ENGINEERING ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-02

Chapter 02 : Vector

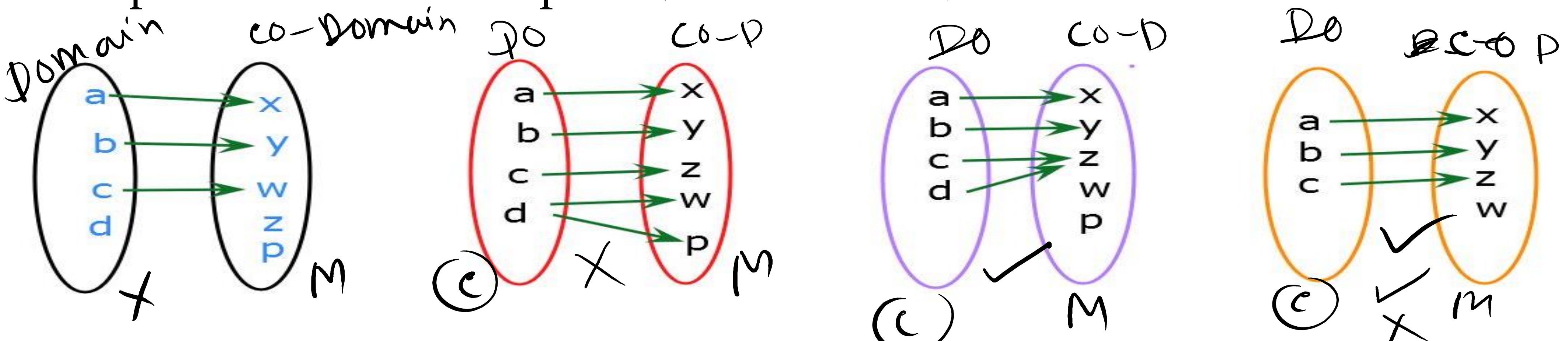
Chapter 08 : Function and Graph of Function

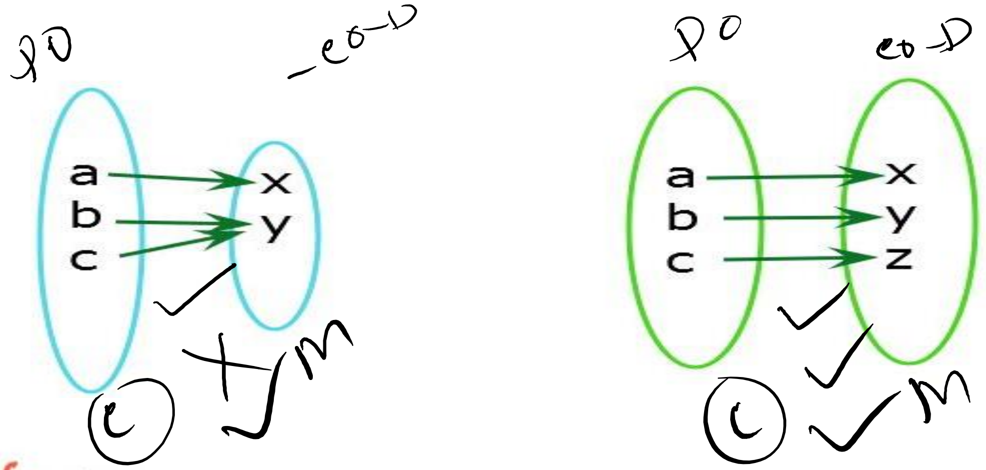
Chapter-08
Function and Graph of Function

Identify Functions from Mapping

একটি input-এর একটিই output; একাধিক output নয়

একটি output-এর একাধিক input হলেও ফাংশন হয়।





injective/one-one → X

surjective/onto : X

Domain → set of indepen-dent variable

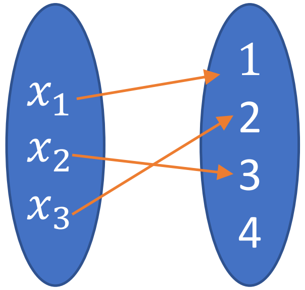
codomain → " " " " dependent variable

Bijective = one-one + onto

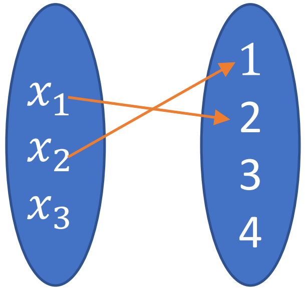
Poll Question 01

Which one is Surjective Function?

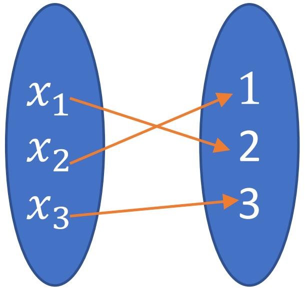
(a)



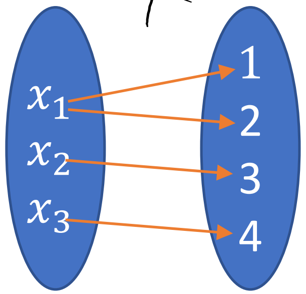
(b)



(C)

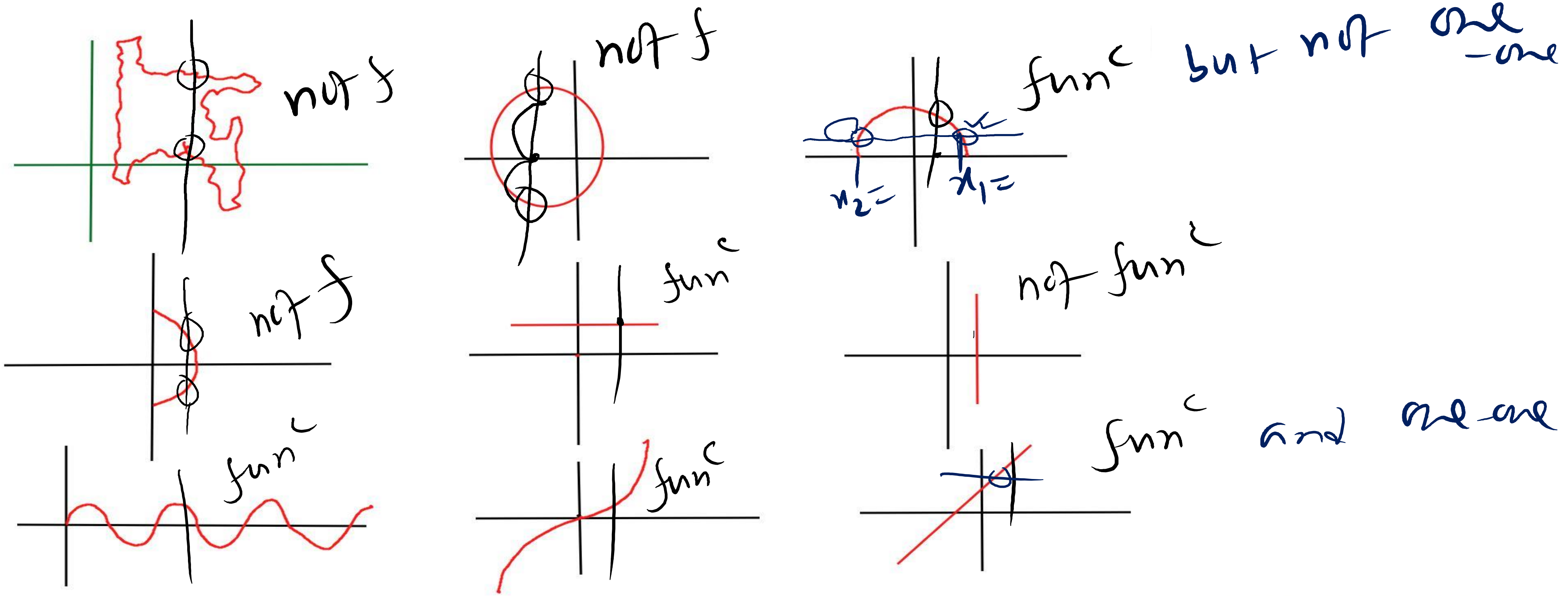


(d)



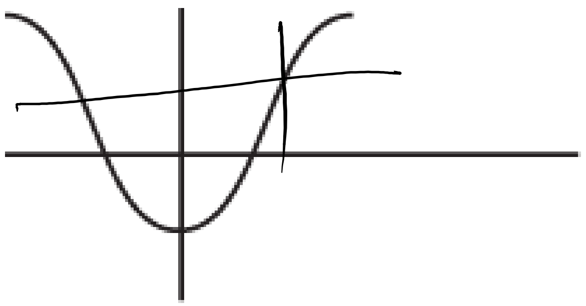
Identify Functions and One-One Function from Graph

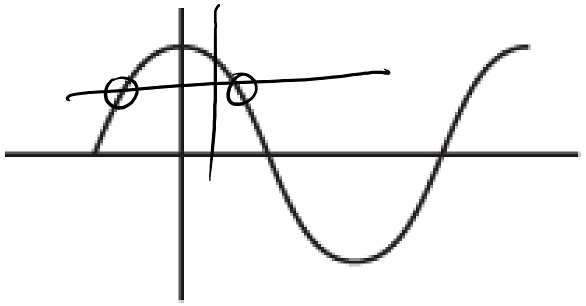
If y –axis or its' parallel line intersects the graph of a relation at one point only, then the relation is called as Function.
If x –axis or its' parallel line intersects the graph of a function at one point only, then the function is called as One-One Function.

