بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

Class 9: Higher Math (Chapter-9.1)

Trigonometric Ratio

Lecture HM-26

Rational & Irrational Exponent

- Structure: Base & Exponent

base $\quad x^{y}$

In any math of power, there's a base, and there's an exponent, which determines how many times the base was multiplied $\quad x^2 \quad x^{-2} \quad x^{3/2}$

- Freedom of exponent

$x^{1.5673----}$

The number representating exponenet can be any free real number, any rational or irrational, positive or negative, anything.

$\frac{1}{x^{-3}} = x^3$

$\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$\sqrt[5]{x} = x^{1/5}$

$x^{-2} = \frac{1}{x^2}$

Recalling of the formulas

$(a^m)^n = (a^n)^m$
$= a^{mn}$

If a∈R and n∈N,

$$a^1 = a, a^{n+1} = a^n . a$$

$x^1 = x$

If a∈R, a≠0 and m,n∈N, m≠n,

$$\frac{a^m}{a^n} = a^{m-n} \text{ when } m > n$$

$$\text{or } \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \text{ when } m < n$$

$(xy)^3$
$= x^3 . y^3$

If a,b∈R and n∈N,

$$(a.b)^n = a^n . b^n$$

Real

If a∈R and m,n∈N,

$$a^m . a^n = a^{m+n}$$

$\sqrt{-2} \rightarrow$ Complex

$x^3 . x^2$
$= x^{3+2}$
$= x^5$

If a∈R and m,n∈N,

$$(a^m)^n = a^{mn}$$

$(x^3)^2 = (x^2)^3$
$= x^6$

If a∈R and a≠0

$$\boxed{a^0 = 1}$$

$$a^{-n} = \frac{1}{a^n}, \text{when } n\in N$$

উদ্ভাস উন্মেষ শিক্ষা পরিবার

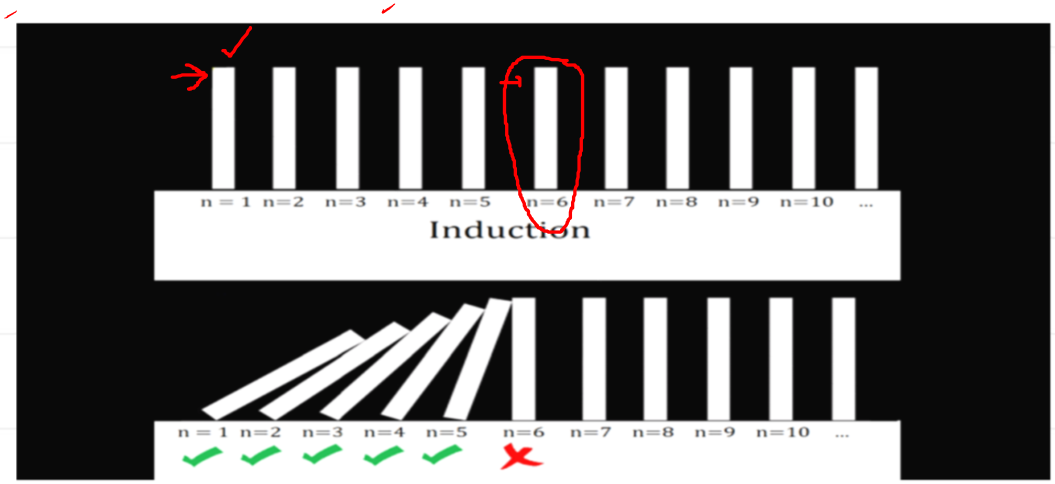
Mathematical induction rule

$(a+b)^2 = a^2 + 2ab + b^2$ ×

• Rule:

If any mathematical statement is true for a unit value, and after assumingit true for a common value,if it is again true for consecutive value, it can be admitted true for every value.

$(a^m)^n = a^{mn}$



Method of mathematical induction

Step

n=1

n=k

- Assuming the value of preferable variable 1 and proving the statement.
- Assuming that for the value of the variable being k, the statementis true.
- Proving the statement for value of the variable being k+1.

$\boxed{n=1}$ True Prove

let, $\boxed{n=k}$

n=k+1 True

Example (Method of Induction)

- Formula-2:

$$a^m.a^n = a^{m+n}$$

এখানে চলক (variable) নিই n কে,

Step-1: So if, n=1, then,

$$a^m.a^1$$
$$= a^m.a$$
$$= a^{m+1}\ [formula\ 1]$$

অর্থাৎ n=1 এর জন্য এটা সত্য।

Step-2: এবার ধরে নিই, n=k এর জন্যও এটা সত্য,

অর্থাৎ, $a^m.a^k = a^{m+k}$

Step-3: এখন ধরি, n=k+1, তাহলে,

$$a^m.a^{k+1}$$
$$= a^m.(a^k.a)\ [formula\ 1]$$
$$= (a^m.a^k).a$$
$$= a^{m+k}.a\ [from\ step\ 2]$$
$$= a^{m+k+1}\ [formula\ 1]$$

অর্থাৎ এটা n=k+1 এর জন্যও সত্য

সুতরাং, এটা n এর সব মানের জন্য সত্য

$(a^m)^n = a^{mn}$ —— (1)

Step 1: for n = 1,

L.H.S = $(a^m)^1 = a^m$

R.H.S = $a^{m.1} = a^m$

∴ L.H.S = R.H.S

∴ for n = 1, (1) is true.

