ENGINEERING ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-01

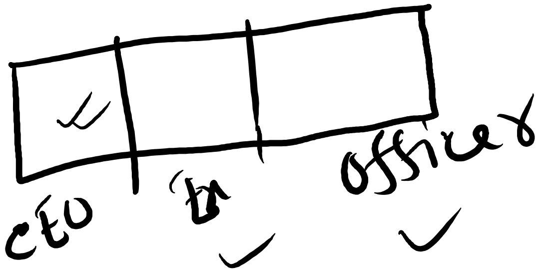
Chapter 5 : Permutation and Combination

Primary Discussions

Relation betn Permutation and combination

10

$^{10}P_3$



$^{10}C_3$

$^{10}C_3 \times 3! = {}^{10}P_3$ → Permutation

combination

$$\boxed{^nC_r \times \lfloor r = \boxed{^nP_r}}$$

$$\therefore \; ^nC_r = \frac{^nP_r}{\lfloor r}$$

Poll Question-01

In how many ways 50 guests can handshake among themselves?

(a) $^{50}P_2$

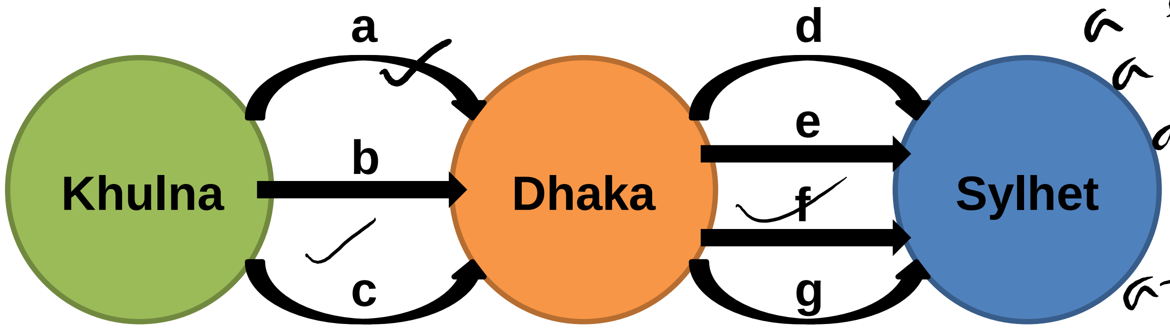
(b) 50×2

(c) $^{50}C_2$

(d) 50^2

Fundamentals of Permutation

Rule of Product:



No of Roads = 3   No of Roads = 4

3×4

ad
ae
af
ag

a → 4
b → 4
c → 4

12

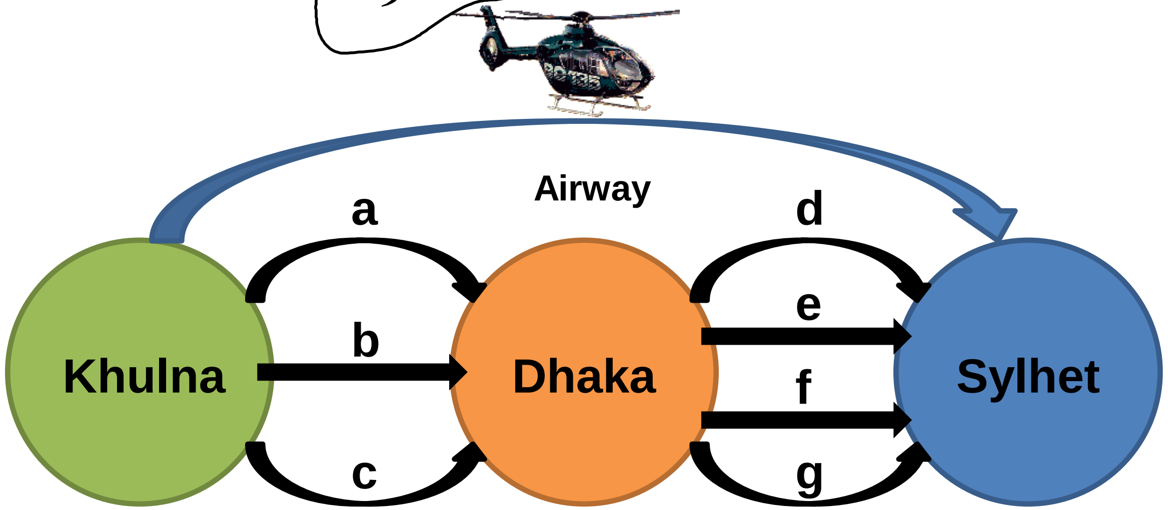
① incomplete / dependent events

and → ×

Rule of Sum:

5

complete or



3

No of Roads = 3   No of Roads = 4

Type – 01: Related To The Equations of Permutation

Concept: $^nP_r = \frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots\ldots(n-r+1)$

i.e.: $^4P_2$ Permutations = $\underbrace{4 \times 3}_{} \rightarrow$ 2 Terms

$^4P_2 = \frac{4!}{2!} = \frac{4.3.\cancel{2!}}{\cancel{2!}} = 4.3$

$^{10}P_2 = 10.9$

Example: Find The value n if $^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12$.