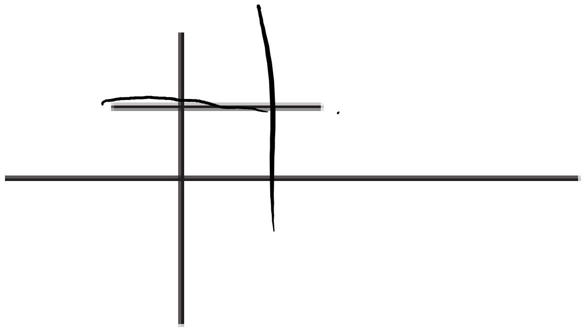


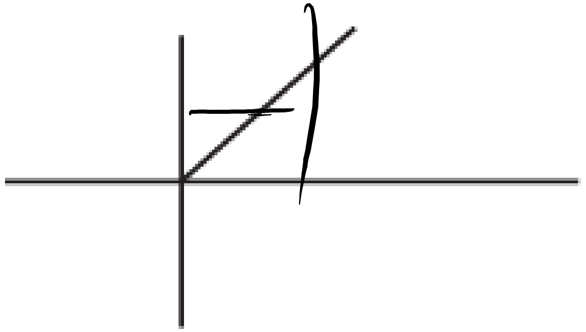
Poll Question 02

Which is One-One Function?

(a) 

(b) 

(c) 

(d) 

Determination of Domain

For, $y = f(x)$

For which set of real values of x, the values of y or $f(x)$ will be real, is called as Domain of $f(x)$.

Determination of Domain of Different Types of Functions

Domain   co domain

$f: A \longrightarrow B; f(x) =$ Any relation

$(0, 1, 2, 4/5 \ldots)$

Example: $f: R_{+} \longrightarrow R; f(x) = 2x + 1$

R_{+}

Determination of Domain of Different Types of Functions

$$f(x) = 2x + 1$$

Sol^n:

$D_f = R$

Determination of Domain of Different Types of Functions

$$f(x) = \frac{2x+1}{5x-3}$$

Sol^n:

D_f: $5x-3 \neq 0$

$5x \neq 3$

$x \neq 3/5$

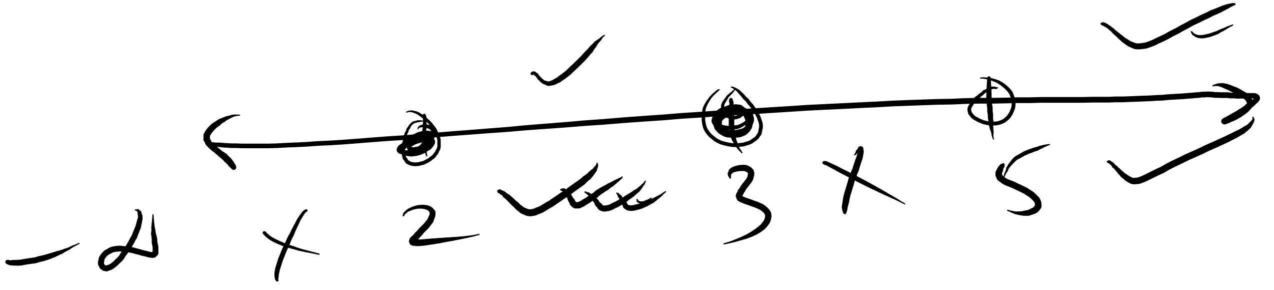
D_f: $\mathbb{R} - \{3/5\}$

Determination of Domain of Different Types of Functions

$$f(x) = \sqrt{\frac{(x-2)(x-3)}{(x-5)}}$$

$$\frac{(x-2)(x-3)}{x-5} \geq 0$$

Sol^n:



$$\boxed{[2,3] \cup (5,\infty)}$$

$2 \leq x \leq 3$ or $5 < x < \infty$

$[2,3)$

(a,b)

$(2,3) \rightarrow]2,3[$

Determination of Domain of Different Types of Functions

$$f(x) = \sqrt{4 - x^2}$$

Sol^n**:**

$4 - x^2 \geq 0$

$-(x^2 - 4) \geq 0$

$x^2 - 4 \leq 0$

$(x+2)(x-2) \leq 0$

$-\infty \quad \times \quad -2 \quad \checkmark \quad 2 \quad \times \quad +\infty$

$-2 \leq x \leq 2$

$\sqrt{2^2 - x^2}$

$-2 \leq x \leq 2$

$[-2, 2]$

$\sqrt{a^2 - x^2}$

$-a \leq x \leq a$

$[-a, a]$

$\sqrt{5^2 - x^2}$

$-5 \leq x \leq 5$

$\sqrt{b - ax^2}$

$D_f = \left[-\sqrt{\frac{b}{a}}, \sqrt{\frac{b}{a}}\right]$

$\sqrt{b - ax^2}$

$\left(-\sqrt{\frac{b}{a}}, \sqrt{b/a}\right) +, -$

$b - ax^2 = 0$

$b = ax^2$

$x^2 = b/a$

Poll Question 03

Find the Domain of $f(x) = \frac{1}{\sqrt{9-25x^2}}$

(a) $\left(-\frac{3}{5}, \frac{3}{5}\right)$

(b) $\left[-\frac{3}{5}, \frac{3}{5}\right]$

(c) $\left(-\frac{5}{3}, \frac{5}{3}\right)$

(d) $\left[-\frac{5}{3}, \frac{5}{3}\right]$

$\frac{1}{\sqrt{b-ax^2}}$

$\left(-\sqrt{b/a}, \sqrt{b/a}\right)$

$$f(x) = \sqrt{9x^2 - 25}$$

$\sqrt{ax^2 - b^2} \rightarrow D_f: (-\infty, \sqrt{\frac{b}{a}}] \cup [\sqrt{\frac{b}{a}}, \infty)$

Sol^n:

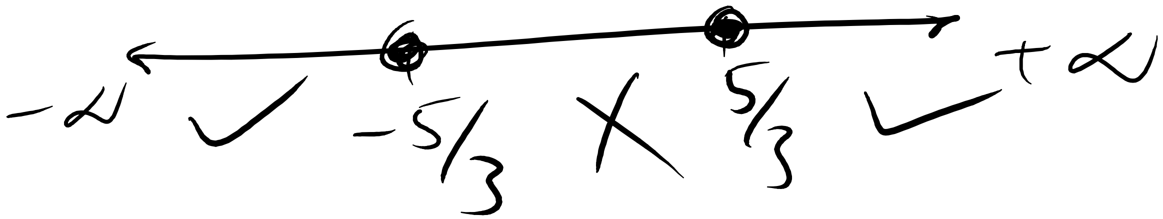
$9x^2 - 25 \geq 0$

$(3x+5)(3x-5) \geq 0$

$+ \quad + \quad +$

$- \quad -$

$3x+5=0$



$$(-\infty, -5/3] \cup [5/3, \infty)$$

Poll Question 04

Fine the Domain of $f(x) = \frac{1}{\sqrt{25x^2-16}}$

$\sqrt{ax^2-b^2}$

(a) $\left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$

(b) $\left(-\infty, -\frac{4}{5}\right) \cup \left(\frac{4}{5}, \infty\right)$

(c) $\left]-\infty, -\frac{5}{4}\right] \cup \left[\frac{5}{4}, \infty\right[$

(d) $\left]-\infty, -\frac{4}{5}\right] \cup \left[\frac{4}{5}, \infty\right[$

$\left(-\infty, -\sqrt{\frac{b}{a}}\right) \cup \left(\sqrt{\frac{b}{a}}, \infty\right)$