Step 2: let, for n = K, (1) is true

$(a^m)^K = a^{mK}$ —— (11)

Step 3:- for, n = K+1

L.H.S = $(a^m)^{K+1}$

$= (a^m)^K (a^m)^1$

$= a^{mK}.a^m$ [Using (11)]

$= a^{mK+m}$

$= a^{m(K+1)}$

R.H.S $= a^{m(K+1)}$

L.H.S = R.H.S

Example (Method of Induction)

Example 4

If m,n∈N , then,

$$\frac{a^m}{a^n} = a^{m-n}, when\ a \neq 0$$

এখানে তিনটা কেস আছে

Case:1- যখন m>n, তাহলে

$$\frac{a^m}{a^n} = a^{m-n} \quad [formula\ 3]$$

Case:2- যখন m<n, তাহলে

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad [formula\ 3]$$

$$= a^{-(n-m)} \quad \left[a^{-n} = \frac{1}{a^n}\right]$$

$$= a^{m-n}$$

Case:3- যখন m=n, তাহলে,

$$\frac{a^m}{a^n} = \frac{a^n}{a^n} = 1 = a^0 = a^{m-m} = a^{m-n}$$

অর্থাৎ, m,n এর যেকোনো মানের জন্য $\frac{a^m}{a^n} = a^{m-n}$

The discussion on roots

The structure of root considered as this thing,

If $x^n = a$

Then x will be called the nth root (n−তম মূল) of a [only if n∈N, and n>1]

x can be written as $\sqrt[n]{a}$.

Some points to be noted:

- If a>0 and n∈N, n>1, then there are two cases. If n is odd(বিজোড়), then the nth root will be simply $\sqrt[n]{a}$. But if n is even(জোড়), then there will be two nth roots for a. One is $\sqrt[n]{a}$, and another is $-\sqrt[n]{a}$
- If a<0 and n∈N, n>1, then there are two cases again. If n is odd, then the nth root will be $-\sqrt[n]{a}$. But if n is even, there is no nth root for a.

Relation between root and exponent:

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Some tips for math

- $If\ a^x = 1, where\ a > 0\ and\ a \neq 1, then\ x = 0$
- $If\ a^x = 1, where\ a > 0\ and\ x \neq 0, then\ a = 1$
- $If\ a^x = a^y, where\ a > 0\ and\ a \neq 1, then\ x = y$
- $If\ a^x = b^x, where \frac{a}{b} > 0\ and\ x \neq 0, then\ a = b$

$x = 0$

$a^x = 1$

$a = 1$

$3^x = 1$

$3^x = 3^0$

$\therefore x = 0$

$a^3 = 1$

$\Rightarrow a^3 = 1^3$

$a = 1$

Example 11

3^{xy}

3^{y}

$(3^x)^y = (3^y)^x$

If $x^{x\sqrt{x}} = (x\sqrt{x})^x$, *then what is the value of* x?

$\Rightarrow (x^x)^{\sqrt{x}} = (x^1 \cdot x^{\frac{1}{2}})^x$

$\Rightarrow (x^x)^{\sqrt{x}} = (x^{1+\frac{1}{2}})^x$

$\Rightarrow (x^x)^{\sqrt{x}} = (x^{3/2})^x$

$\Rightarrow (x^x)^{\sqrt{x}} = (x^x)^{3/2}$

$\therefore \sqrt{x} = \frac{3}{2}$

$\Rightarrow (\sqrt{x})^2 = (\frac{3}{2})^2$

$\Rightarrow x = \frac{9}{4}$ Ans

Example 13

$(x^2)^3$

x^6

Prove that, $\left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \times \left(\frac{x^a}{x^b}\right)^{a+b} = 1$

$$\text{L.H.S} = \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} \times \left(\frac{x^a}{x^b}\right)^{a+b}$$

$$= \left(x^{b-c}\right)^{b+c} \times \left(x^{c-a}\right)^{c+a} \times \left(x^{a-b}\right)^{a+b}$$