Solve:

$\frac{\cancel{(n-1)}(n-2)(n-3)}{(n+1)\,n\,\cancel{(n-1)}} = \frac{5}{12}$

$5n^2 + 5n = 12(n^2 - 5n + 6)$

$\therefore n = 8$

Type – 01: Related To The Equations of Permutation

$3 : 5$

Example: Find The value n if ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 5 : 3.$

Solve:

$$\frac{\dfrac{(2n+1)!}{((2n+1)-(n-1))!}}{\dfrac{(2n-1)!}{(2n-1-n)!}} = 3/5$$

$\therefore \quad n = 4$

If some objects are similar

- If ‘p’, ‘q’ & ‘r’ number of objects are similar in total ‘n’ number of objects, then the permutation number will be $\frac{n!}{p!\times q!\times r!}$

- In how many ways can the word ‘ENGINEERING’ be arranged?

$$\frac{\lfloor 11}{\lfloor 3 \lfloor 3 \lfloor 2 \lfloor 2}$$

Type – 02: Related To Formation Of Words (A)

This Type has lots of divisions/sub-types. We're about to discuss them together. It'll help us to better understand those problems and their differences.

A. Taking all the letters from the word **DIRECTOR**

1. In how many ways it can be arranged?

   Among those arrangements,

2. In how many words the vowels will be together?
3. In how many words the vowels won't be together?
4. In how many words 2 vowels won't be sitting side by side?
5. In how many words the vowels won't change their position?
6. (a) In how many words the vowels won't change their order?

   (b) In how many words "I" will come before "E"?

   (c) In how many words "I" will come before "E" and "E" will come before "O"?
7. In how many words the vowels and the consonants won't change their relative position?

Type – 02: Related To Formation Of Words (A)

Taking all the letters from the word DIRECTOR

1. In how many ways it can be arranged?
   Solve:
   Total no. of Letters (D,I,R,E,C,T,O,R) =
   No. of Vowels (I,E,O)=
   No. of Consonants (D,R,C,T,R) =
   No. of copies of R =
   No. of ways to Arrange the word DIRECTOR taking all the letters =

We have to consider the letters needed to be kept together a single letter.

2. In how many words the vowels will be together?
   Solve:
   Vowels I, E, O (IEO, OEI, EIO.) will arrange among themselves in =
   We consider (I,E,O) a single letter. Then, along with the consonants (D, R, C, T, R) total no. of letter becomes = 6 where R comes twice.
   Those 6 letters can be arranged among themselves in =
   No. of words where the vowels will be together =

$\frac{6!}{2!} \times 3!$

Type – 02: Related To Formation Of Words (A)

DIRECTOR

3. In how many words the vowels won't be together?
   Solve: Total no. of Arrangements =
   No. of arrangements keeping vowels together =
   No. of Arrangements not keeping vowels together =

$\frac{\lfloor 8}{\lfloor 2} - \frac{6!}{2!} \times 3!$

> Row Permutation
> The ones which can't be kept side by side needs to be arranged later or have to be put in the blank spaces among the other set of letters.
>
> No. of blank spaces among "n" people and their two sides = n – 1 + 2 = n + 1

✓D✓R✓ C✓T✓R✓

4. In how many words 2 vowels won't be sitting side by side?
   Solve: No. of ways the Consonants can be arranged =
   Total no of blank spaces =
   No. of ways the vowels can be put in the blank spaces =
   Total No. of arrangements where 2 vowels won't be sitting side by side =

$^6P_3 \times \frac{\lfloor 5}{\lfloor 2}$

Type – 02: Related To Formation Of Words (A)

5. In how many words the vowels won't change their position?

Solve: Not changing the positions of the vowels D I R E C T O R can be organized in = no. of ways.

$\frac{\lfloor 5}{\lfloor 2}$

6. In how many words the vowels and the consonants won't change their relative position?

$\lfloor 3 \times \frac{\lfloor 5}{\lfloor 2}$

DIRECTOR has 8 letters.

