



ENGINEERING ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-03

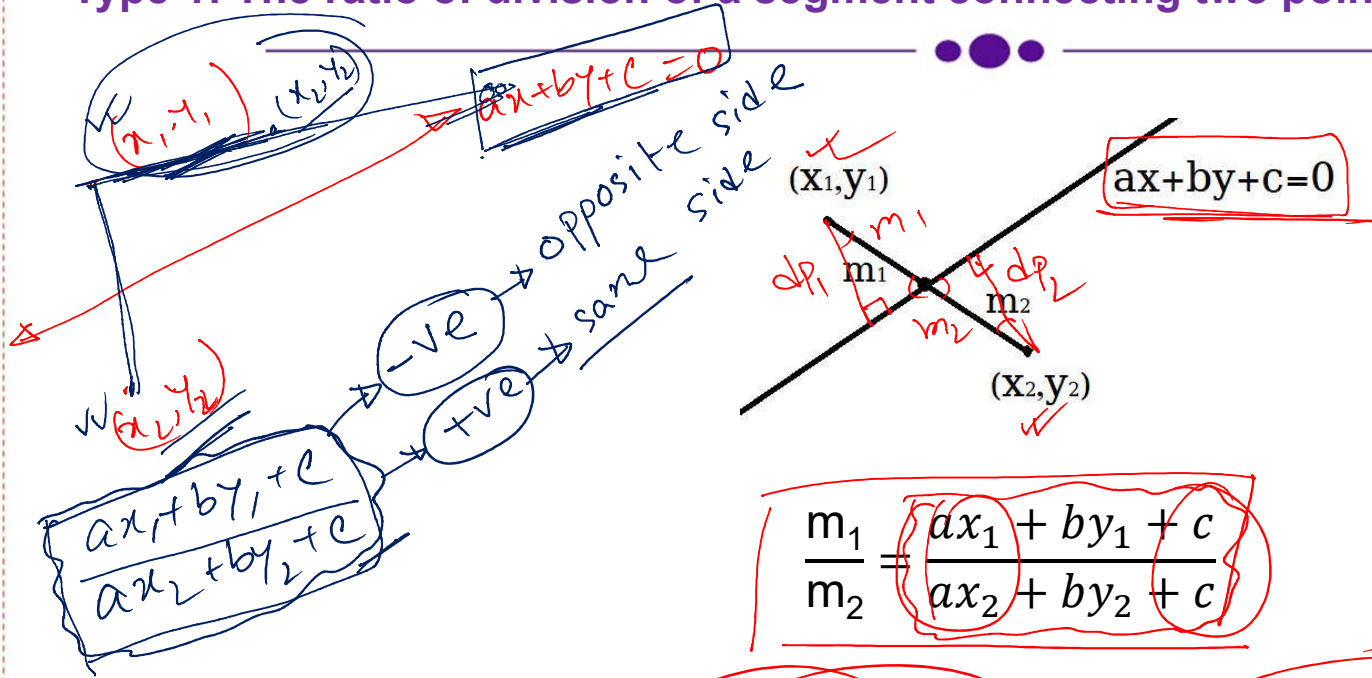
Chapter 03 : Straight line 1st Part



$$x = \sqrt{\frac{c^2}{a^2} + c} - \frac{b}{2}$$



Type 1: The ratio of division of a segment connecting two points by another straight line



$$\frac{m_1}{m_2} = \frac{dP_1}{dP_2} = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{|ax_2+by_2+c|}{\sqrt{a^2+b^2}} = \left(\frac{\pm}{\pm}\right) \frac{|ax_1+by_1+c|}{|ax_2+by_2+c|}$$

$$\frac{m_1}{m_2} = \frac{ax_1+by_1+c}{ax_2+by_2+c}$$

[- implies internal division, + implies external division]

- ☐ The ratio of division by x axis, $\frac{m_1}{m_2} = \frac{y_1}{y_2}$ [ratio of their ordinates]
- ☐ The ratio of division by y axis, $\frac{m_1}{m_2} = \frac{x_1}{x_2}$ [ratio of their abscissas]

Type 1: The ratio of division of a segment connecting two points by another straight line

Example 1: In what ratio x and y axis divide the segment connecting the points (3,2) and (5,-7)?

x-axis

$$\frac{m}{n} = \frac{2}{-7} \rightarrow \text{ve}$$

$$\therefore m:n = 2:7 \text{ (Int.)}$$

st. line

y-axis:

$$\frac{p}{q} = \frac{3}{5} \rightarrow \text{+ve}$$
$$p:q = 3:5 \text{ (Ext.)}$$

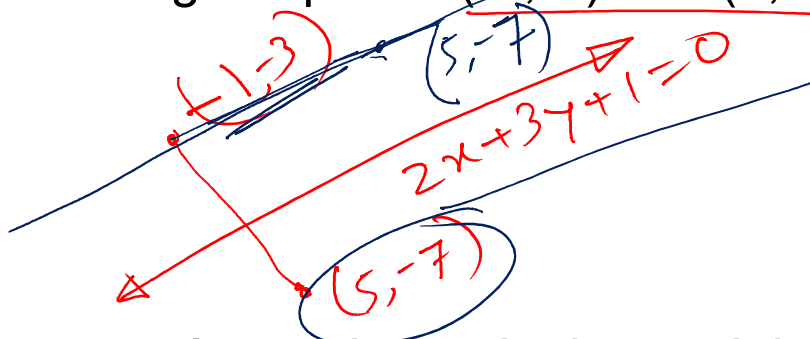
Example 2: In what ratio the straight line $2x+3y+1=0$ divides the segment connecting the points (3,2) and (5,-7)?

$$\frac{m}{n} = \frac{2(3)+3(2)+1}{2(5)+3(-7)+1} = \frac{13}{-10} \rightarrow \text{-ve}$$

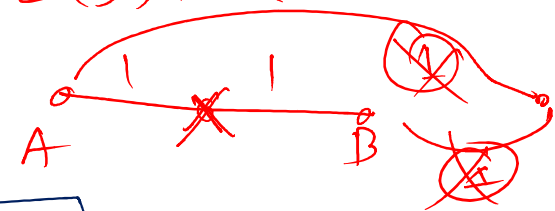
$$\therefore m:n = 13:10 \text{ (Int.)}$$

Type 1: The ratio of division of a segment connecting two points by another straight line

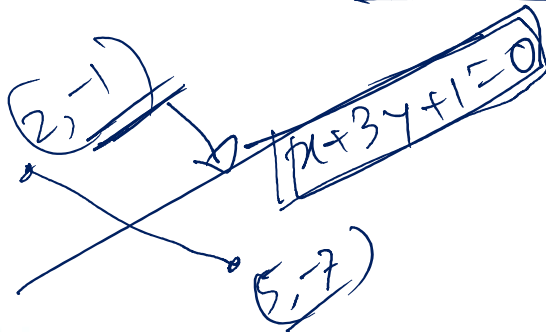
Example 3: In what ratio the straight line $2x+3y+1=0$ divides the segment connecting the points $(-1,-3)$ and $(5,-7)$?



$$\frac{m}{n} = \frac{2(-1) + 3(-3) + 1}{2(5) + 3(-7) + 1} = \frac{-10}{-10} = 1$$



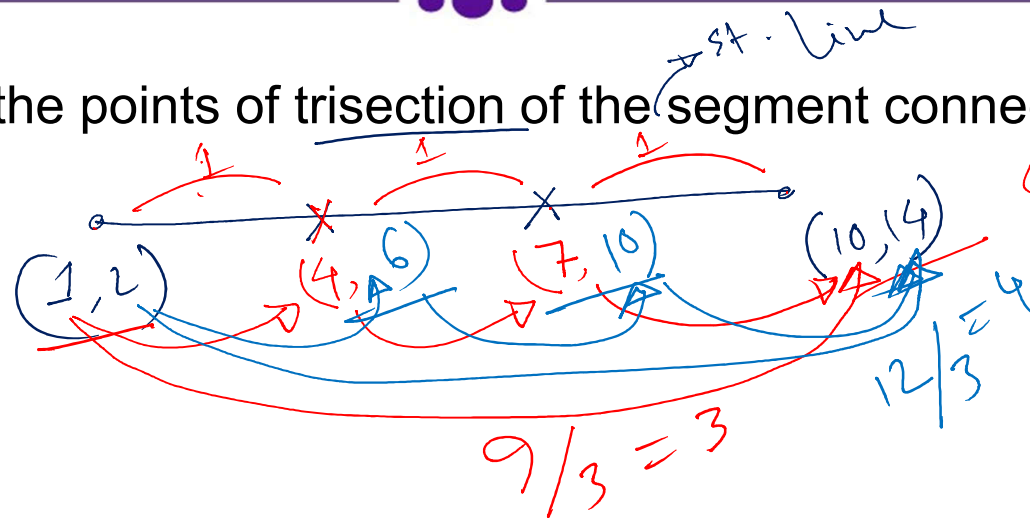
Example 4: In what ratio the straight line $x+3y+1=0$ divides the segment connecting the points $(2,-1)$ and $(5,-7)$?