$$= x^{(b-c)(b+c)} \times x^{(c-a)(c+a)} \times x^{(a-b)(a+b)}$$

$$= x^{b^2-c^2} \times x^{c^2-a^2} \times x^{a^2-b^2}$$

$$= x^{b^2-c^2+c^2-a^2+a^2-b^2}$$

$$= x^0$$

$$= 1$$

Example 14

If $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ and $abc = 1$,

then show that $\boxed{x + y + z = 0.}$

$$a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = K$$

$a^{\frac{1}{x}} = K$

$\Rightarrow \left(a^{\frac{1}{x}}\right)^x = K^x$

$\Rightarrow a^1 = K^x$

$\Rightarrow a = K^x$

$b^{\frac{1}{y}} = K$

$\Rightarrow \left(b^{\frac{1}{y}}\right)^y = K^y$

$\Rightarrow b^1 = K^y$

$\Rightarrow b = K^y$

$c^{\frac{1}{z}} = K$

$\Rightarrow \left(c^{\frac{1}{z}}\right)^z = K^z$

$\Rightarrow c^1 = K^z$

$\Rightarrow c = K^z$

$abc = 1$

$\Rightarrow K^x \cdot K^y \cdot K^z = 1$

$\Rightarrow K^{x+y+z} = 1 = K^0$

$\Rightarrow x + y + z = 0$

[Showed]

Example 15

Simplify:

$$\frac{1}{1+a^{y-z}+a^{y-x}}+\frac{1}{1+a^{z-x}+a^{z-y}}+\frac{1}{1+a^{x-y}+a^{x-z}}$$

$$\frac{x}{y}=\frac{2x}{2y}$$

$$\frac{a^{-y}}{a^{-y}(1+a^{y}a^{-z}+a^{y}a^{-x})}=\frac{a^{-y}}{a^{-y}+a^{-z}+a^{-x}}$$

$$=\frac{1}{1+a^{y}a^{-z}+a^{y}a^{-x}}+\frac{1}{1+a^{z}.a^{-x}+a^{z}.a^{-y}}+\frac{1}{1+a^{x}.a^{-y}+a^{x}a^{-z}}$$

$$=\frac{a^{-y}}{a^{-y}+a^{-z}+a^{-x}}+\frac{a^{-z}}{a^{-z}+a^{-x}+a^{-y}}+\frac{a^{-x}}{a^{-x}+a^{-y}+a^{-z}}$$

$$=\frac{a^{-y}+a^{-z}+a^{-x}}{a^{-x}+a^{-y}+a^{-z}}=\underline{\underline{1}}$$

Example 17

Solve for x

$4^x - 3.2^{x+2} + 2^5 = 0$ (সমাধান করো)

$x^2 - 5x + 6 = 0$

$(2^2)^x - 3.2^x.2^2 + 32 = 0$

$\Rightarrow (2^x)^2 - 3.2^x.4 + 32 = 0$

$\Rightarrow (2^x)^2 - 12.2^x + 32 = 0$

let, $2^x = y$

$y^2 - 12y + 32 = 0$

$\Rightarrow y^2 - 8y - 4y + 32 = 0$

$\Rightarrow y(y-8) - 4(y-8) = 0$

$\Rightarrow (y-8)(y-4) = 0$

$y - 8 = 0$

$y = 8$

$2^x = 2^3$

$\therefore x = 3$

$y - 4 = 0$

$y = 4$

$2^x = 2^2$

$x = 2$

Ans

$x = 3, 2$

Poll Question 01

$$\frac{a^3b^3}{a^2b^{-3}} = ?$$

$$a^{3-2}\,b^{3-(-3)}$$

$$= a^1 b^6 = ab^6$$

(a) a

(b) a^3b^6

(c) ab^6

(d) ab^0

Poll Question 02

Mathematical induction is used-

(a) To invent new formula

(b) To prove a formula

(c) None

Poll Question 03

$\sqrt{a} \times \sqrt[3]{a} \times \sqrt[4]{a} \times \sqrt[5]{a} \times \ldots\ldots\ldots = ?$

$a^{\frac{1}{2}} \cdot a^{1/3} \cdot a^{1/4} \cdot a^{1/5} = a^{1/2 + 1/3 + 1/4 + 1/5}$

(a) $(\sqrt{a})^{1\times2\times3\times..}$

(b) $(a^{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots})$

(c) a

(d) 1

Exercise 01

If $a^b = b^a$, then show that, $\left(\frac{a}{b}\right)^{\frac{a}{b}} = a^{\frac{a}{b}-1}$ and from this prove that, if $a = 2b$, then $b = 2$

Exercise 02

If $a = 2 + 2^{\frac{2}{3}} + 2^{\frac{2}{3}}$, that show that, $a^3 - 6a^2 + 6a - 2 = 0$

Exercise 03

If $a^m . a^n = (a^m)^n$, then prove that, $m(n-2) + n(m-2) = 0$

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

D™¢vm-D‡b¥l শিক্ষা
পরিবার

Thank You