3 Vowels (V) [I, E, O]

5 Consonants (C) [D, R, C, T, R]

2 "R"s

Solve:	D	I	R	E	C	T	O	R
	Con.	Vow.	Con.	Vow.	Con.	Con.	Vow.	Con.

No. of ways to arrange the word DIRECTOR where the vowels and the consonants won't change their relative positions are = $\frac{5!}{2!} \times 3!$

Type – 02: Related To Formation Of Words (A)

III EEE

About not Changing the Order:
If some letters are similar (or same) then it's not possible to change their order.
So the ones whose order can't be changed are to be considered same letter and the permutation is to be done. Then in the newly formed words their order can't be changed.

❖ Permutation is done taking all the letters of the word DIRECTOR. Among the arrangements -

(a) In how many words the vowels won't change their order? $\frac{8!}{2!\,3!}$

(b) In how many words "I" will come before "E"? → $\frac{8!}{2!\,2!}$

→ $\frac{8!}{3!\,2!}$

(c) In how many words "I" will come before "E" and "E" will come before "O"?

Type – 02: Related To Formation Of Words (B)

B. In how many ways Permutation can be done taking all the letters of the word **DIRECTOR** so that -

1. D always sits in the first place.
2. D never sits in the first place.
3. R always sits in the last place.
4. R always sits in the last place.
5. D always sits in the first and R always sits in the last place.
6. D sits in the first or R sits in the last place.
7. D sits in the first but R never sits in the last place.
8. The vowels always sit in the even positions.
9. The vowels always sit in odd the positions.

Type – 02: Related To Formation Of Words (B)

❖ In how many ways Permutation can be done taking all the letters of the word DIRECTOR so that -

01. D always sits in the first place. $7/\lfloor 2$

Solve:

02. D never sits in the first place. $\frac{\lfloor 8}{\lfloor 2} - \frac{\lfloor 7}{\lfloor 2}$

Solve:

03. R always sits in the last place. $\lfloor 7$

Solve:

Total Permutations =

No. of permutations where D sits in the first place =

No. of permutations where D doesn’t sit in the First place =

Type – 02: Related To Formation Of Words (B)

04. R always sits in the last place.

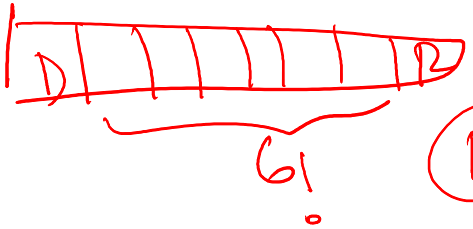
Solve: Total Permutations =

No. of permutations where R sits in the last place =

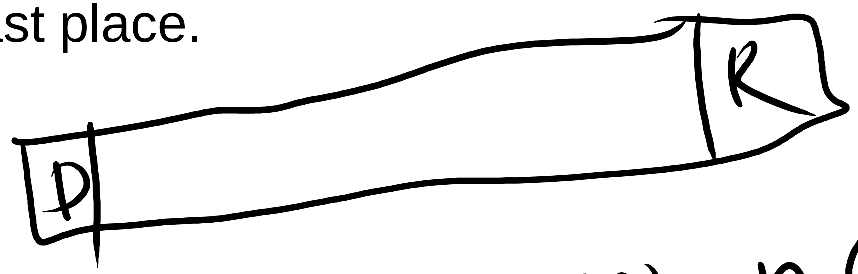
No. of permutations where R doesn’t sit in the last place =

05. D always sits in the first and R always sits in the last place.

Solve:



R → Once



06. D sits in the first or R sits in the last place.

Solve:

$$n(D \cup R) = n(D) + n(R) - n(D \cap R)$$
$$= \frac{7!}{2!} + 7! - 6!$$

Type – 02: Related To Formation Of Words (B)

07. D sits in the first but R never sits in the last place.

Solve:

08. The vowels always sit in the even positions.

Solve:

$^4P_3 \times \frac{5!}{2!}$

09. The vowels always sit in odd the positions.

Solve:

same

Poll Question–02

In how many ways Permutation can be done taking all the letters of the word DIRECTOR so that R always sits in the first and the last place?

(a) 8!/2!

(b) 6!

(c) 7!/2!

(d) $^8P_6$

Type – 03: Related To Rearrangements

Problem:

Taking all the letters from the word **DIRECTOR** in how many ways:

1. In how many ways can it be rearranged?
2. In how many ways can it be rearranged keeping I in the first place?
3. In how many ways can it be rearranged keeping R in the last place?