$$\frac{m}{n} = \frac{2 + 3(-1) + 1}{5 + 3(-7) + 1} = \frac{0}{-15}$$

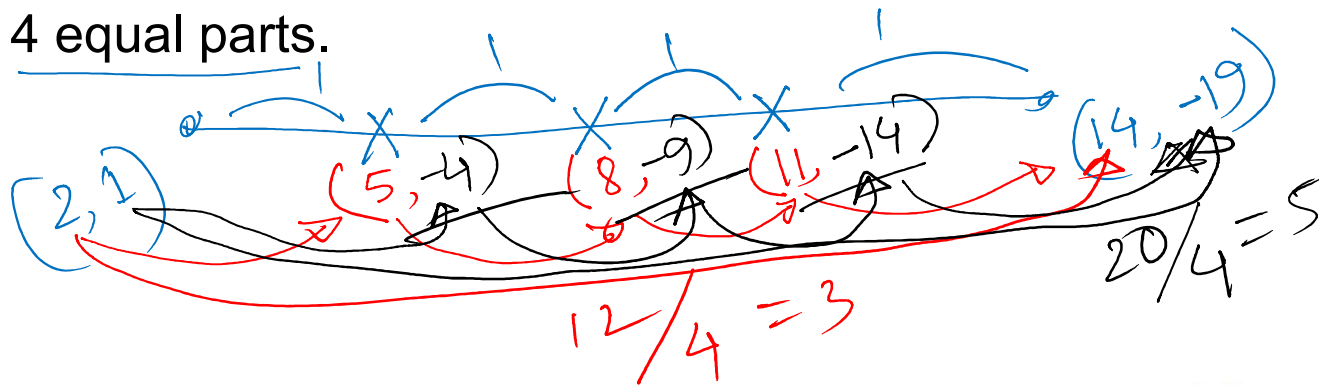
Type 2 : Determination of points of trisection and more equal sections

Example 1: Find the points of trisection of the segment connecting the points (1,2) and (10, 14).



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কয়কণে বিভক্ত
করা হলে
আমরা কতক
কাজ করতে হবে

Example 2: Find the points that divide the segment connecting the points (2,1) and (14,-19) into 4 equal parts.



Poll Question-01

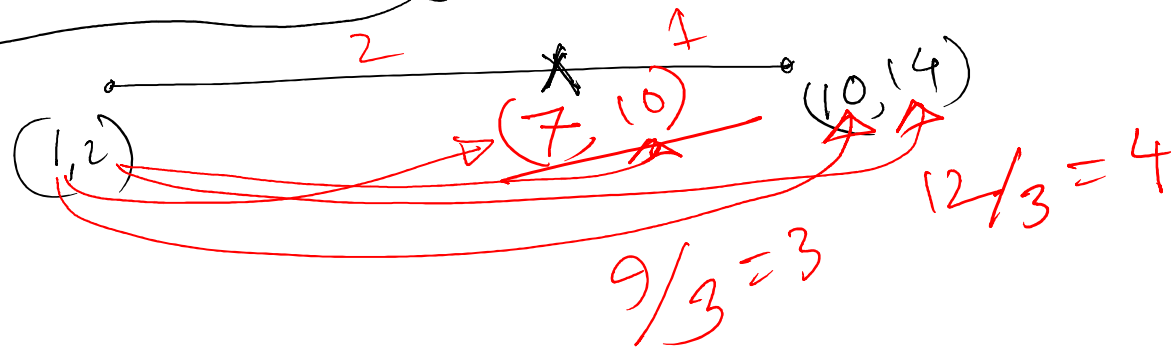
Find the coordinate of the point that internally divides the segment connecting the points $(1,2)$ and $(10,14)$ at a ratio of $2:1$?

(a) $(7,9)$

(b) $(10,7)$

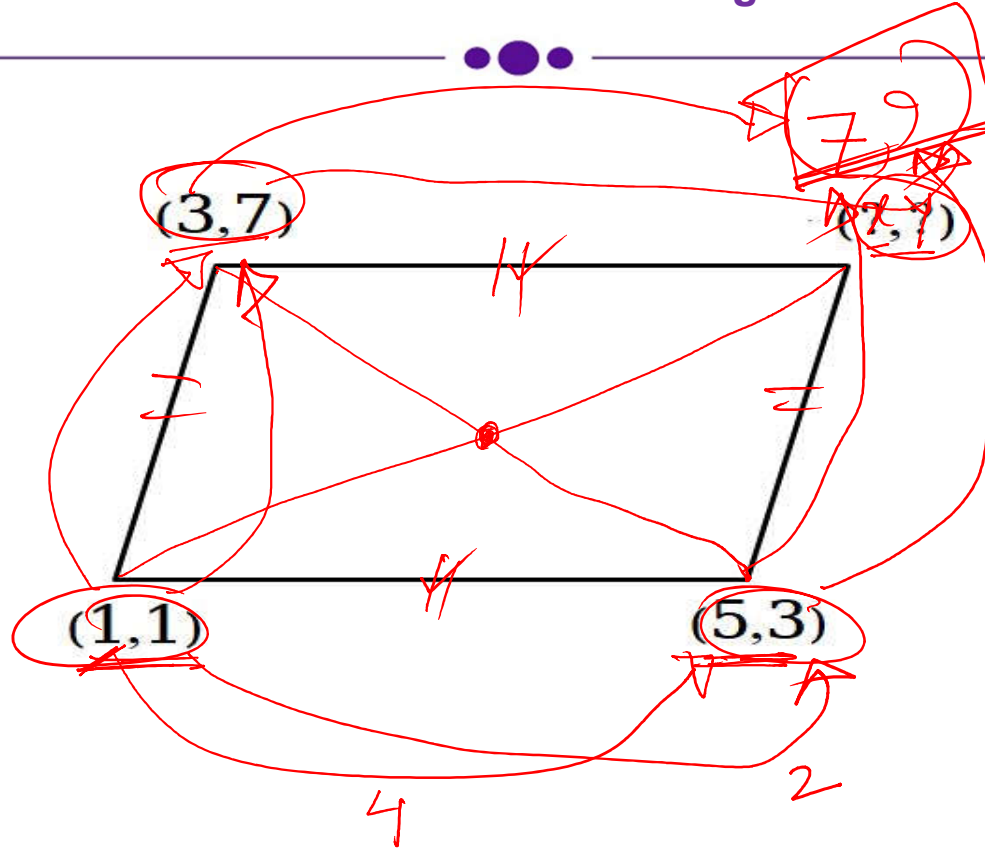
(c) $(7,10)$

(d) none



Type 3: Determination of the 4th vertex of a Parallelogram/Rectangle/Rumbas/Square

MCQ



Ans.
written!