$\left]-\infty, \sqrt{\frac{b}{a}}\right[\cup \left]\sqrt{\frac{b}{a}}, \infty\right[$

Domain of logarithmic functions:

$\log_a x$

1. x>0
2. a>0 & $a \neq 1$

We also need to know, if, $\log_a x = p$

then, $x = a^p$

$\log_a x$ $x>0$

a base

$a \neq 0$
$a = 1$
$a < 0$

Domain of logarithmic functions:

$$f(x) = \log(9 - x^2)$$

Sol^n:

$9 - x^2 > 0$

$-(x^2 - 9) > 0$

$x^2 - 9 < 0$

$(x+3)(x-3) < 0$

$-3 < x < 3$

$(x-2)(x-5) < 0$

$2 < x < 5$

$x^2 - a^2 < 0$

$-a < x < a$

$x^2 - 5^2 < 0$

$-5 < x < 5$

$x^2 - 6^2 < 0$

$-6 < x < 6$

Domain of logarithmic functions:

$f(x) = \boxed{\log_2}(\log_3(\log_4 x)))$

$\log_a(10x+5)$

$f(x) = \log_a(\log_b(\log_c x))$

Sol^n:

$\log_3(\log_4 x) > 0$

$\log_4 x > 3^0$

$\log_4 x > 1$

$x > 4$

$\boxed{\begin{array}{l} \log_a x = 3 \\ x = a^3 \end{array}}$

$\log_b(\log_c x) > 0$

$\log_c x > b^0$

$\log_c x > 1$

$x > c$

Determination of Range

For, $y = f(x)$

For which set of real values of $y or\ f(x)$, the values of x will be real and belong to dom, f, is called as Range of $f(x)$.

Determination of Range

$$f(x) = \frac{ax+b}{cx+d}$$

Solve:

let,

$$y = \frac{ax+b}{cx+d}$$

$$cxy + dy = ax + b$$

$$cxy - ax = b - dy$$

$$x(cy - a) = b - dy$$

$$\therefore x = \frac{b-dy}{cy-a}$$

$$y = \frac{ax+b}{cx+d}$$

Range:

$$cy - a \neq 0$$

$$cy \neq a$$

$$y \neq a/c$$

$$\therefore R_f = \mathbb{R} - \{a/c\}$$

Range:

$$\mathbb{R} - \left\{\frac{a}{c}\right\}$$

$$f(x) = \frac{5x+6}{2x+4}$$

$$\mathbb{R} - \{5/2\}$$

Determination of Range

$$f(x) = \sqrt{4 - x^2}$$

Range

min^m
max^m

$\sqrt{a^2 - x^2}$

$Df: -a \le x \le a$

$\sqrt{2^2 - x^2}$

$-2 \le x \le 2$

Solve:

$\sqrt{4 - x^2}$
$= \sqrt{2^2 - x^2}$

$Df: \boxed{-2 \le x \le 2}$

x	$y = \sqrt{4 - x^2}$
-2	0 ✓
-1	$\sqrt{3}$
0	2 ✓
1	$\sqrt{3}$
2	0

Range: $[0, 2]$

$f(x) = \sqrt{a^2 - x^2}$
$[0, a]$

$f(x) = \sqrt{5^2 - x^2}$
$[0, 5]$

Poll Question 05

Find the Domain and Range of $f(x) = \frac{x}{|x|}$

$\frac{0}{0}$ ×

$\{1, -1\}$

(a) $d_f = \mathbb{R}, R_f = \mathbb{R}$

(b) $d_f = \mathbb{R}\backslash\{0\}$, $R_f = \{-1, +1\}$

(c) $d_f = \mathbb{R}_+, R_f = [-1, +1]$

(d) $d_f = \mathbb{R}_-$, $R_f = \{0\}$

$= \frac{2}{|2|} = 1$

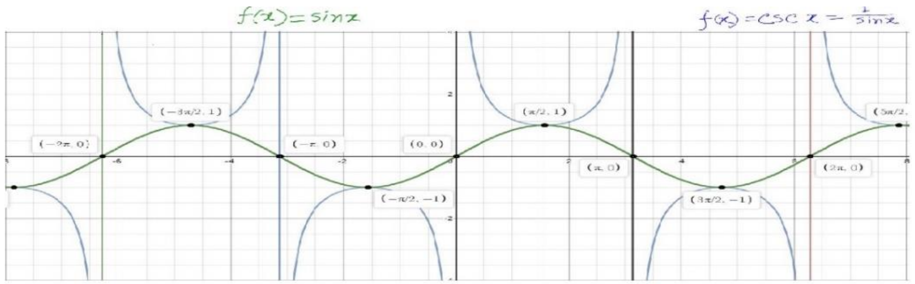
$\frac{-2}{|-2|} = \frac{-2}{2} = -1$

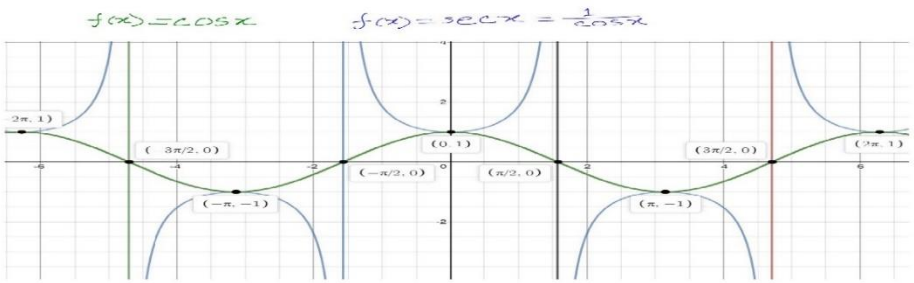
$\frac{-3}{|-3|} = \frac{-3}{3} = -1$

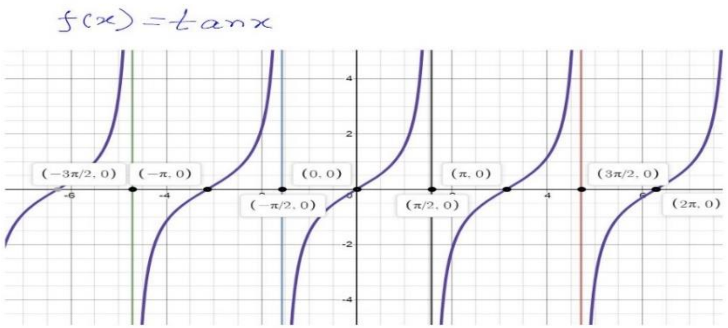
$\frac{3}{|3|} = \frac{3}{3} = 1$

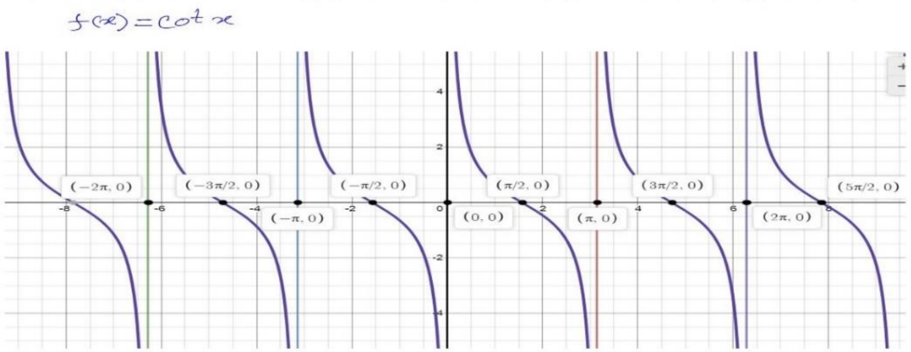
Domain & Range of Trigonometric Function:

Function	Domain	Range
$\sin^{-1}x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$
$\tan^{-1}x$	$\mathbb{R}$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
$\cot^{-1}x$	$\mathbb{R}$	$(0,\pi)$
$\operatorname{cosec}^{-1}x$	$\mathbb{R}-(-1,1)$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$
$\sec^{-1}x$	$\mathbb{R}-(-1,1)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$