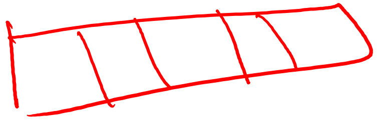
It's not obvious to subtract 1 whenever you see the word rearrangement.

Solution:

1. Total no. of arrangements taking all the letters of the word **DIRECTOR** = $\lfloor 8 / \lfloor 2$ -1
   So, No. of rearrangements =
2. No. of arrangements keeping I in the first place = $\lfloor 7 / \lfloor 2$
   So, No. of rearrangements =
3. No. of arrangements keeping R in the last place = $\lfloor 7$ -1
   So, No. of rearrangements =

Type-4: Inclusion-exclusion Of Particular Objects

❖ In how many ways can 5 objects among 10 objects be arranged where 2 Particular objects always be present?



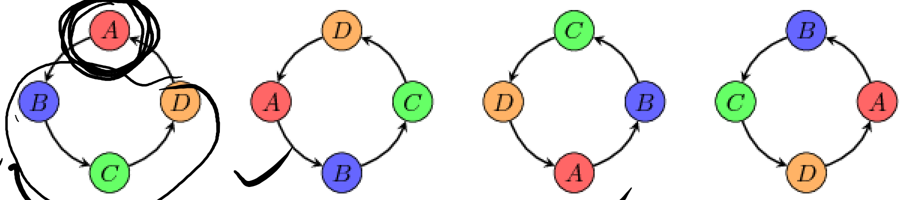
$^{8}P_{3} \times {^{5}P_{2}}$

or $^{10}C_{3} \times 5!$

Type 5: Cyclic Permutation

Permutation < Linear permutate / cyclic

{ order → sequence



C R

$4 \rightarrow 4!$

$n \rightarrow n!$

$4 \rightarrow$ 1 fixed $\rightarrow (4-1)! = 3!$

$n \rightarrow (n-1)!$

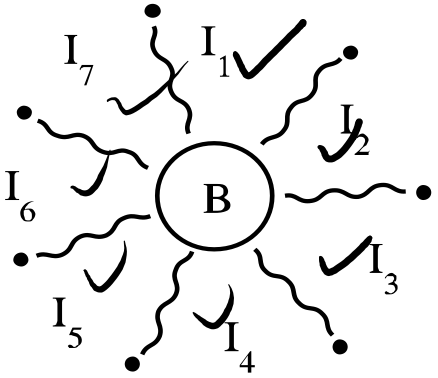
$\dfrac{(n-1)!}{2!} \rightarrow$

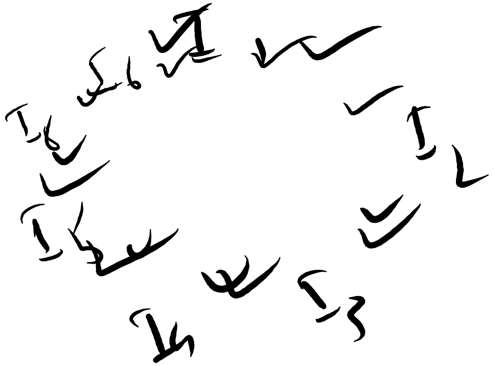
Type 5: Cyclic Permutation

$(n-1)!$

(1) 5 Bangladeshis & 7 Indians wants to seat in roundtable discussion. In how many ways the discussion can take place so that 2 Bangladeshis never sit side by side?

Sol^n:





$\rightarrow (7-1)! \times {}^7P_5$

(2) In how many ways a pearl necklace can be made using 12 different kinds of pearl?

Sol^n:

$\frac{(n-1)!}{2}$

$\frac{(12-1)!}{2}$

Type 6: Permutation With Repetition

<u>Problem:</u> In how many ways 5 letters can be posted in 5 postboxes?

post box = 5

letter = 5

5

^nP_r → only one time

n^r

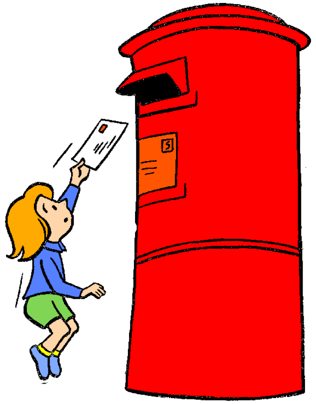
n^r

post box = 4

letter = 5

(পাত্র)

n = post box



To Remember easily: $\mathbf{n^r}$

Where, n = The object which can contain multiple things
r = The object that can move to multiple places