$$\frac{x+1}{2} = \frac{3+5}{2}$$

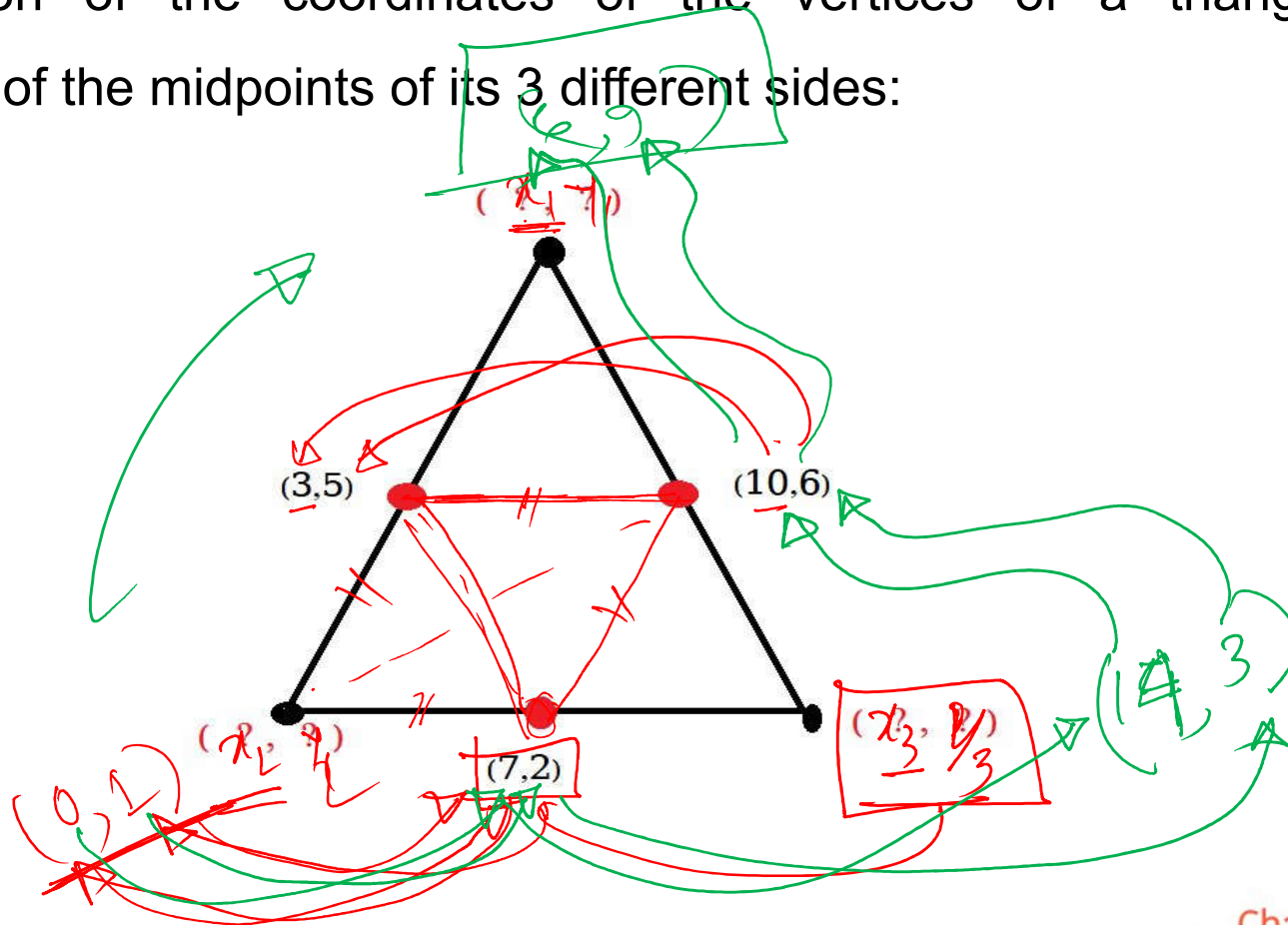
$$\Rightarrow x = 3+5-1$$

$$\frac{y+1}{2} = \frac{7+3}{2}$$

$$\Rightarrow y = 7+3-1$$

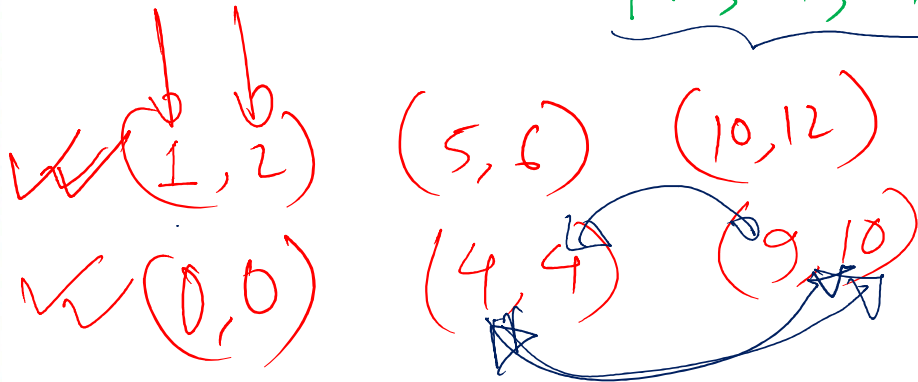
Type 3: Determination of the 4th vertex of a Parallelogram/Rectangle/Rumbas/Square

Determination of the coordinates of the vertices of a triangle using the coordinates of the midpoints of its 3 different sides:



Type - 4: Area

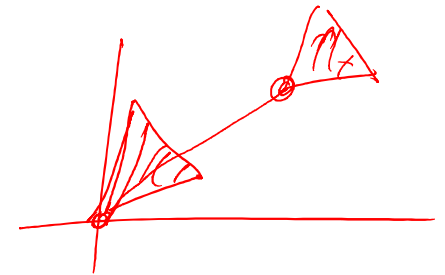
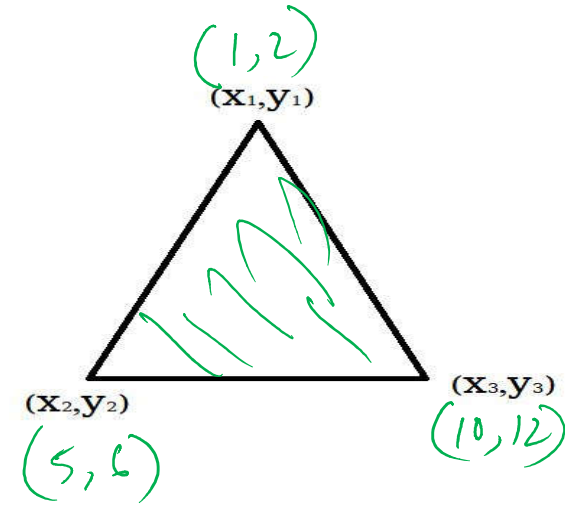
Area of Triangle = $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ sq. units



$$\frac{1}{2} \begin{vmatrix} 4 \times 10 - 4 \times 9 \\ \dots \end{vmatrix} \text{ sq. units}$$

$$= \frac{1}{2} \times 4 \text{ sq. units}$$

$$= 2 \text{ sq. units}$$



Type - 5: Locus

Example: Determine the loci of the points given below:

- i. $(a \cos \theta, a \sin \theta)$
- ii. $(a \cos \theta, b \sin \theta)$
- iii. $(a \cos \theta + c, b \sin \theta + d)$
- iv. $(3 \tan \theta, 2 \sec \theta + 1)$

v. $(\theta + \frac{1}{\theta}, \theta - \frac{1}{\theta})$

v) $x = \theta + \frac{1}{\theta}$

$y = \theta - \frac{1}{\theta}$

$x + y = 2\theta$

$x - y = \frac{2}{\theta}$

Solⁿ:

i) $\begin{cases} x = a \cos \theta \\ y = a \sin \theta \end{cases}$

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1 \Rightarrow x^2 + y^2 = a^2$
 \hookrightarrow circle

$(x+y)(x-y) = 2\theta \cdot \frac{2}{\theta}$

$\therefore x^2 - y^2 = 4$

$\Rightarrow \frac{x^2}{2^2} - \frac{y^2}{2^2} = 1$
 \hookrightarrow Rect. Hyp.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
 Hyp.

$a = b \rightarrow$ Rect. Hyp.

$e = \sqrt{2}$

$(\theta \rightarrow \text{variable})$
 (x, y) (x, θ)

Poll Question-02

What is the equation of locus of the point $(at^2, 2at)$?

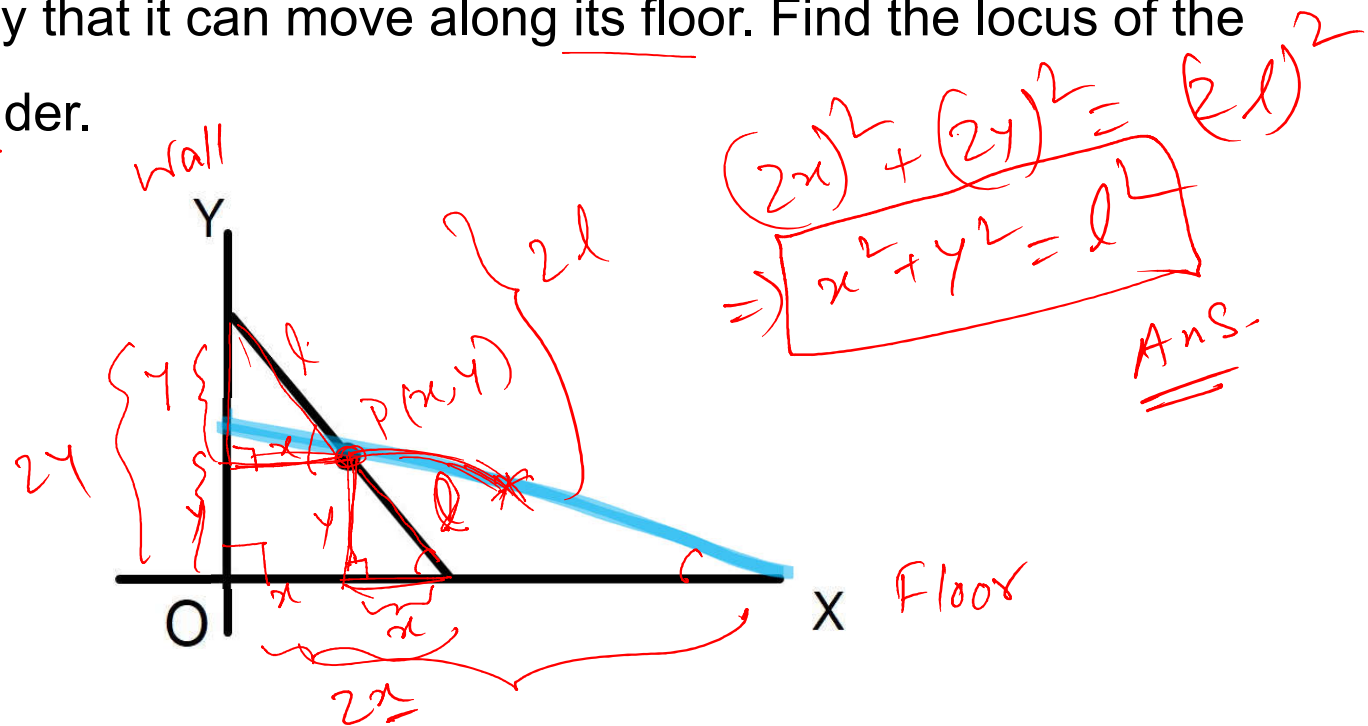
\rightarrow ($t \rightarrow$ variable)

- (a) $x^2=4ay$
- (b) $y^2=4ax$
- (c) $y^2= - 4ax$
- (d) $x^2= - 4ay$

$$\begin{aligned} & \left. \begin{aligned} x &= at^2 \\ y &= 2at \end{aligned} \right\} \\ & y^2 = 4a^2 t^2 \\ & \quad = 4a(at^2) \\ & \quad = 4ax \end{aligned}$$

Type - 5: Locus

✓ **Example 2:** A ladder of $2l$ length is placed between a wall and the floor of a room in such a way that it can move along its floor. Find the locus of the midpoint of the ladder.