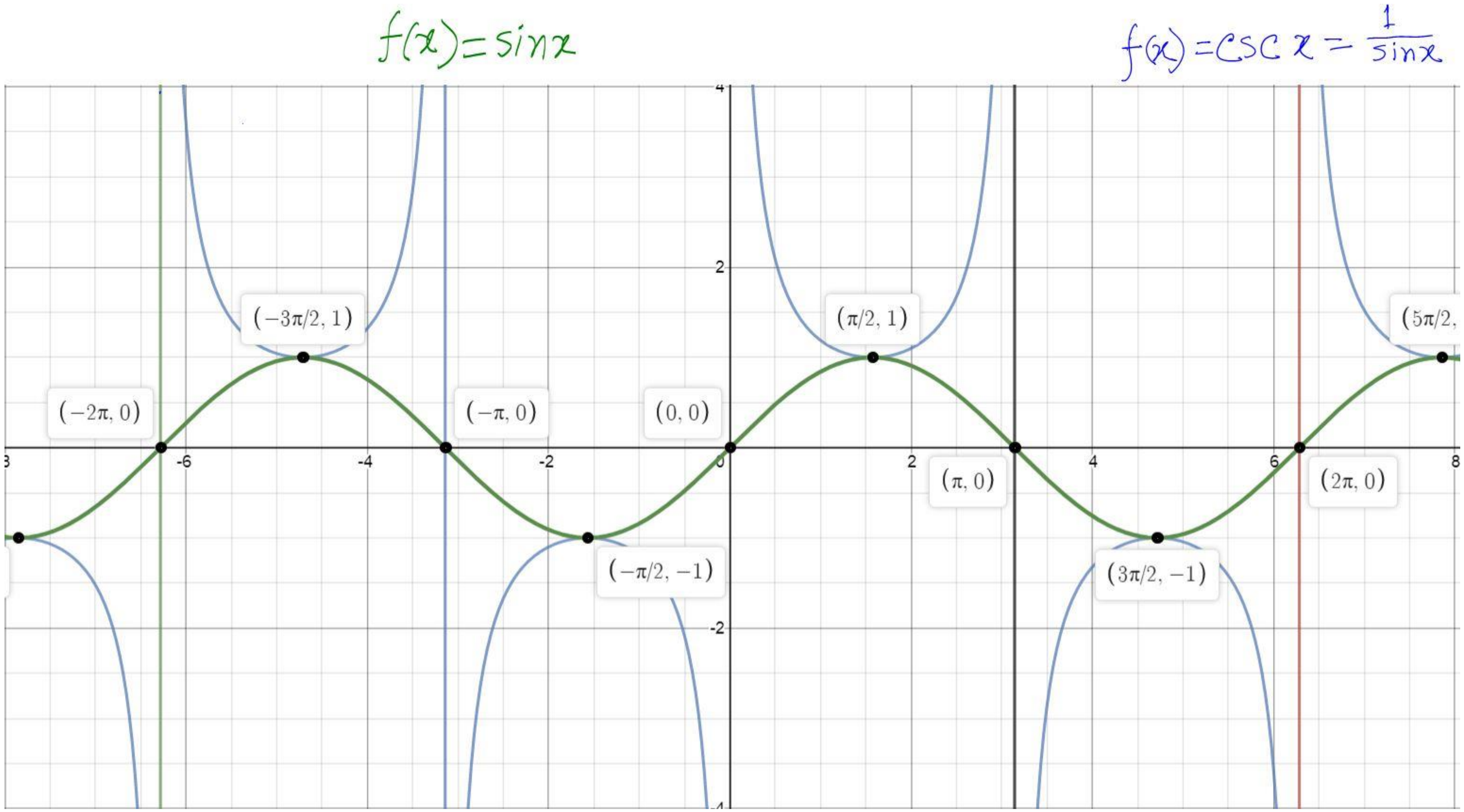






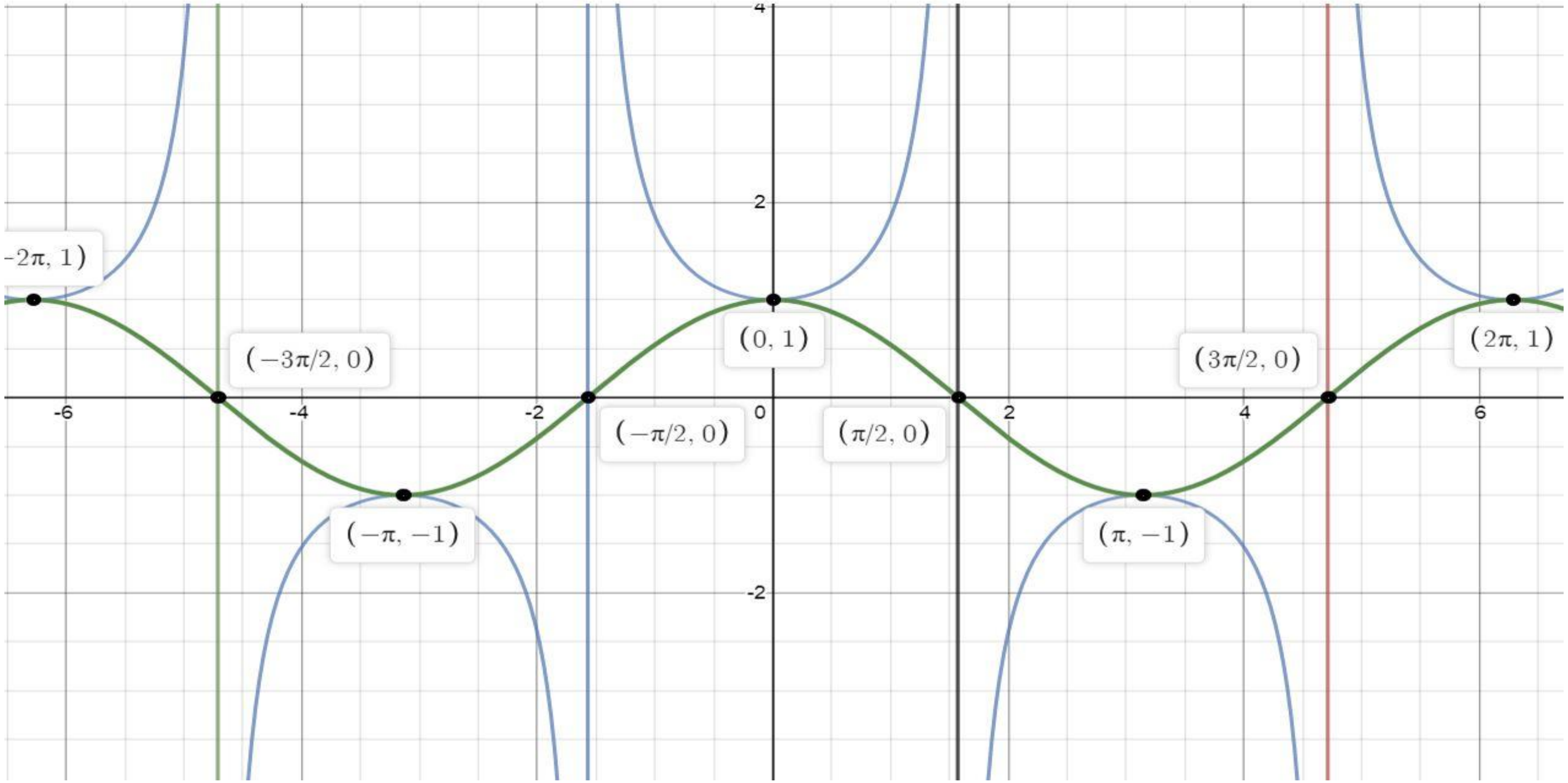


Domain & Range of Trigonometric Function:



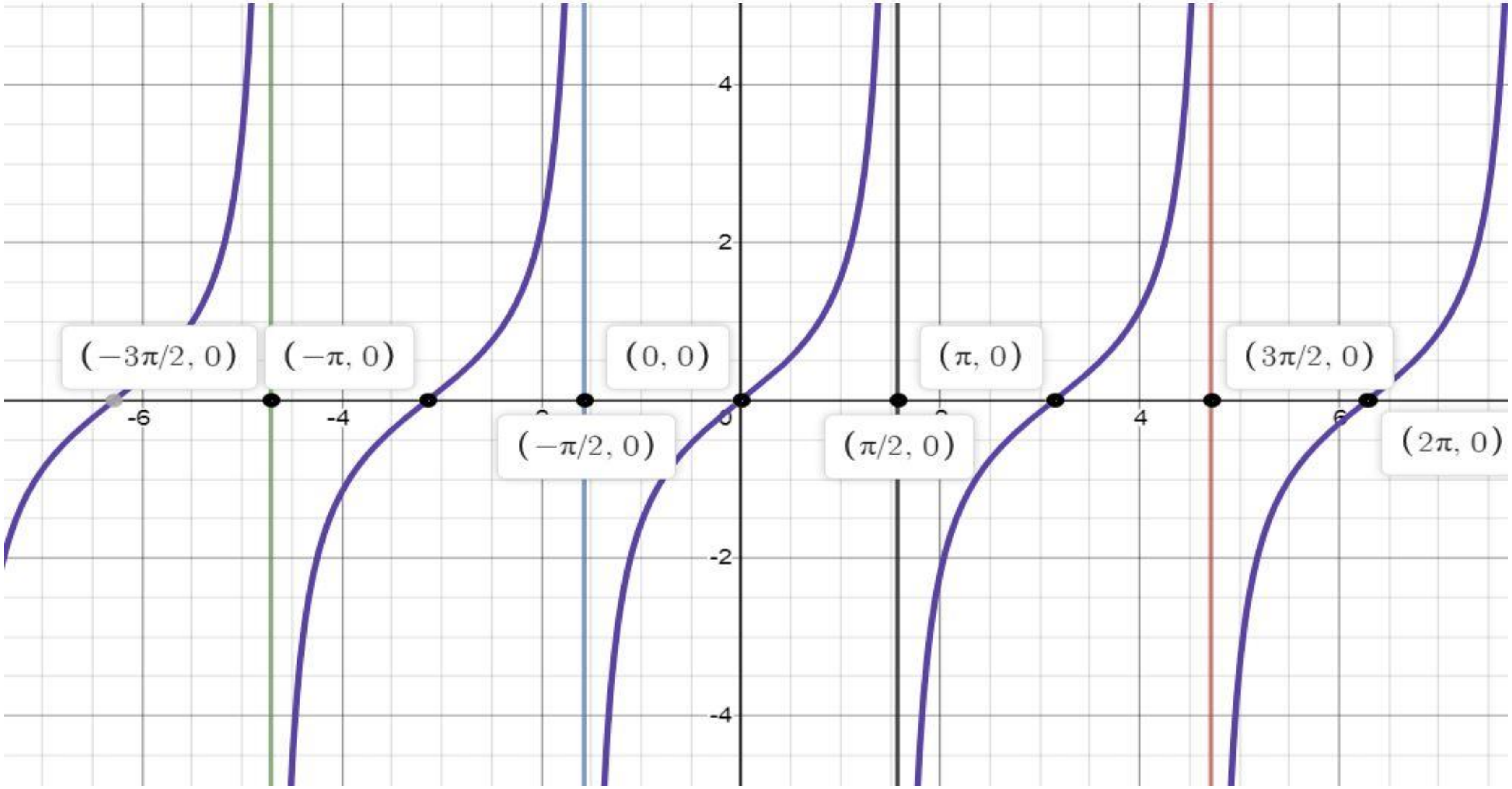
Domain & Range of Trigonometric Function:

$f(x) = \cos x$ $\qquad$ $f(x) = \sec x = \frac{1}{\cos x}$



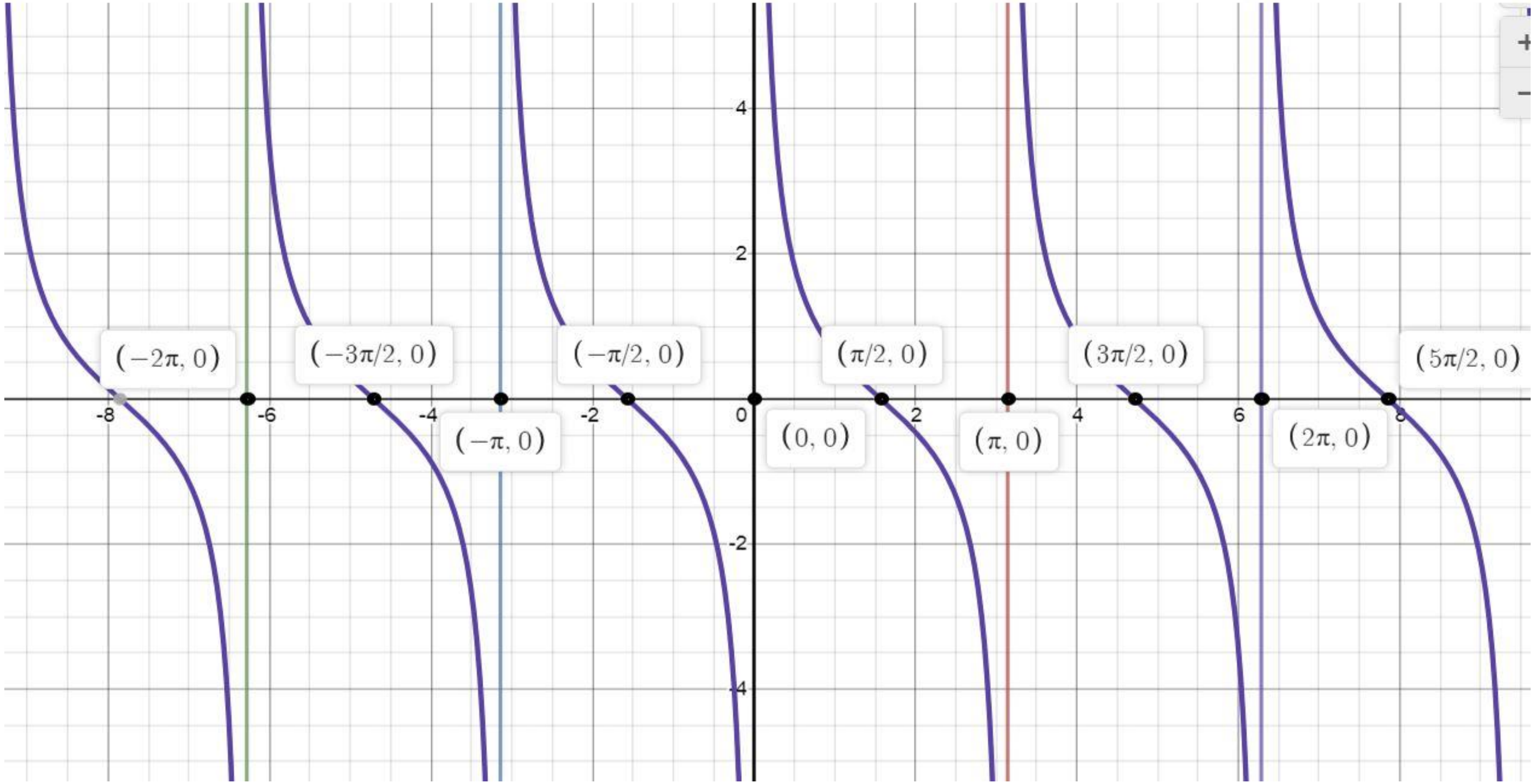
Domain & Range of Trigonometric Function:

$f(x) = \tan x$



Domain & Range of Trigonometric Function:

$f(x) = \cot x$



Domain & Range of Trigonometric Function:

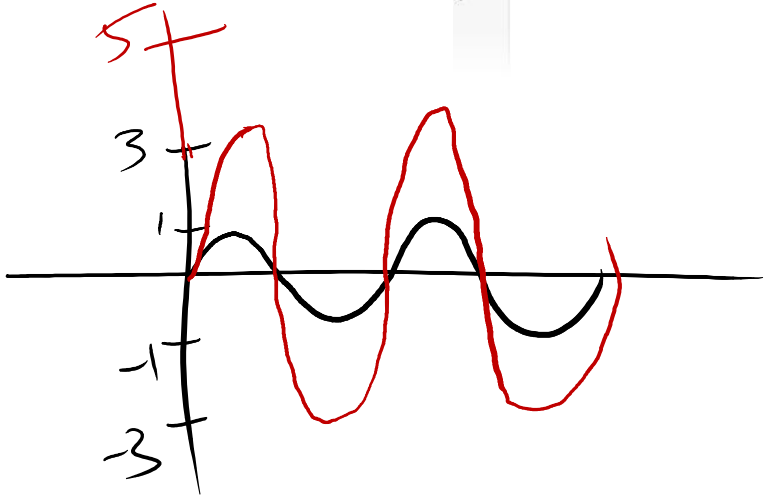
Find the domain & range of $f(x) = 2 + 3\sin x$.