If r no. of items are arranged from n no. of items (n<r) where one item can be repeated limitless number of time then total no. of arrangements will become = $\mathbf{n^r}$

Type 6: Permutation With Repetition

Example: Arif got 13 rings as a gift on his wedding. In how many ways can he wear those rings in his 10 fingers? [Consider, Arif can wear many rings (even 13 rings!) in each of his fingers. Do not consider the order of multiple rings worn in one finger]

Ring = 13

finger = 10

10^{13}

Example: In the annual sports of a school, 3 different prizes will be awarded (for good behavior, improvement and sports). In how may ways those prizes can be awarded among 10 students?

prize = 3

student = 10

10^{3}

Poll Question-03

A Grameenphone number starts with 017 or 013.

How many connections can Grameenphone provide?

(a) ${}^{10}P_8$

(b) 10^8

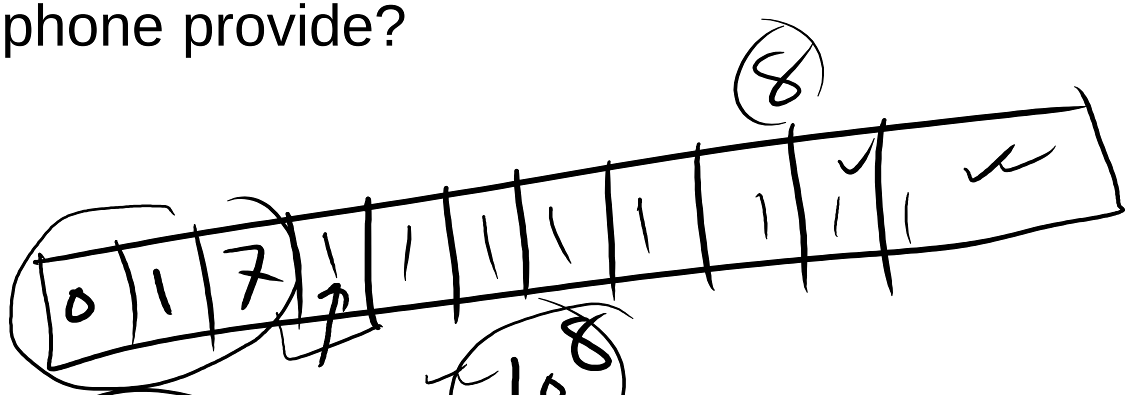
(c) ${}^{10}C_8$

(d) 2×10^8 ✓

0–9

digit = 10

ghor = 8



10^8

013

2×10^8

Type 07: Related To Formation Of Numbers

Non zero concept

n^r

Problems:

From the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using a digit i) once, ii) multiple times-

Condition/Question	i) Using once ^nP_n	ii) Using multiple times
(a) (a) How many 4 digit numbers can be formed?	$^9P_1 \times {}^9P_3$	$9^1 \times 10^3$ digit =
(b) (b) How many odd 4 digit numbers can be formed?	9 $^8P_1 \times {}^8P_2 \times {}^5P_1$	$9^1 \times 10^2 \times 5^1$
(c) (c) How many even 4 digit numbers can be formed?	a − b	(a − b)

Type – 07: Related To Formation Of Numbers

Problems: From the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using a digit i) once, ii) multiple times-   Repe

Condition/Question	i) Using once ^nP_r	ii) Using multiple times n^r
(d) How many 4 digit numbers can be formed which are divisible by 5?	$^9P_3 \times {}^1P_1 + {}^8P_1 \times {}^8P_2 \times {}^1P_1$	0/5 $9^1 \times 10^2 \times 2^1$
(e) How many numbers can be formed which are greater than 4,000 but less than 7,000?		
(f) How many numbers can be formed which are greater than 4,000 but less than 70,000?		

Concept Of Combination

01. In how many ways a group of 15 people can be made choosing from 40 people?

Case – 1: If Ashik is in the group: $^{39}C_{14}$

Case – 2: If Ashik is not in the group: $^{39}C_{15}$

Type 8: Related To The Formation Of Team/Group

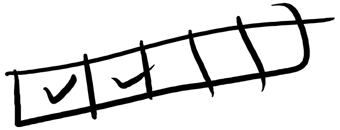
Example: In how many ways a group of 5 people (which includes at least one woman) can be formed choosing from another group of 10 people (which includes 4 women)?

women 4	other 6	
1	4	$\rightarrow {}^4C_1 \times {}^6C_4$
2	3	$\rightarrow {}^4C_2 \times {}^6C_3$
3	2	$\rightarrow {}^4C_3 \times {}^6C_2$
4	1	$\rightarrow {}^4C_4 \times {}^6C_1$
		+

Type – 9: Related To Problems With A Few Selected/ Rejected

Example: (i) By always selecting (ii) By always rejecting two boys, in how many ways can 5 boys be selected out of 12 boys?