Type 6: Equation of perpendicular and parallel straight line

Example: Find the equation of a straight line which is parallel to $2x+3y+5=0$ and passes through the point $(1,6)$.

$$ax+by+c=0$$

Parallel $\rightarrow ax+by+k=0$

Perpendicular $\rightarrow bx-ay+k=0$

$$2x+3y+k=0$$

$$(1,6) \rightarrow 2 \times 1 + 3 \times 6 + k = 0$$

$$\Rightarrow k = -(2 \times 1 + 3 \times 6)$$

Solⁿ

$$2x+3y - (2 \times 1 + 3 \times 6) = 0$$

$$\Rightarrow 2x+3y-20=0 \quad \text{Ans.}$$

Perpendicular

$$3x-2y - (3 \times 1 - 2 \times 6) = 0$$

$$\Rightarrow \text{H.W.} = 0$$

Poll Question-03

Find the equation of a straight line which is perpendicular to $2x+3y+5=0$ and passes through the point $(7,9)$.

(a) $3x-2y-3=0$

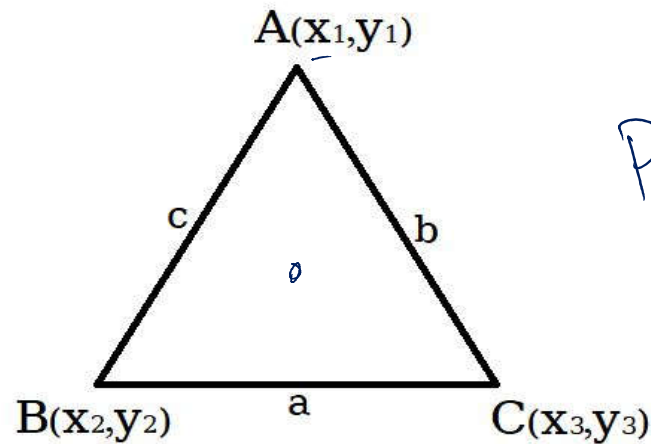
(b) $2x+3y-41=0$

(c) $3x-2y=3$

(d) Option a and c both

$$3x-2y - (3 \times 7 - 2 \times 9) = 0$$
$$\Rightarrow 3x-2y - 3 = 0$$
$$\Rightarrow 3x-2y = 3$$

Type 7: Determination of different Centers of a Triangle



$$P \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Centroid:

Example: Find the centroid of a triangle having vertices (2,2) , (6,2) and (4,4).

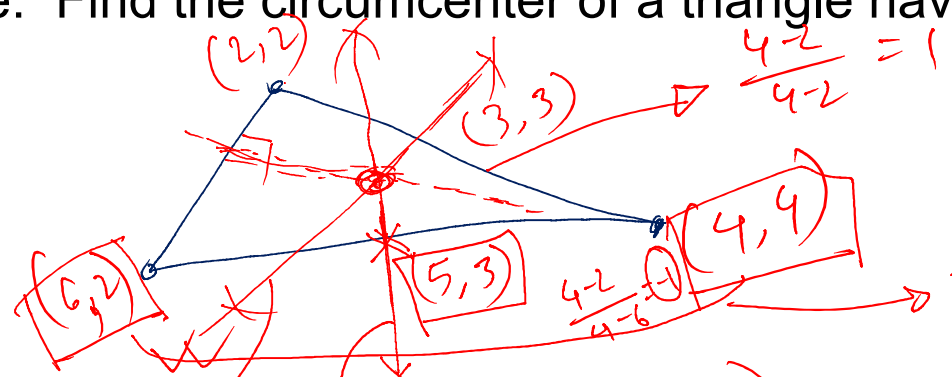
$$P \equiv \left(\frac{2+6+4}{3}, \frac{2+2+4}{3} \right)$$

$$\equiv \left(\boxed{4}, \boxed{2} \right) \quad \boxed{A.V.S}$$

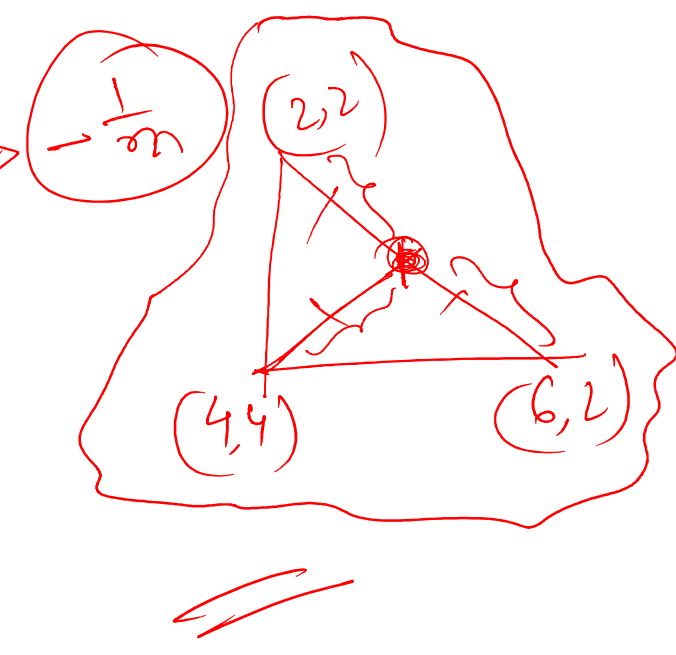
Type 7: Determination of different Centers of a Triangle of perpendicular bisectors of the sides.

Circumcenter: Intersecting of the perpendicular bisectors of the sides.

Example: Find the circumcenter of a triangle having vertices $(2,2)$, $(6,2)$ and $(4,4)$.



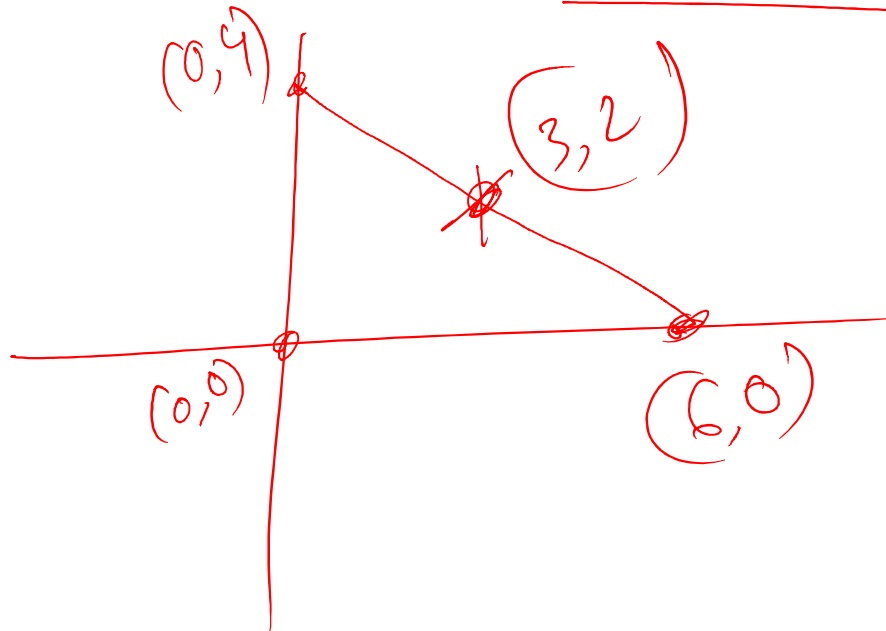
$y-3 = -1(x-3)$
 $\Rightarrow x+y-6=0$
 $y-3 = +1(x-5)$
 $\Rightarrow x-y-2=0$
 intersecting pt.
Circumcenter



Poll Question-04

What is the circumcenter of a triangle having vertices $(0,0)$, $(0,4)$, $(6,0)$?

- (a) $(3,2)$
- (b) $(2,3)$
- (c) $(0,2)$
- (d) $(3,0)$



Type 7: Determination of different Centers of a Triangle

Orthocenter:

Example: Find the orthocenter of a triangle having vertices $(2,2)$, $(6,2)$ and $(4,4)$.