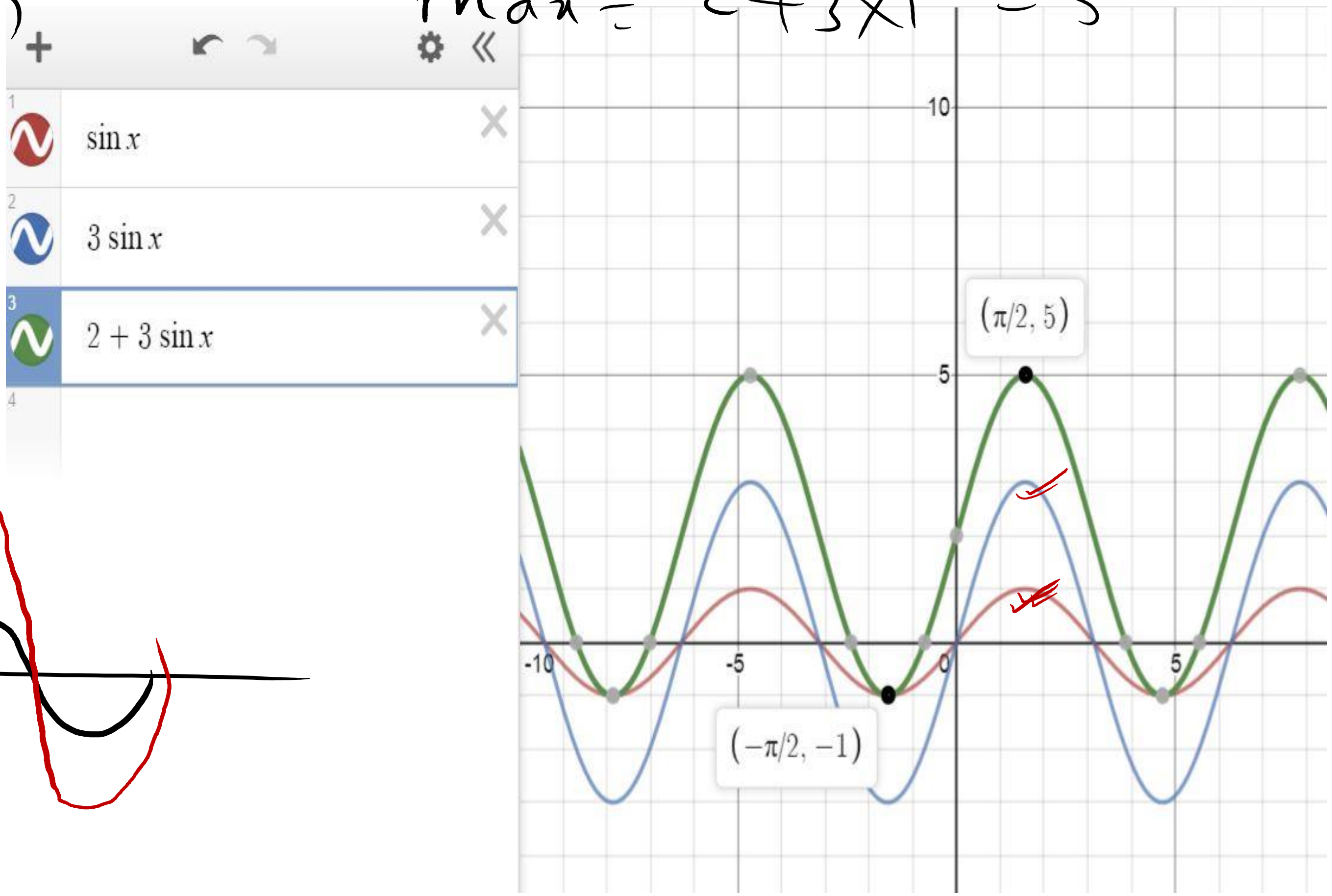
$\min = 2 + 3 \times (-1) = -1$

$\max = 2 + 3 \times 1 = 5$

$\sin x \rightarrow [-1, 1]$

$[-1, 5]$





Domain & Range of Trigonometric Function:

Find the domain & range of $f(x) = 3sinx + 4cosx + 2020.$

$$f(x) = 5\left(\frac{3}{\sqrt{3^2+4^2}}\sin x + \frac{4}{\sqrt{3^2+4^2}}\cos x\right) + 2020$$

$$= 5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) + 2020$$

$$= 5(\cos\theta \cdot \sin x + \sin\theta \cdot \cos x) + 2020$$

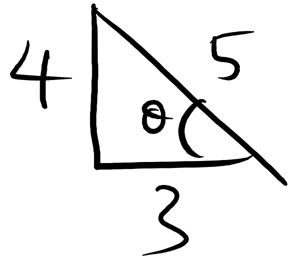
$$= 5\sin(x+\theta) + 2020$$

$$\downarrow$$

min: $5 \times (-1) + 2020 = 2015$

max: $5 \times 1 + 2020 = 2025$

$[2015, 2025]$



$a\sin x + b\cos x = c$

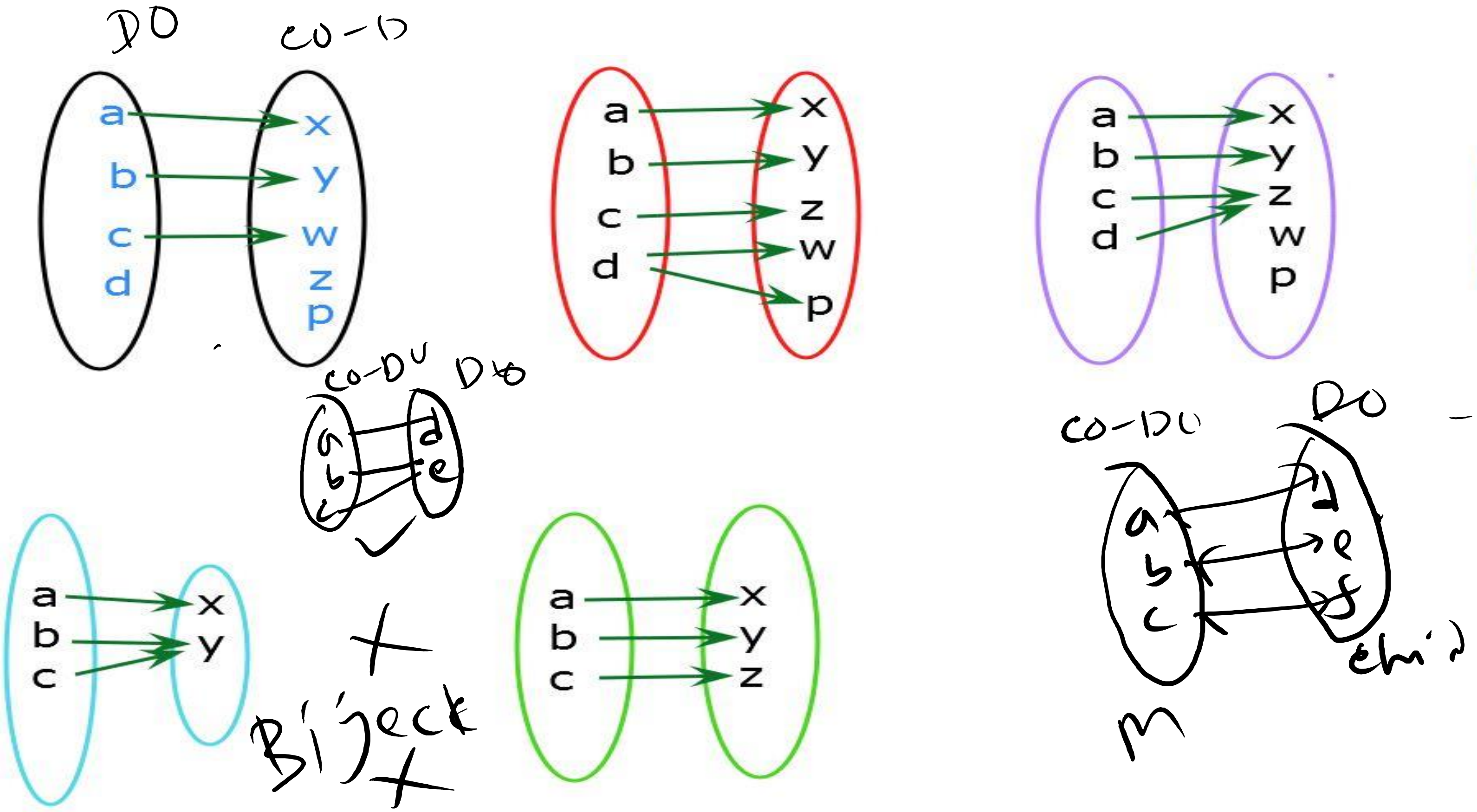
$\frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$