Sol^n:



(i) Required selection number = ${}^{10}C_3$

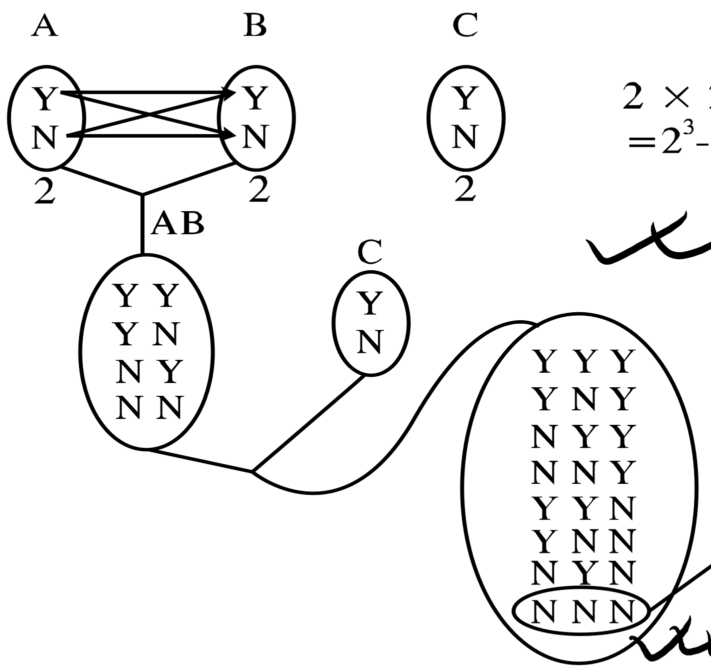
(ii) Required selection number = ${}^{10}C_5$

Type – 10: Related to the Selection of One or Multiple Objects

Say, Abul, Babul and Kabul are 3 friends of a person. In how many ways can he invite one or multiple of his friends?