S_{Δ}^m

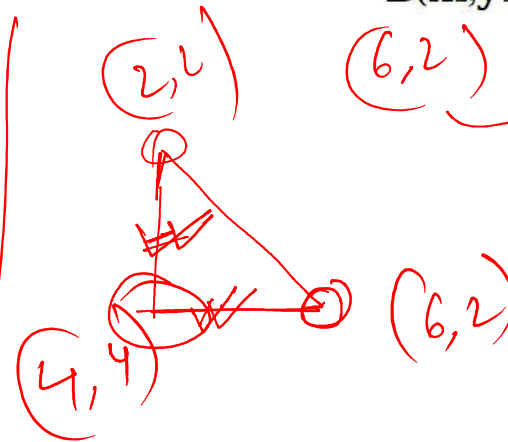
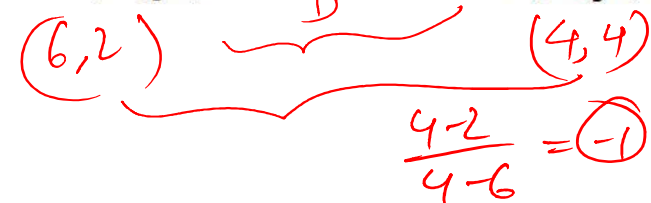
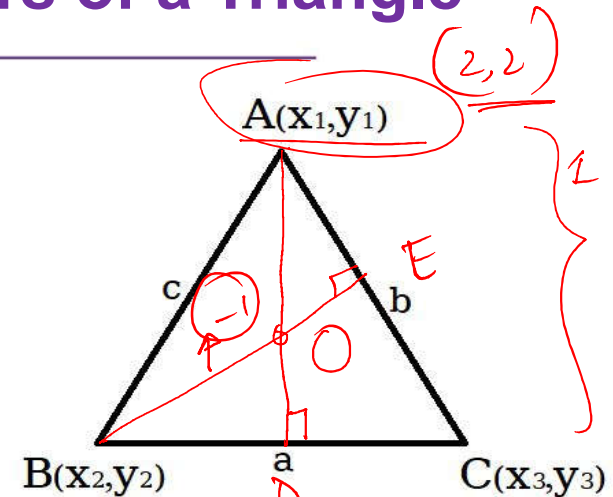
$$AD \equiv y-2 = m(x-2)$$

$$\Rightarrow x - y = 0$$

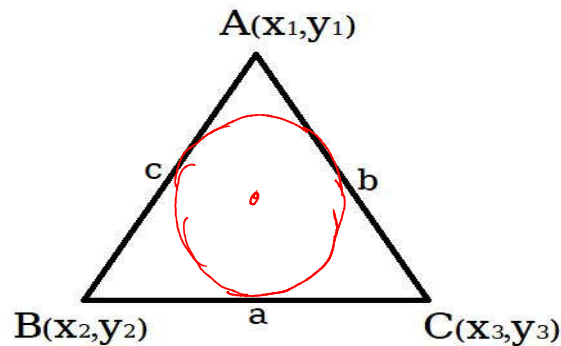
$$BE \equiv y-2 = -1(x-6)$$

$$\Rightarrow x + y - 8 = 0$$

$$O = (4,4)$$



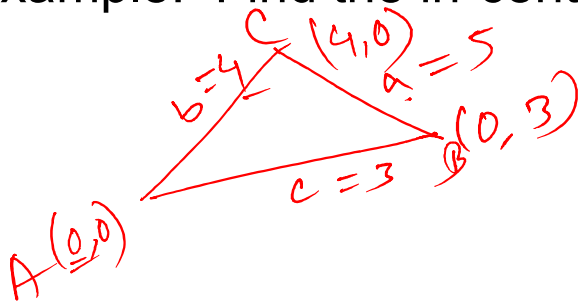
Type 7: Determination of different Centers of a Triangle



In-center:

$$I = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

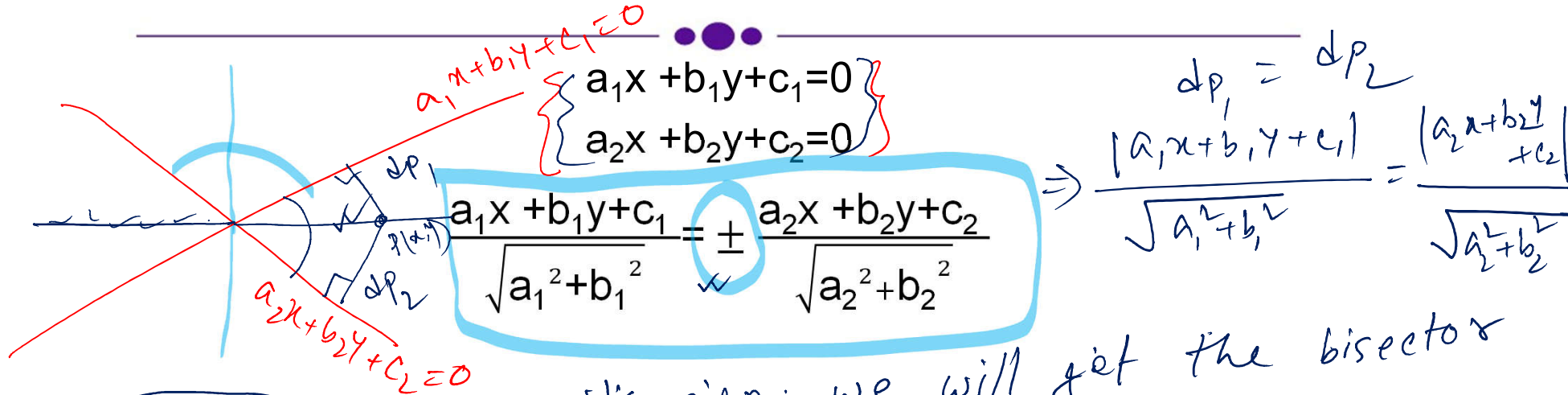
Example: Find the in-center of a triangle having vertices $(0,0)$, $(0,3)$ and $(4,0)$.



$$I = \left(\frac{5 \times 0 + 4 \times 0 + 3 \times 4}{5 + 4 + 3}, \frac{5 \times 0 + 4 \times 3 + 3 \times 0}{5 + 4 + 3} \right)$$

$$= (H.W) \quad \underline{\underline{Ans}}$$

Type 8: Equations of the angular bisectors of two intersecting straight lines



- i) $(a_1, a_2 + b_1, b_2)$ → using its sign; we will get the bisector of obtuse angle.
 → using its opposite sign, we will get the bisector of acute angle.
- ii) $a_1, a_2 + b_1, b_2 = 0$ → Given lines are perpendicular.
- iii) (c_1, c_2) → using its sign, we will get the bisector of origin containing angle.

Type 8: Equations of the angular bisectors of two intersecting straight lines

Example : Find the equations of the angular bisectors of the acute and obtuse angles between the straight lines $2x+3y-1=0$ and $x-2y+3=0$, which one of them is the bisector of the angle that contains the origin?

acute:

$$\frac{2x+3y-1}{\sqrt{2^2+3^2}} = \boxed{+}$$

$$\frac{x-2y+3}{\sqrt{1^2+(-2)^2}}$$

$$\frac{x-2y+3}{\sqrt{1^2+(-2)^2}}$$

contains origin

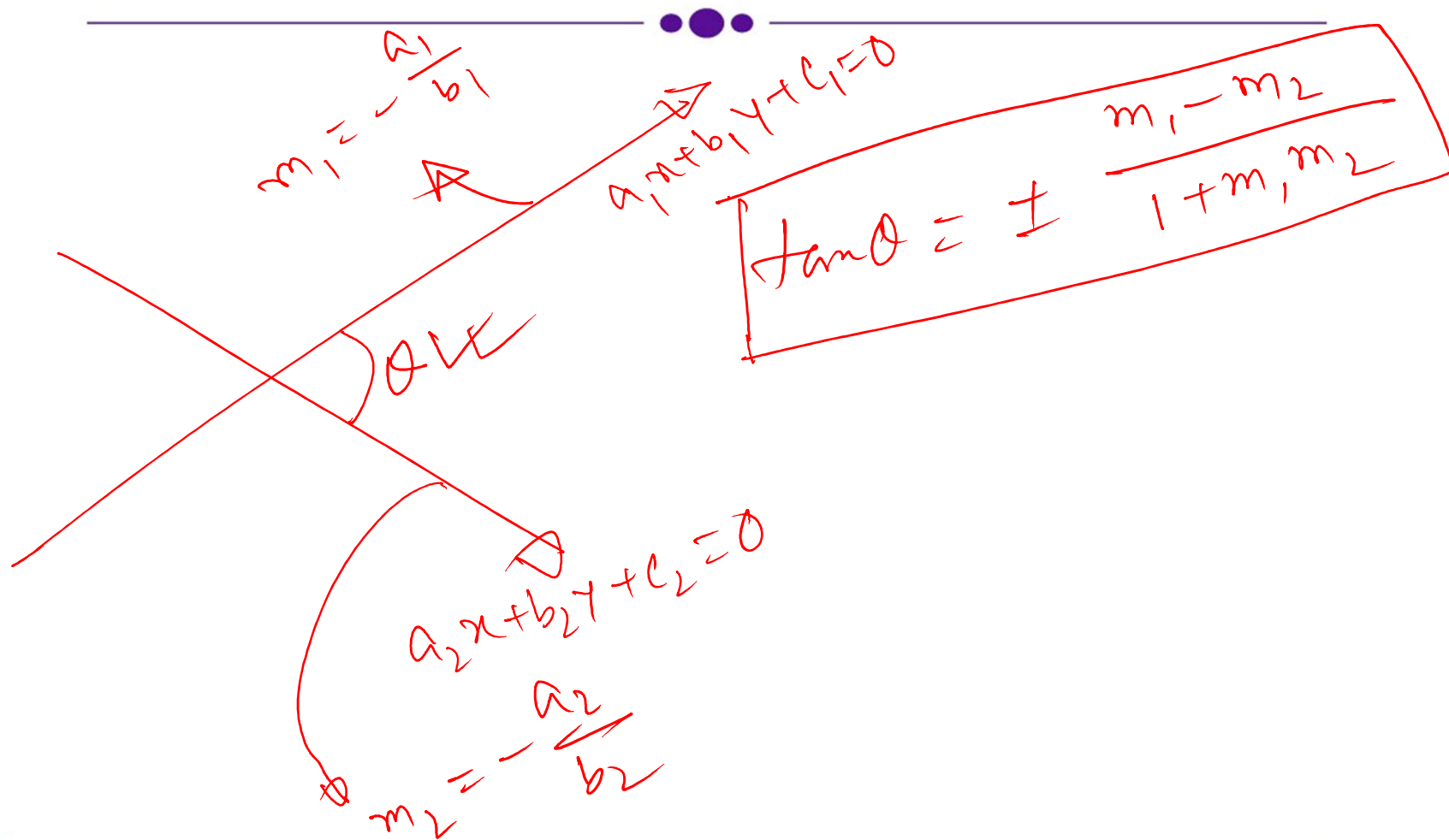
obtuse:

$$\frac{2x+3y-1}{\sqrt{2^2+3^2}} = \boxed{-}$$

$$\begin{aligned} a_1 a_2 + b_1 b_2 &= 2 \times 1 + 3 \times (-2) \\ &= 2 - 6 \\ &= -4 \rightarrow -ve \end{aligned}$$

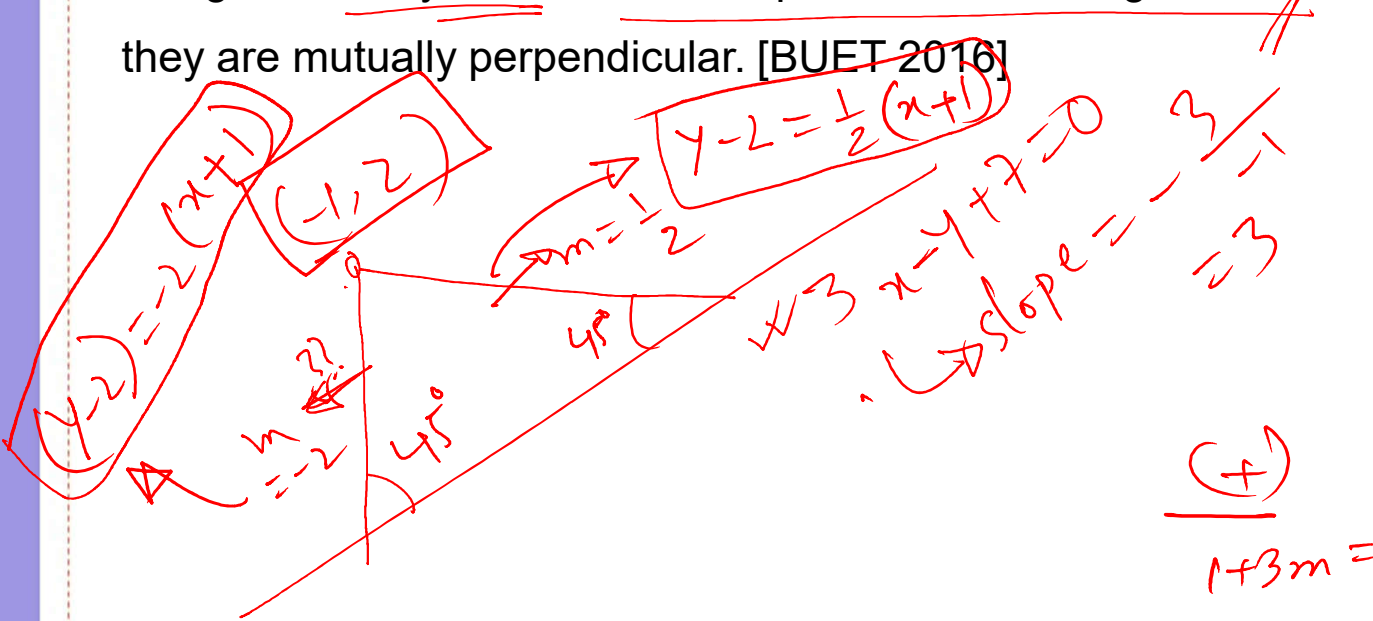
$$\begin{aligned} c_1 c_2 &= (-1)(+3) \\ &= -3 \rightarrow -ve \end{aligned}$$

Type 9: Finding the equation of the straight line which makes a certain angle with one of the axis or another straight line



Type 9: Finding the equation of the straight line which makes a certain angle with one of the axis or another straight line

Example 1: Two straight lines pass through the point $(-1, 2)$ and make an angle of 45° with the straight line $3x - y + 7 = 0$. Find the equation of the straight lines and using their equations show that they are mutually perpendicular. [BUET 2016]



$$\tan 45^\circ = \pm \frac{m - 3}{1 + m \cdot 3}$$

$$\Rightarrow 1 = \pm \frac{m - 3}{1 + 3m}$$

$$\Rightarrow 1 + 3m = \pm (m - 3)$$

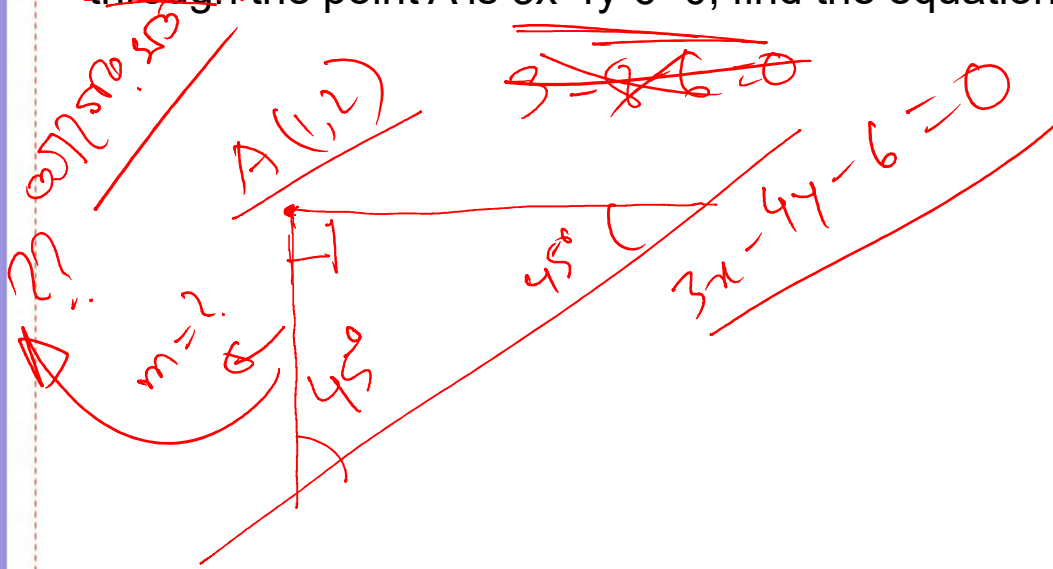
$$\begin{aligned} (+) \\ 1 + 3m &= m - 3 \\ \Rightarrow 2m &= -4 \\ \therefore m &= -2 \end{aligned}$$

$$\begin{aligned} (-) \\ 1 + 3m &= -(m - 3) \\ \Rightarrow 4m &= 2 \\ \therefore m &= \frac{1}{2} \end{aligned}$$

Type 9: Finding the equation of the straight line which makes a certain angle with one of the axis or another straight line

Example 2: If one of the vertex of a square is $A(1,2)$ and the equation of its diagonal passing through the point A is $3x-4y-6=0$, find the equations its sides that pass through the point A.

