3^3 &= 513 - 3(ab)(3) \\ ab &= \frac{513 - 3^3}{9} = \boxed{?}\end{aligned}$$

EXERCISE 3.2....

6) If $a = 15$, then what is the value of $8a^3 + 60a^2 + 150a + 130$?

$$\left\{ (2a)^3 + 3 \cdot (2a)^2 \cdot 5 + 3 \cdot (2a) \cdot (5)^2 + 5^3 \right\} + 5$$

$$= (2a + 5)^3 + 5$$

$$= 35^3 + 5$$

$$= \boxed{?}$$

EXERCISE 3.2....

7) If $a + b = m$, $a^2 + b^2 = n$ and $a^3 + b^3 = p^3$, then show that $m^3 + 2p^3 = 3mn$?

$$(a+b)^3 = m^3$$

$$\Rightarrow a^3 + b^3 + 3ab(a+b) = m^3$$

$$\Rightarrow p^3 + 3m \cdot \frac{1}{2}(m^2 - n) = m^3$$

$$\Rightarrow 2p^3 + 3m(m^2 - n) = 2m^3$$

$$\Rightarrow 2p^3 + 3m^3 - 3mn = 2m^3$$

$$\Rightarrow m^3 + 2p^3 = 3mn$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$\Rightarrow m^2 = n + 2ab$$

$$\Rightarrow 2ab = m^2 - n$$

$$\Rightarrow ab = \frac{1}{2}(m^2 - n)$$

[shown]

Poll Question: 05

If $2x - \frac{2}{x} = 3$ then, $8(x^3 - \frac{1}{x^3}) = ?$

(a) 32

(b) 24

(c) 64

✓(d) 63

$$\begin{aligned} 8x^3 - \frac{8}{x^3} &= (2x)^3 - \left(\frac{2}{x}\right)^3 \\ &= \left(2x - \frac{2}{x}\right)^3 + 3 \cdot 2x \cdot \frac{2}{x} \cdot \left(2x - \frac{2}{x}\right) \\ &= 3^3 + 3 \cdot 4 \cdot 3 \\ &= 27 + 36 = 63 \end{aligned}$$

$$\begin{aligned} a^3 - b^3 &= (a-b)^3 + 3ab(a-b) \end{aligned}$$

EXERCISE 3.2....

13) If $a + b + c = 0$, then should that

a) $a^3 + b^3 + c^3 = 3abc$

b) $\frac{(b+c)^2}{3bc} + \frac{(c+a)^2}{3ca} + \frac{(a+b)^2}{3ab} = 1$

EXERCISE 3.2....

16) If $a = \sqrt{6} + \sqrt{5}$, what is the value of $\frac{a^6 - 1}{a^3}$?

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

উদ্ভাস-উন্মেষ শিক্ষা পরিবার