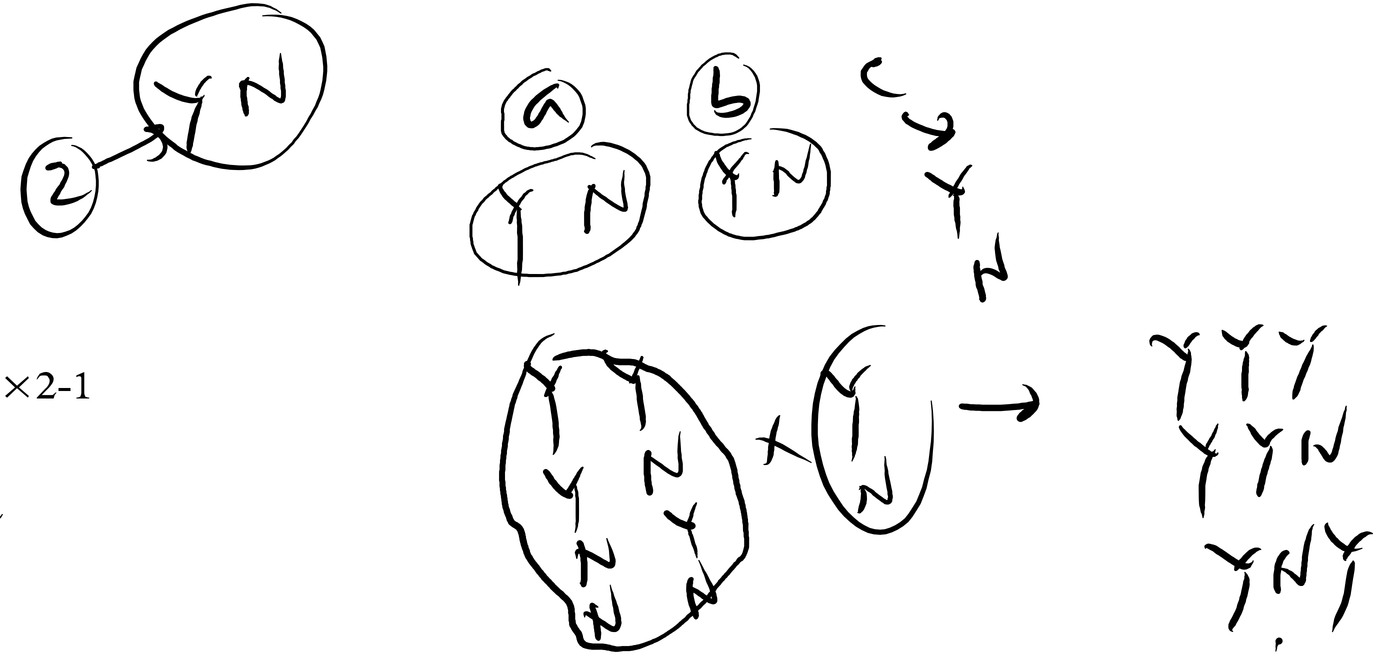
Ways to invite

A B C



$2 \times 2 \times 2 - 1$
$= 2^3 - 1$

Total events = 8
Total ways to invite = 8 – 1 = 7



$= 2^3 - 1$

$2^n - 1$

$2^5 - 1$

$2^{10} - 1$

Type – 11: Related To The Selection Of One Or Multiple Objects

01. One has 7 friends. In how many ways he can invite one or multiple friends of him?

$2^7 - 1$

02. How many ways are there to choose one or multiple questions from 8 questions (each question have an alternate)?

Options 3

Question 1 [ ] on [ ] 3 options

$3^8 - 1$ Ans

Type – 11: Related To The Selection Of One Or Multiple Objects

❖

No. of Mangos = 5
No. of Bananas = 4
No. of Apples = 3

In the case of selecting one or multiple fruits, 2 kinds of events can be formed. They are:

(i) Of Same Brand:

(ii) Of Different Brand:

M → 5   B → 4   A → 3

0, 1, 2, 3, 4, 5   ×   0, 1, 2, 3, 4   ×   0, 1, 2, 3

$6 \times 5 \times 4 - 1$

$(0, 0, 0)$

(ii) Mango

M_1 M_2 M_3 M_4 M_5

Y N   Y N   Y N   Y N   Y N

$(2^5 \times 2^4 \times 2^3) - 1$

Type – 11: Related To The Selection Of One Or Multiple Objects

Example: Find the number of factors and Real factors of 1500.

$$\begin{array}{r|l} 2 & 1500 \\ \hline 2 & 750 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$1500 = 2^2 \times 3^1 \times 5^3$

$2^0 \quad 2^1 \quad 2^2$

$3^0 \quad 3^1$

$5^0 \quad 5^1 \quad 5^2 \quad 5^3$

$3 \times 2 \times 4 = \underline{\underline{24}}$

Real factor $= 24 - 1$

****Very Important****

Concept:

01. There are 20 straight lines on a plane. None of them are parallel to each other and 3 straight lines never intersects. How many Intersection points can be found?

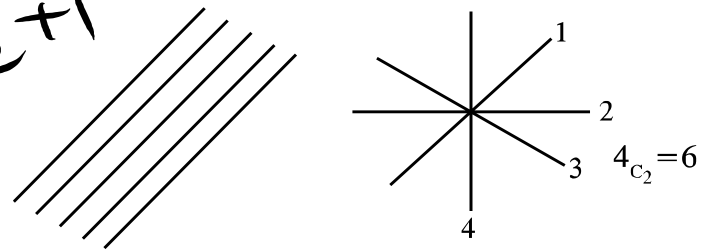
$^{20}C_2$

Type – 12: Related To Intersection Point, Straight Line, Triangle, Quadrilateral, Circle

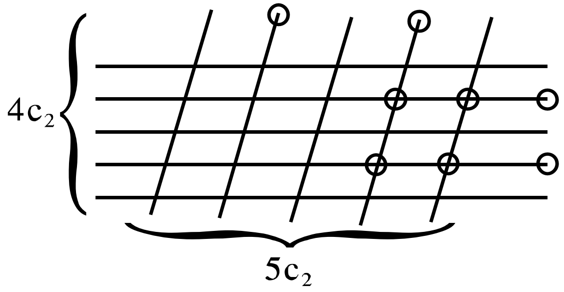
****Very Important****

02. There are 20 straight lines on a plane. 5 of them are parallel to each other and 4 of them are concurrent. How many intersection points can be found?

$^{20}C_2 - {^5C_2} - {^4C_2} + 1$



03. 4 parallel lines intersects 5 parallel lines as shown in the figure. How many parallelograms can be found?



How many parallelograms can be seen in the figure?