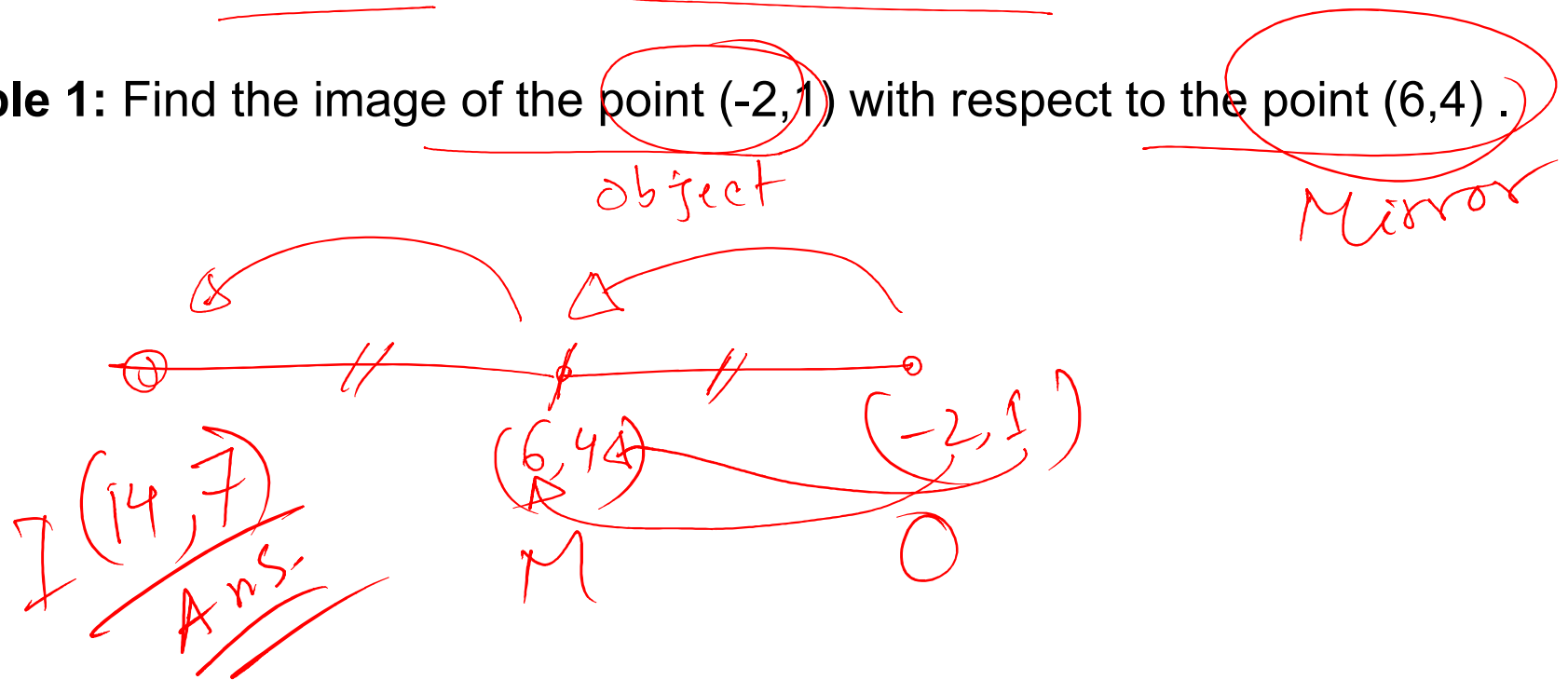
[BUET '18]



Type 10: Image

Case 1: Image of a point with respect to another point

Example 1: Find the image of the point $(-2, 1)$ with respect to the point $(6, 4)$.



Poll Question-05

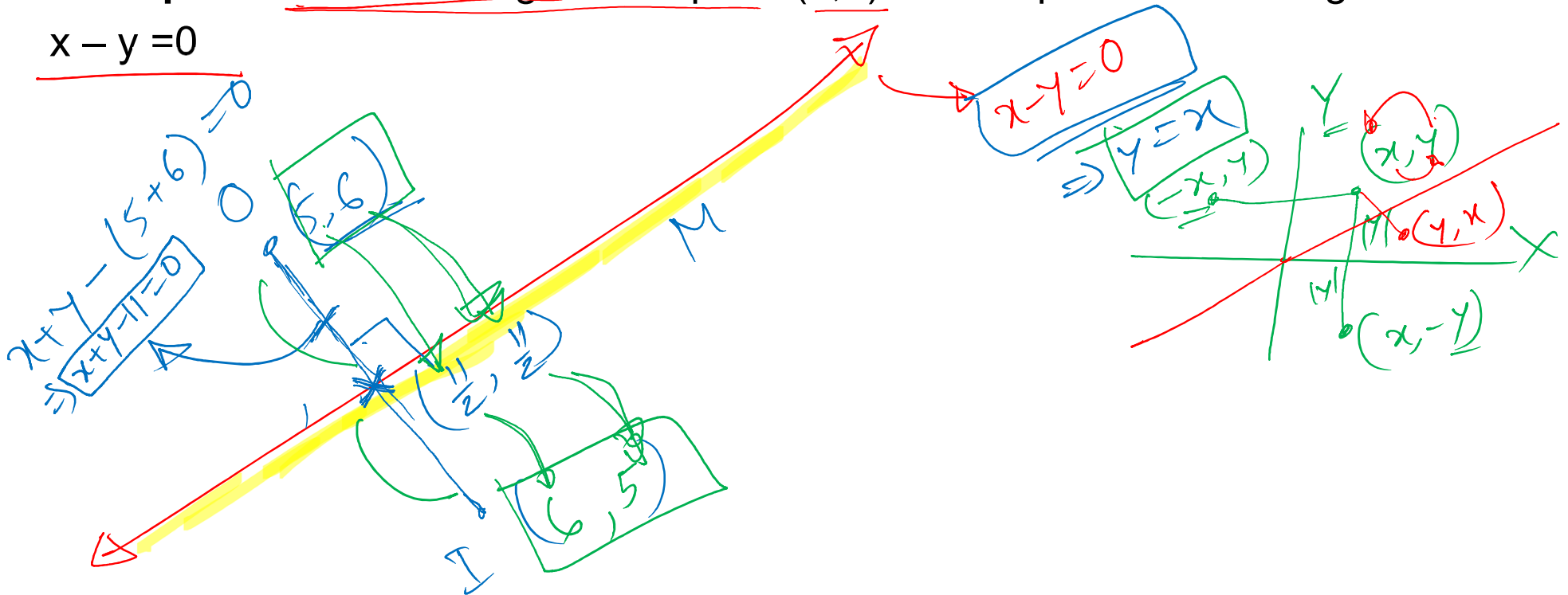
Which of the following is the image of the point $(-2,2)$ with respect to the point $(-5,5)$?

- (a) $(5,-5)$
- (b) $(-8,8)$
- (c) $(5,-1)$
- (d) none



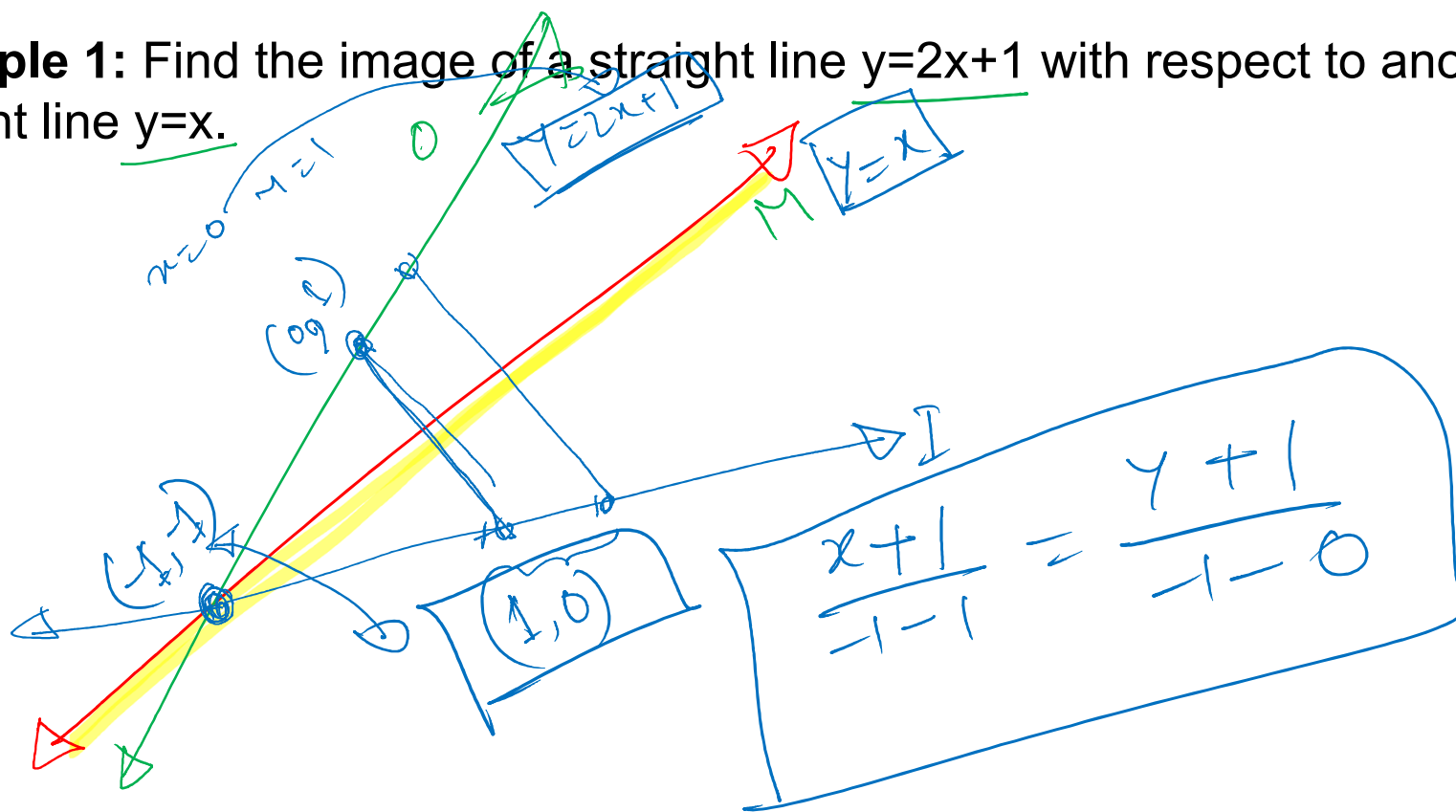
Case 2: Image of a point with respect to a straight line

Example 1: Find the image of the point $(5,6)$ with respect to the straight line $x - y = 0$