$-\sqrt{a^2+b^2} + c$

$-\sqrt{3^2+4^2} + 2020$

Inverse Function:

If $f: A \rightarrow B;\ f^{-1}: B \rightarrow A$ (Only Bijective Function has its’ Inverse Function)



Determination of Inverse Function

If $f: R\backslash\{1/2\} \rightarrow R$; $f(x) = \dfrac{x+3}{2x-1}$, then $f^{-1}(x) = ?$

one-one / injective

surjective / on-to

$x_1 = x_2 \rightarrow f(x_1) = f(x_2)$

$x_1 \neq x_2 \rightarrow f(x_1) \neq f(x_2)$

$x_1, x_2 \in R\backslash\{\frac{1}{2}\}$

$f(x_1) = f(x_2)$

$\vdots$

$x_1 = x_2$

$f^{-1}(x)$ doesn't exist

surjective

$y = \dfrac{x+3}{2x-1}$

$2xy - y = x+3$

$2xy - x = y+3$

$x(2y-1) = y+3$

$\therefore x = \dfrac{y+3}{2y-1}$

Range:

$2y - 1 \neq 0$

$y \neq \frac{1}{2}$

$R - \{\frac{1}{2}\}$

Range $\neq$ co-do-

Determination of Inverse Function

If $f: R\setminus\{3\} \to R\setminus\{1\}$; $f(x) = \frac{x-2}{x-3}$, then $f^{-1}(x) = ?$

A.W

$f(x) = 2x + 5$

$f(x) = y$

$\boxed{x = f^{-1}(y)}$

$y = 2x + 5$

$y - 5 = 2x$

$x = \frac{y-5}{2}$

$f^{-1}(y) = \frac{y-5}{2}$

$f^{-1}(x) =$

Poll Question 06

If $f: R \to R; f(x) = 2x + 3$ then what will be the value of $f^{-1}(x)$?

$y = 2x + 3$

$y - 3 = 2x$

$x = \frac{y-3}{2}$

(a) $\frac{x-1}{3}$

(b) $\frac{x-2}{3}$

(c) $\frac{x-3}{2}$ ✓

(d) None

Composite Function:

Composite Function:

If $f(x) = \sqrt{x-1}, (x \geq 1), g(x) = x^2 + 2,$ then find $(gof^{-1})(x) = ?$ [BUET'18-19]

$(gof^{-1})x$

$= g(f^{-1}(x))$

$= g(x^2+1)$

$= (x^2+1)^2 + 2$

$= x^4 + 2x^2 + 1 + 2$

$= x^4 + 2x^2 + 3$

$f(x) = \sqrt{x-1}$

let $y = \sqrt{x-1}$

$y^2 = x-1$

$y^2 + 1 = x$

$x = y^2 + 1$

$f^{-1}(y) = y^2 + 1$

$f^{-1}(x) = x^2 + 1$

Problem related to the function value:

If $f(x) + 3f(-x) = 2x + 3$, then $f(2) = ?$ [SUST'12-13]

$f(x) + 3f(-x) = 2x + 3 \quad \cdots (\mathrm{i})$

$x = -x$

$f(-x) + 3f(x) = -2x + 3 \quad \cdots (\mathrm{ii})$

$(\mathrm{ii}) \times 3$

$3f(x) + 9f(x) = -6x + 9 \quad \cdots (\mathrm{iii})$

$(+) \quad (-)$

$(\mathrm{i}) - (\mathrm{iii})$

$-8f(x) = 8x - 6$

$f(x) = \frac{8x - 6}{-8} = -5/4 \quad x = 2$

Some special functions:

- **Even function:** $f(x) = f(-x)$ $\quad f(x) = x^2,\ x^4$

$$f(x) = \cos x$$

$$f(-x) = \cos(-x) = \cos x = f(x)$$

- **Odd function:**

$$f(x) = -f(x)$$

$$f(x) = \sin x$$

$$\therefore f(-x) = \sin(-x) = -\sin x$$
$$= -f(x)$$

- **Identity Function:** $f(x) = x$
- **Constant function:** $f(x) = 5$

$$f(x) = c$$

Poll Question 07

Which one is even function?

(a) $f(x) = tanx$

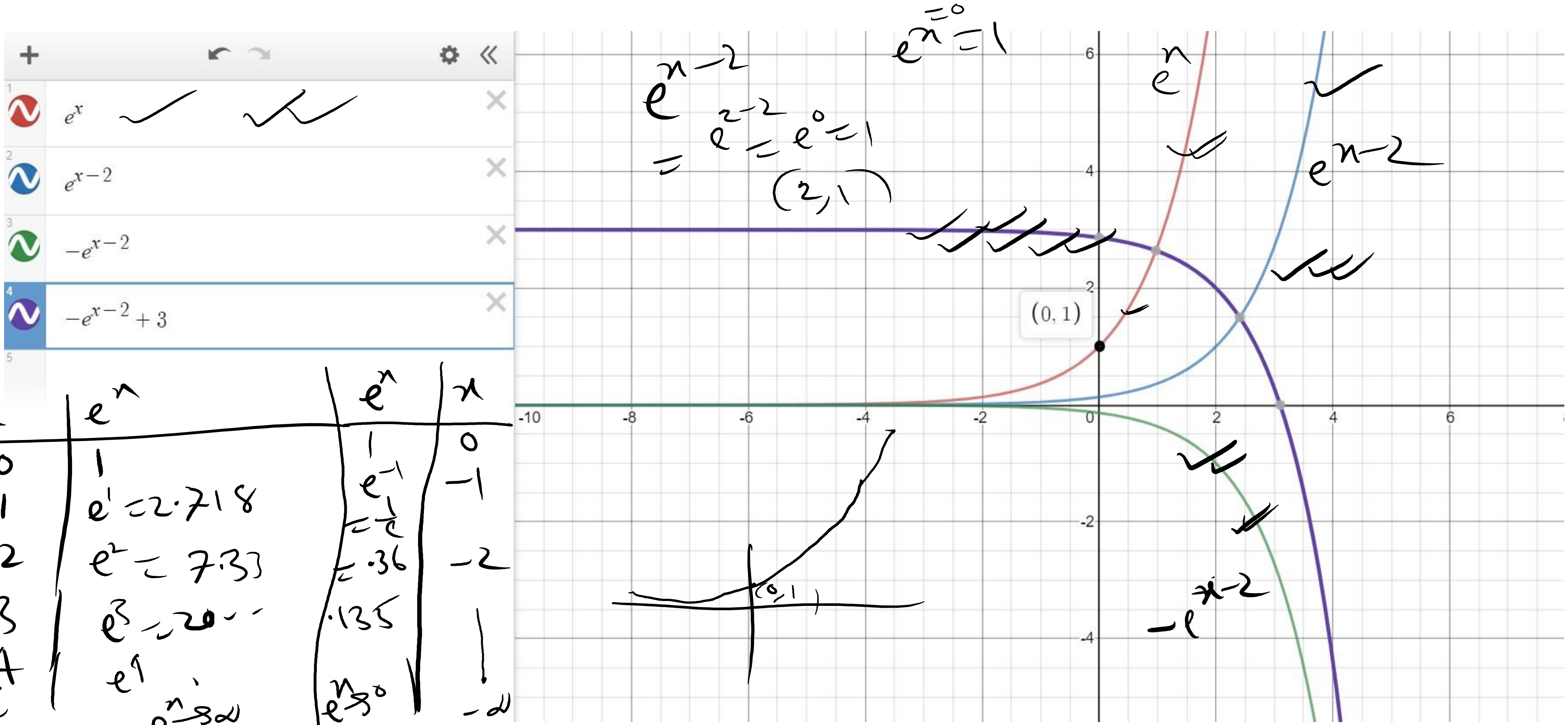
(b) $f(x) = cosx + 2$ ✓

(c) $f(x) = sinx + 2$

(d) None

Graph of Functions:

Draw $f(x) = -e^{(x-2)} + 3$, with the help of $f(x) = e^x$.



x	e^x
0	1
1	$e^1 = 2.718$
2	$e^2 = 7.33$
3	$e^3 = 20\ldots$
4	e^4

$x \to \infty \rightarrow e^x \to \infty$

x	e^x
0	1
-1	$e^{-1} = \frac{1}{e} = .36$
-2	$.135$
$-\infty$	$e^x \to 0$

Chapter-02 : Vector

Determination of magnitude and internal angle

Concept:

(i) For a vector $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$, $|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$