$\therefore$ Total no. of parallelograms $= {^4C_2} \times {^5C_2}$

****Very Important****

Note: No. of Triangles formed using 3 vertices of a polygon of n no. of sides = $^nC_3$

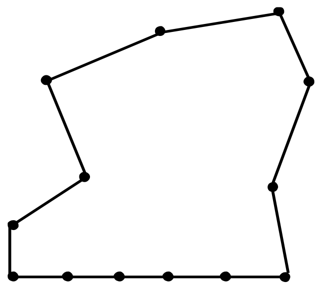
- Number of diagonals in a polygon with n number of angular points = $^nC_2 - n$
- Number of diagonals in a hexagon = $^6C_2 - 6$

[Diagonal means the lines formed connecting two vertices of a polygon minus the sides]

****Very Important****

There are 13 points on a plane. 6 of them are collinear. How many triangles and straight lines can be formed connecting those straight lines?

- No. of Triangles formed = $^{13}C_3 - {^6C_3}$
- No. of Straight Lines formed = $^{13}C_2 - {^6C_2} + 1$



Poll Question-04

Among 20 points floating in space, 7 are coplanar. No 3 points are collinear. How Many planes can be formed using these points?

(a) $^{20}C_3$

(b) $^{20}C_3 - {}^{7}C_3$

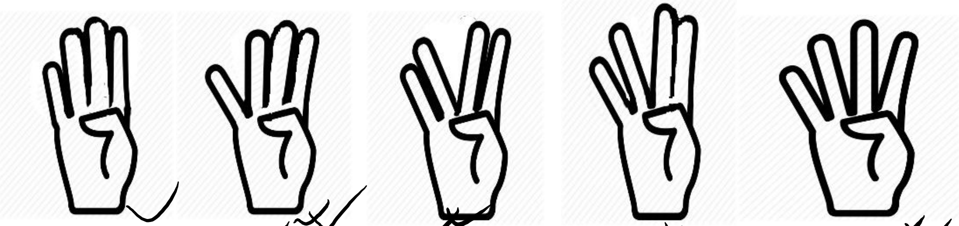
(c) $^{20}C_3 + {}^{7}C_3 - 1$

(d) $^{20}C_3 - {}^{7}C_3 + 1$

Type 13 – Related To The Combined Problems Of Permutation & Combination

Example: Find the no. of words that can be formed taking 4 letters from the word "ENGINEERING".

Ways	Examples	Number of Combinations ^nc_r	Number of Permutations $^nP_r = {}^nc_r \times \lfloor r$
(i) 4 are different	E N G I	$^5c_4$	$^5c_4 \times \lfloor 4$
(ii) 2 diff, 2 same	E E N G	$4 \times {}^4c_2$	$^4c_1 \times {}^4c_2 \times \frac{\lfloor 4}{\lfloor 2}$
(iii) 2 are same, another 2 are same	E E N N	$^4c_2$	$^4c_2 \times \frac{\lfloor 4}{\lfloor 2 \lfloor 2}$
(iv) 3 are same, (1)	E E E R	$^2c_1 \times {}^4c_1$	$^2c_1 \times {}^4c_1 \times \frac{\lfloor 4}{\lfloor 3}$



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(i) EEE, NNN

(ii) GG II

(iii) R

Find the no. of words that can be formed taking 4 letters from the words PROFESSOR, MISSISSIPPI & EXAMINATION (Practice each differently)

Type 13 – Related To The Combined Problems Of Permutation & Combination

Example: A man has one white, two green and three red flags. Find how many different signals can he make, each containing 5 and 6 flags arranged one above the other?

(1) 1st por: $\frac{6!}{2!\,3!}$

Type 14: Related To Division Into Teams/ Groups

Example: Find the no. of ways to equally divide 52 cards among 4 bridge players.

$${}^{52}C_{13} \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13}$$

$$= \frac{\lfloor 52}{\lfloor 13 \, \cancel{\lfloor 39}} \times \frac{\cancel{\lfloor 39}}{\lfloor 13 \, \cancel{\lfloor 26}} \times \frac{\cancel{\lfloor 13}}{\lfloor 13 \, \lfloor 0}$$

$$= \frac{\lfloor 52}{(\lfloor 13)^4} \quad \text{Ans}$$

Poll Question–05

How many ways are there to equally divide 52 cards?

(a) $\frac{52!}{(13!)^4}$

(b) $\frac{52!}{(13!)^4 \times 4!}$

(c) $\frac{52!}{4!}$

(d) $\frac{52!}{13! \times 4!}$

না বুঝে
মুখস্থ করার
অভ্যাস প্রতিভাকে
ধ্বংস করে