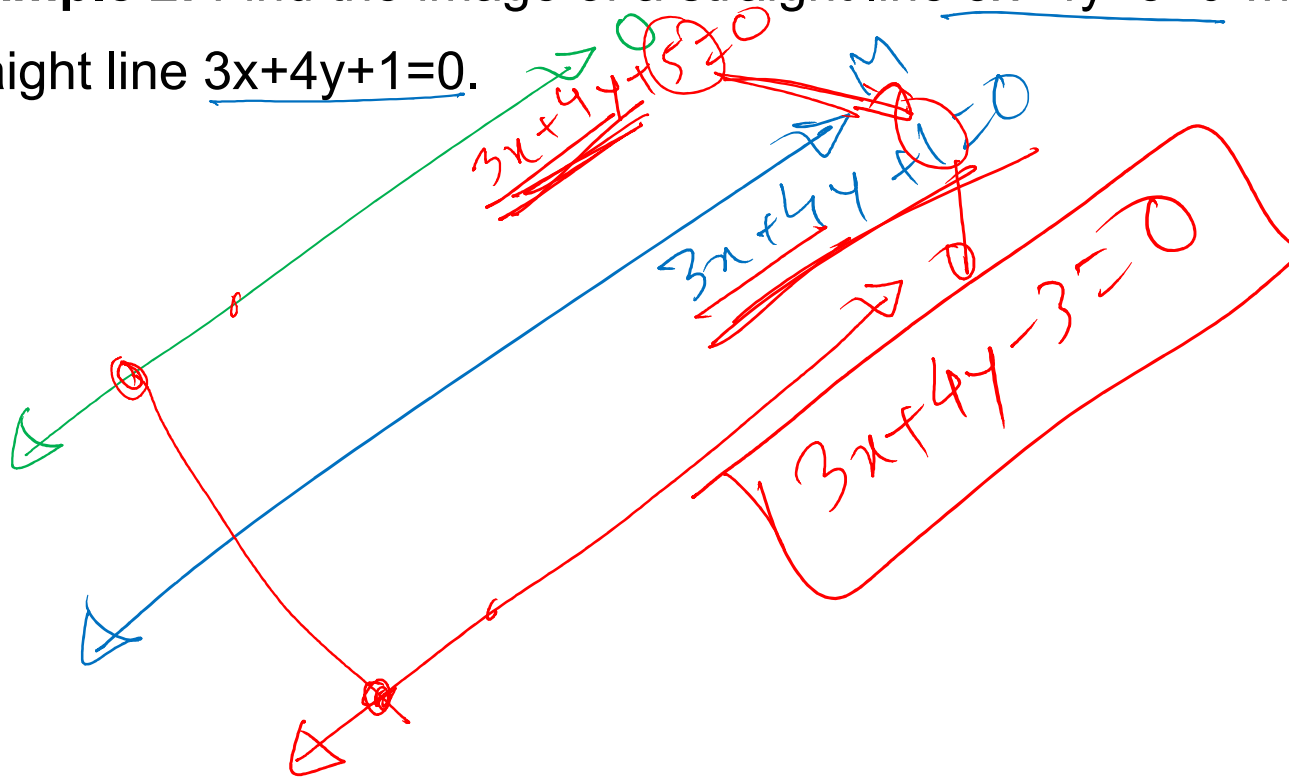
Case 3: Image of a straight line with respect to another straight line

Example 1: Find the image of a straight line $y=2x+1$ with respect to another straight line $y=x$.



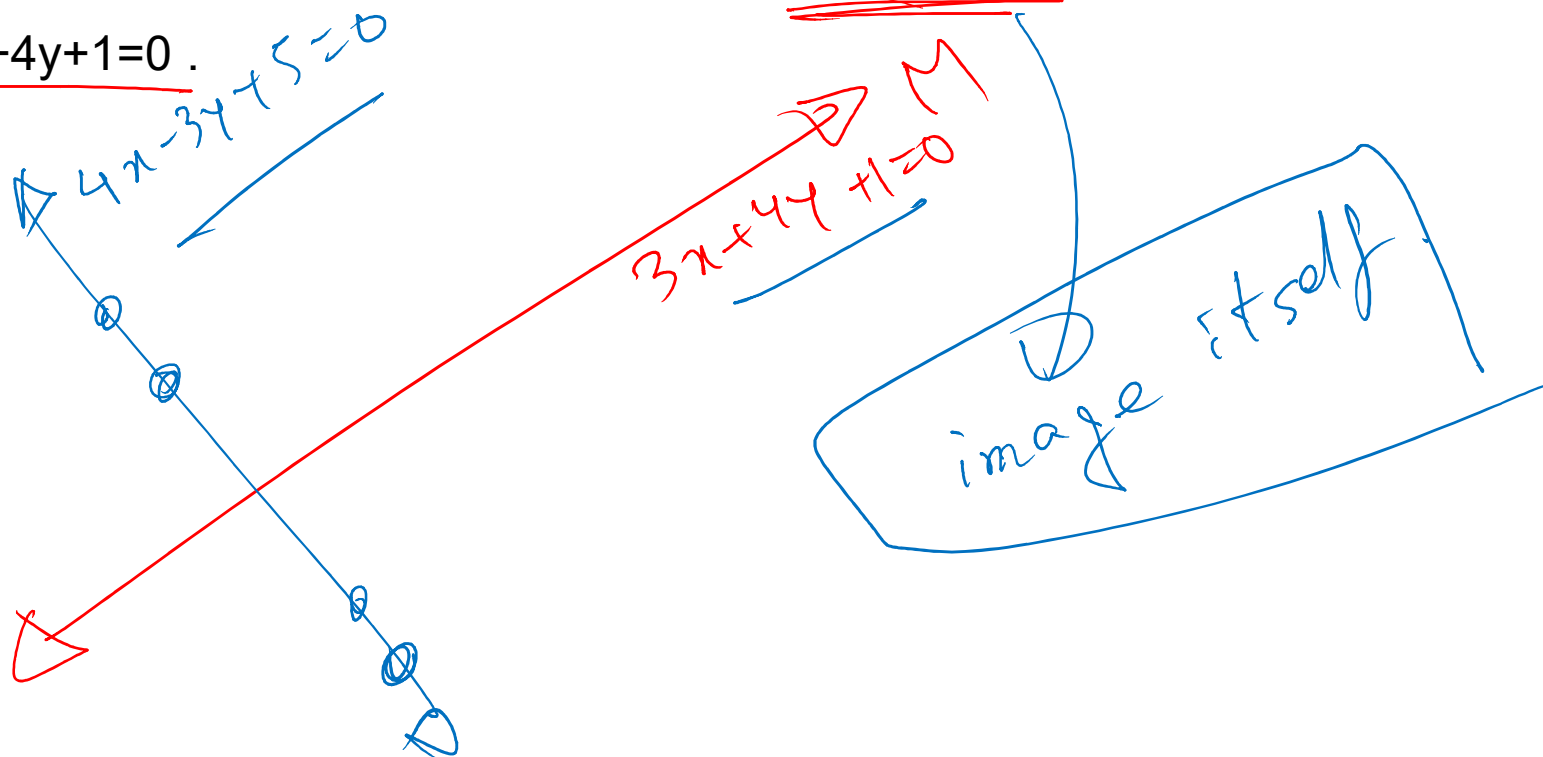
Case 3: Image of a straight line with respect to another straight line

Example 2: Find the image of a straight line $3x+4y+5=0$ with respect to another straight line $3x+4y+1=0$.



Case 3: Image of a straight line with respect to another straight line

Example 2: Find the image of a straight line $4x-3y+5=0$ with respect to another straight line $3x+4y+1=0$.



Special Cases:

Image with respect to x or y axis

Example 1: Find the image of the point $(4,3)$ with respect to x axis.

Ans! $(4, -3)$

Example 2: Find the image of the straight line $5x+7y+1=0$ with respect to x axis. Ans! $5x-7y+1=0$

Image with respect to the straight line $y=x$

Example 1: Find the image of the point $(4,3)$ with respect to the straight line $x=y$.

Ans! $(3,4)$

Example 2: Find the image of the straight line $5x+7y+1=0$ with respect to the straight line $x=y$.

Ans! $5y+7x+1=0$

Example 3: Find the image of the circle $x^2+y^2-2x+7y+1=0$ with respect to the straight line $x=y$.

Ans! $y^2+x^2-2y+7x+1=0$

Poll Question-06

✓ Which of the following is the image of the circle $x^2 + y^2 + 6x + 4y + 1 = 0$ with respect to y axis?

~~$x \rightarrow -x$~~ $x \rightarrow -x$

$$(-x)^2 + y^2 + 6(-x) + 4y + 1 = 0$$

$$\Rightarrow x^2 + y^2 - 6x + 4y + 1 = 0$$

(a) ~~$x^2 - y^2 + 6x - 4y + 1 = 0$~~

✓ (b) $x^2 + y^2 - 6x + 4y + 1 = 0$

(c) $x^2 + y^2 - 6x - 4y + 1 = 0$

(d) $x^2 + y^2 + 6x - 4y + 1 = 0$

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মুখস্থ করার
অভ্যাস প্রতিভাকে
ধ্বংস করে

$$X = caP \frac{V^2}{2S}$$

$$X = caP \frac{V^2}{2S}$$

$$E = mc^2$$

$$x = \sqrt{\frac{a^2}{c} + c} - \frac{b}{2}$$



উদ্ভাস

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