(ii) If the angle between two vectors $\vec{A}$ and $\vec{B}$ is θ,

then $\vec{A}.\vec{B} = AB\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{AB}$;

Determination of magnitude and internal angle

If $\vec{P} = 4\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{Q} = 4\hat{i} - 2\hat{j} - \hat{k}$ then what's the angle between $\vec{P}$ and $\vec{Q}$? [KUET'18-19]

$$\cos\theta = \frac{\bar{P}\cdot\bar{Q}}{PQ}$$

$$= \frac{4\times4 + (-2)\times(-2) + 4\times(-1)}{\sqrt{4^2+(-2)^2+4^2}\sqrt{4^2+(-2)^2+(-1)^2}}$$

$$= (\quad)$$

Poll Question 08

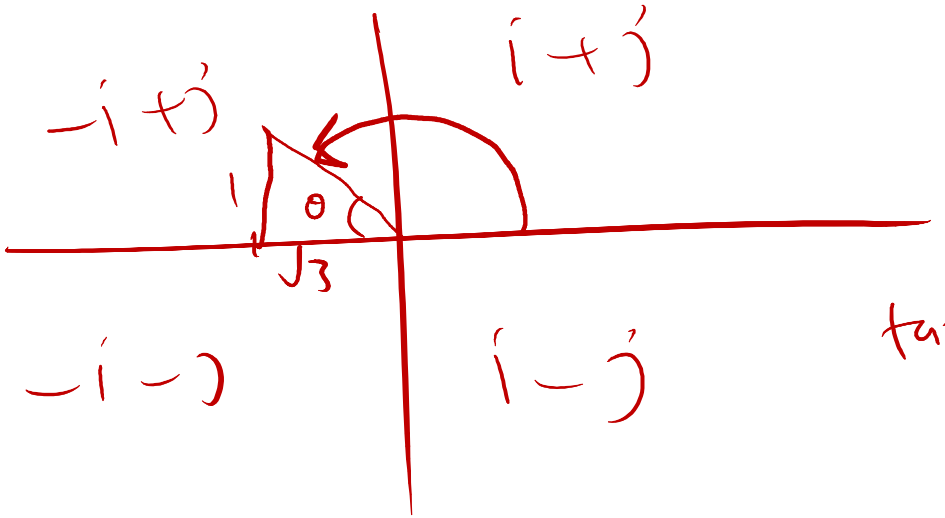
Find the angle that the vector $\overline{A} = -\sqrt{3}i + j$ makes with the positive x-axis. [RUET'18-19]

(a) $150°$

(b) $210°$

(c) Both a & b

(d) None



$\tan\theta = \frac{1}{\sqrt{3}}$

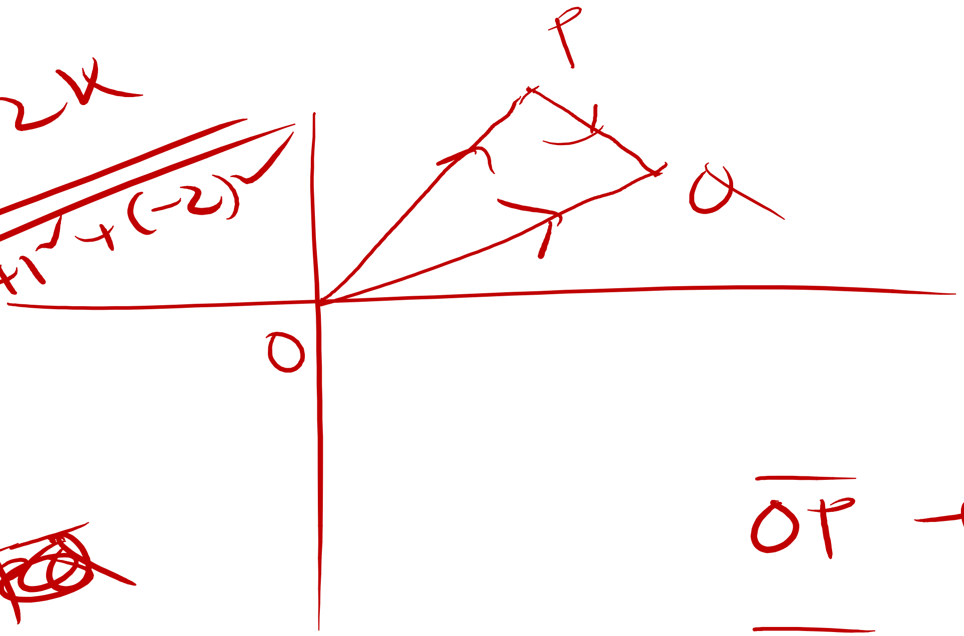
$= \tan 30°$

$\theta = 30°$

Related to unit vector

The coordinates of P and Q are respectively (1,1,1) and (3,2,-1).
Determine the unit vector parallel to $\overrightarrow{PQ}$. [BUET'03-04]

$$\frac{2i+j-2k}{\sqrt{2^2+1^2+(-2)^2}}$$



$\overline{OP} = i+j+k$

$\overline{OQ} = 3i+2j-k$

$\overline{OP} + \overline{PQ} = \overline{OQ}$

$\overline{PQ} = \overline{OQ} - \overline{OP}$

$= 2i+j-2k$

Related to perpendicular or parallel vector

Concept:

(i) Condition on two perpendicular vectors, $\vec{A}.\vec{B} = 0$

(ii) Condition on two parallel vectors, $|\vec{A} \times \vec{B}| = 0$

Shortcut for MCQ : $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$; $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ & if $\vec{A}||\vec{B}$ then $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$

Related to perpendicular or parallel vector

Show that, $\vec{A} = 8\hat{i} + \hat{j} - 6\hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} + 5\hat{k}$ are perpendicular on each other. [BUTex'10-11,07-08,03-04]

$\bar{A} \cdot \bar{B} = 0$

Poll Question 09

For which value of m, $4\hat{i} + 3\hat{j} + 5\hat{k}$ & $8\hat{i} + 6\hat{j} + \frac{m}{3}\hat{k}$ will be parallel?

(a) $\frac{10}{3}$

(b) $\frac{5}{6}$

(c) 30

(d) None

$$\frac{3}{6} = \frac{5}{m/3}$$

$$\frac{3}{6} = \frac{15}{m}$$

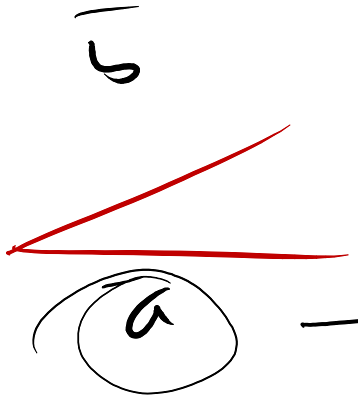
$$3m = 90$$

$$m = 30$$

Related to projection and component

Related to projection and component

If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ & $\vec{b} = 4\hat{i} + 8\hat{j} - \hat{k}$ then find the component of $\vec{b}$ on $\vec{a}$ & projection of $\vec{b}$ along $\vec{a}$. [BUET'08-09, 09-10, 10-11,12-13,13-14; KUET' 05-06,09-10; DU'16−17]



projection $b\cos\theta = \boxed{\dfrac{\bar{a}\cdot\bar{b}}{a}} \rightarrow$ Scaller

component $\left(\dfrac{\bar{a}\cdot\bar{b}}{a}\right) \times \hat{a} \rightarrow$ Vector

Related to area

Concept:

$\vec{A}$ and $\vec{B}$ are two vectors,

- If they indicate two side of a triangle then area, $\Delta = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$
- If they indicate two diagonal of a parallelogram then area, $\Delta = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$
- If they indicate two side of a parallelogram then area, $\Delta = \left|\vec{A} \times \vec{B}\right|$

Related to area

If $\vec{P} = 4\hat{i} - 4\hat{j} + \hat{k}$ & $\vec{Q} = 2\hat{i} - 2\hat{j} - \hat{k}$ is expressed as two adjacent sides of a parallelogram, then find it's area. [CUET'15-16, DU'17-18]

$$\bar{P} \times \bar{Q} = \begin{vmatrix} i & j & k \\ 4 & -4 & 1 \\ 2 & -2 & -1 \end{vmatrix}$$

$$= i(-4\times-1+2) - j(-4-2) + k(-8+8)$$

$$= (\quad)$$

$$|\bar{P} \times \bar{Q}|$$

$$= \sqrt{\quad}$$

না বুঝে
মুখস্থ করার
অভ্যাস প্রতিভাকে
ধ্বংস করে

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার