



VARSITY 'KA' ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-01

Chapter 05 : Permutation and Combination



$$x = \sqrt{\frac{a^2}{c} + c} - \frac{b}{2}$$



উদ্ভাস

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Introduction

abc

$ab \leftrightarrow ba$
 $bc \leftrightarrow cb$
 $ca \leftrightarrow ac$

0
1
2
3
 4 digit

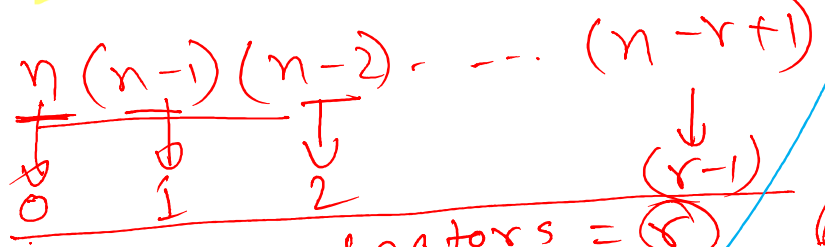
$7P_2 = 7 \times 6$
 $100P_3 = 100 \times 99 \times 98$

$${}^{12}C_3 = \frac{12 \times 11 \times 10}{3!} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1}$$

$${}^{365}C_2 = \frac{365 \times 364}{2 \times 1}$$

$${}^nP_r = {}^nC_r \times r! \Rightarrow {}^nC_r = \frac{{}^nP_r}{r!}$$

$${}^nP_r = \frac{n!}{(n-r)!} = \frac{n(n-1)(n-2)\dots(n-r+1)\cancel{(n-r)!}}{(n-r)!}$$



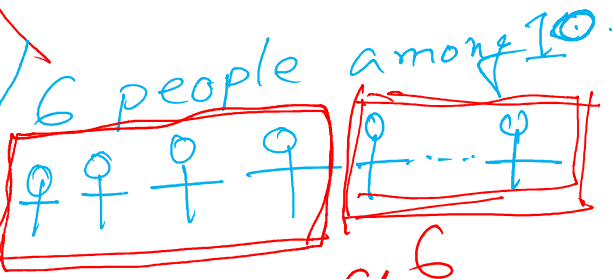
no. factors = r

r (Diff.) $\rightarrow r!$

$1 \rightarrow 10$ Arr.
 $5 \rightarrow 5 \times 10$ Comb.

$$n! = n(n-1)(n-2)\dots 3 \times 2 \times 1$$

$$= n \{ (n-1)! \} = n(n-1)\{ (n-2)! \}$$



v.v. ${}^nC_x = {}^nC_y$ iff

$x = y$

or,

$x + y = n$

Poll Question 01

If ${}^{15}C_3 = {}^{15}C_x$ find value of x-

- (a) 12
- (b) 3
- (c) Both a & b
- (d) None

$${}^{15}C_3 = {}^{15}C_x$$

$$x = 3$$

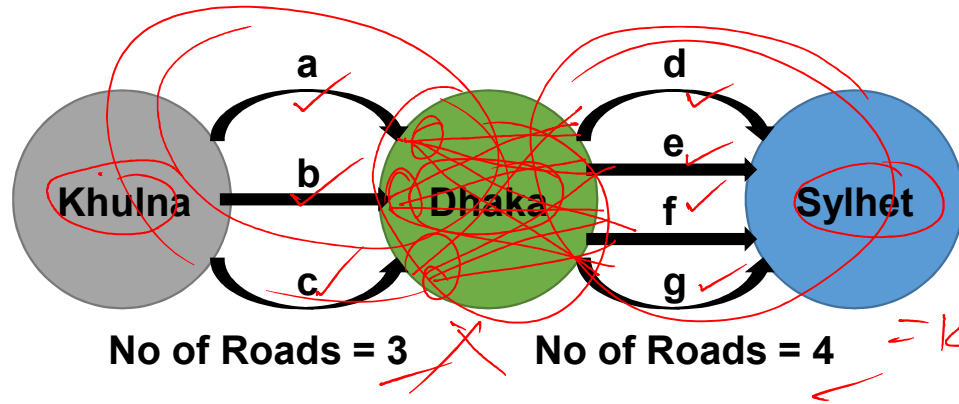
$$3 + x = 15$$

$$\Rightarrow x = 12$$

If ${}^n C_x = {}^n C_y$, then,
 $x = y$ or $x + y = n$

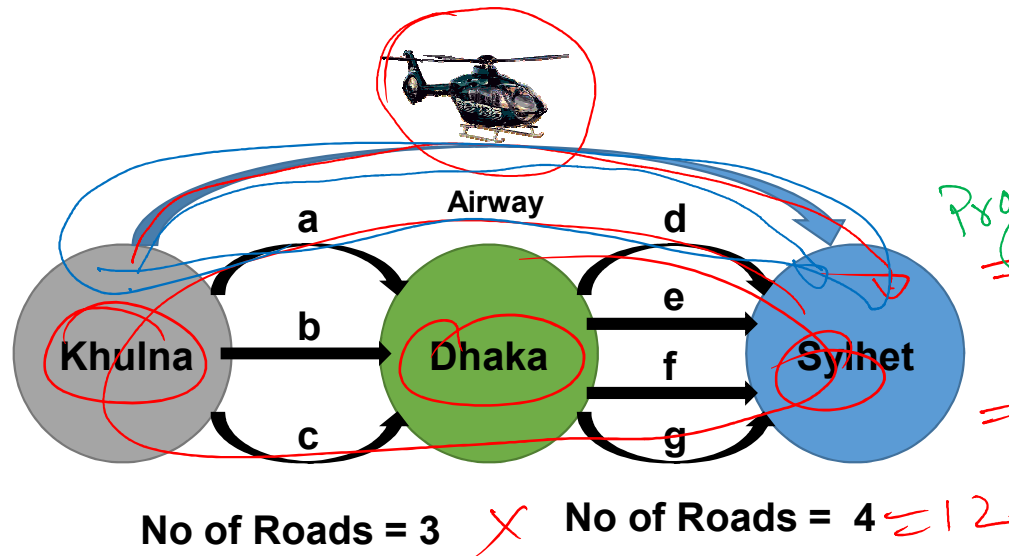
Fundamentals of Permutation

Rule of Product:



UNMESH
 $6 \times 5 \times 4 \times 3 \times 2 \times 1$
 $= 6!$

Rule of Sum:



Product Sum
 $= 3 \times 4 + 1$
 $= 12 + 1$
 $=$

Permutation of objects when all objects are not unique or different or distinct

(A) Number of ways in which n things can be arranged taking them all at a time, when p of the things are exactly alike of 1st type, q of them are exactly alike of a 2nd type, r of them are exactly alike of a 3rd type and the rest all are distinct is: $\frac{n!}{p! \times q! \times r!}$

How many arrangements can be made out of the letters of the word ENGINEERING?

$$\frac{11!}{3! 3! 2! 2! 1!}$$

WRONG!!

$$\frac{11!}{3! 3! 2! 2! 1!}$$

LENGR

Some caste issues

Taking the letters of the word 'DIRECTOR'-

- How many words can be formed?
- How many new words can be formed?
- How many new words can be formed where consonant occupy first position?
- How many new words can be formed where vowel occupy first position?
- How many words can be formed where all vowels remain (come) together?
- How many words can be formed where all vowels don't remain (come) together?
- How many words can be formed where any two vowels don't remain (come) together?
- How many words can be formed where positions of vowels remain unchanged?
- How many words can be formed where vowels and consonants don't change their relative position?
- How many words can be formed where positions of vowels don't change their order?
- How many words can be formed where 'R' occupy last place?

Some caste issues

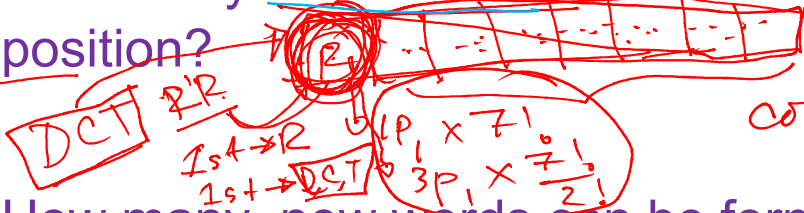
Taking the letters of the word **DIRECTOR**-

- How many words can be formed?
- How many new words can be formed?

$$\frac{8!}{2!}$$

- How many new words can be formed where consonant occupy first position?

$$\frac{8!}{2!} - 1$$



cons. occupy at 1st = $7! + 3P_1 \times \frac{7!}{2!}$

✓ Ans: $7! + 3P_1 \times \frac{7!}{2!} - 1$

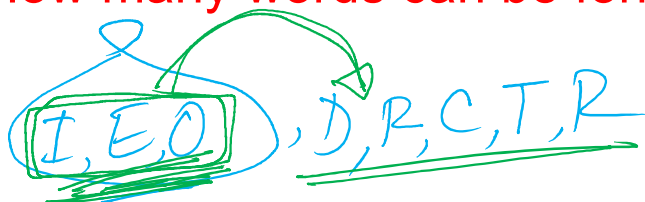
- How many new words can be formed where vowel occupy first position?



Some caste issues

Taking the letters of the word **'DIRECTOR'**-

- How many words can be formed where all vowels remain (come) together?



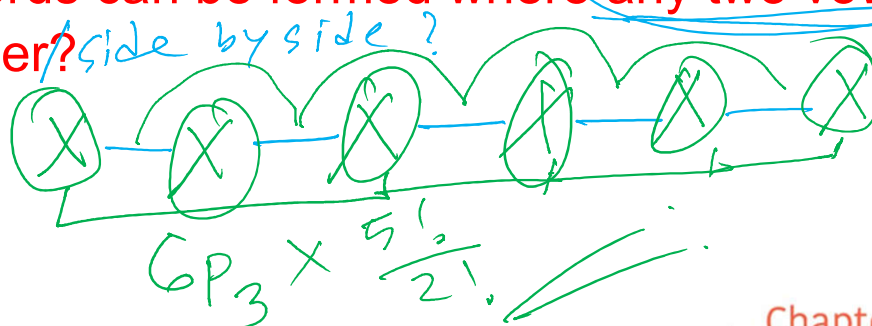
$$\frac{6!}{2!} \times 3!$$

- How many words can be formed where all vowels don't remain (come) together?

$$\frac{8!}{2!} - \frac{6!}{2!} \times 3!$$



- How many words can be formed where any two vowels don't remain (come) together?



Poll Question 02

ⓧ In how many ways the letters of the word 'KACHUA' can be arranged such that all vowels come together?

(a) 24

~~(b) 72~~

(c) 144

(d) 8

$$\begin{aligned} & \text{AUA, K, C, H} \\ & 4! \times 3! \\ & = 24 \times 3 = 72 \end{aligned}$$

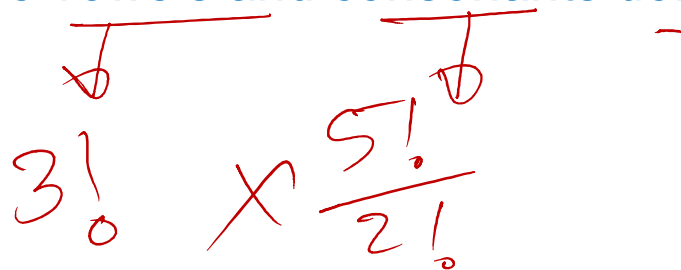
Some caste issues

Taking the letters of the word 'DIRECTOR'-

- How many words can be formed where positions of vowels remain unchanged?

$$\frac{5!}{2!}$$


- ✓✓ How many words can be formed where vowels and consonants don't change their relative position?

$$3! \times \frac{5!}{2!}$$


Some caste issues

Taking the letters of the word 'DIRECTOR' -



- ✓ How many words can be formed where positions of vowels don't change their order?

$$\frac{8!}{2!3!}$$

- How many words can be formed where 'R' occupy last place?



Permutation with repetition



❖ Number of permutations of n distinct things taking r at a time when each thing may be repeated any number of times is n^r

• In how many different ways 5 member can vote 3 person?

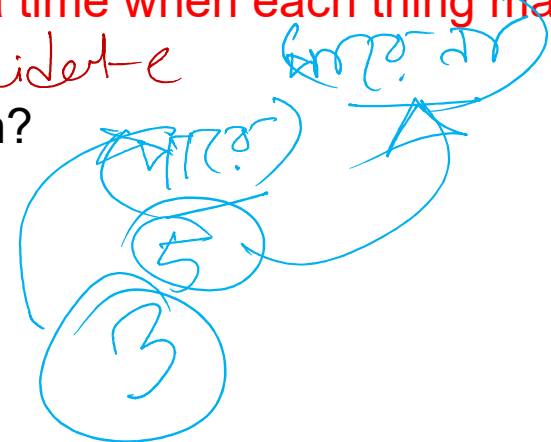
$V = a, b, c, d, e$

can 1. $\rightarrow 1, 2, 3$

1	2	3
a	b	a
b	c	b
c	d	c
d	e	d
e		e

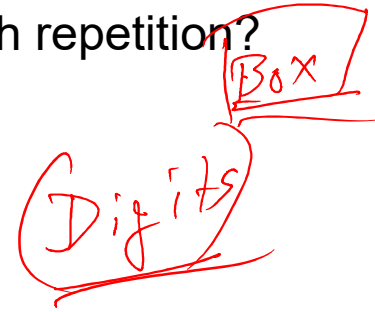
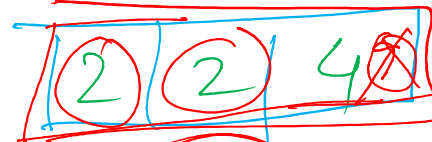
a	b	c	d	e
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

$3 \times 3 \times 3 \times 3 \times 3 = 3^5$



• How many 3 digit numbers can be formed from digits 1, 2, 4, 6, 8, 9 with repetition?

$6 \times 6 \times 6 = 6^3$



Poll Question 03

A Grameenphone number starts with 017 or 013.

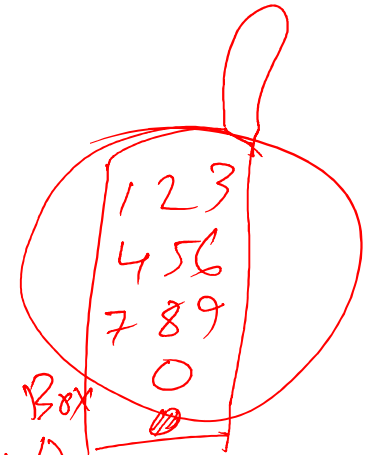
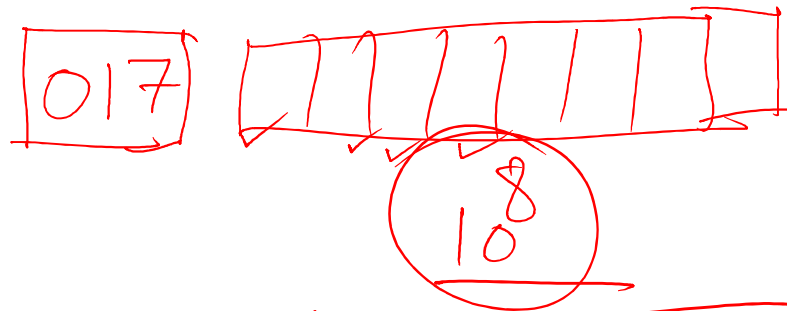
How many connections can Grameenphone provide?

(a) $^{10}P_8$

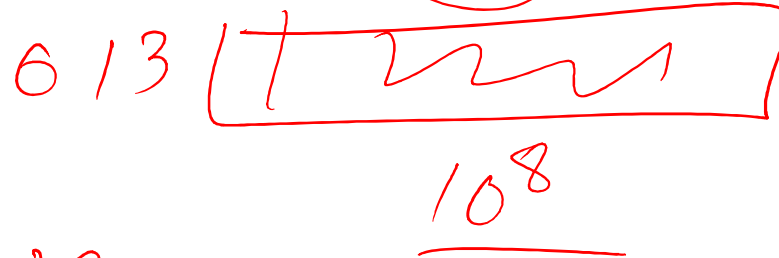
(b) 10^8

(c) $^{10}C_8$

(d) 2×10^8



(Digit)







$$10^8 + 10^8 = 2 \times 10^8$$

$$a + a = 2a$$

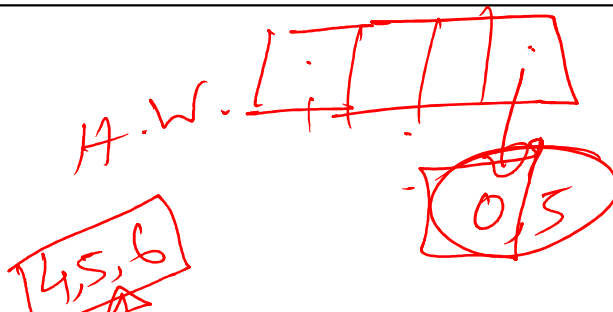
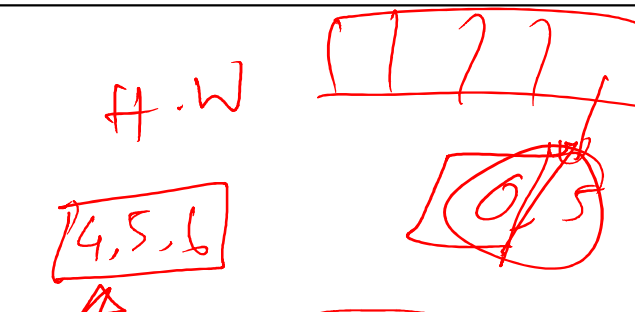
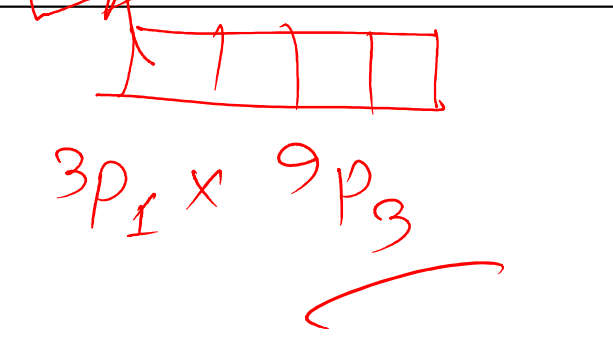
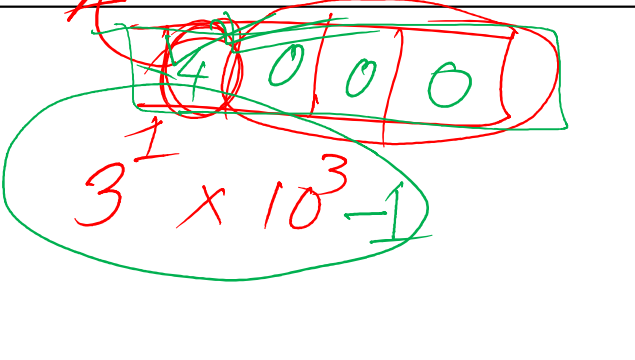
Related To Formation Of Numbers

From the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using a digit- (i) once, (ii) multiple times-

Condition/Question	i) Using once (${}^n P_r$)	ii) Using multiple times (n^r)
(a) How many <u>4 digit numbers</u> can be formed?	 ${}^9 P_1 \times {}^9 P_3$	 $9^1 \times 10^3$
(b) How many <u>odd 4 digit numbers</u> can be formed?	 ${}^8 P_1 \times {}^8 P_2 \times {}^5 P_1$	 $9^1 \times 10^2 \times 5^1$
(c) How many <u>even 4 digit numbers</u> can be formed?	H.W.	H.W.

Related To Formation Of Numbers

From the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 using a digit- (i) once, (ii) multiple times-

Condition/Question	i) Using once	ii) Using multiple times
(d) How many 4 digit numbers can be formed which are divisible by 5?	<p style="text-align: center;">A.W. </p>	<p style="text-align: center;">A.W. </p>
<p>(e) How many numbers can be formed which are <u>greater than 4,000</u> but less than <u>7,000</u>?</p>	<p style="text-align: center;"></p> <p style="text-align: center;">$3P_1 \times 9P_3$</p>	<p style="text-align: center;"></p> <p style="text-align: center;">$3^1 \times 10^3 - 1$</p>

Circular permutations:

- Number of circular permutations (arrangements) of n distinct things = $(n - 1)!$ [When it will be observed by only one side]
- Number of circular permutations (arrangements) of n distinct things, when clockwise and anticlockwise arrangements are not different (i.e., when observations can be made from both sides) = $\frac{(n-1)!}{2}$ [when it will be observed by 2 sides]

- In how many ways 13 person can be seated around a round table?

$$(13-1)!$$



- How many chain can be made using 10 stones of distinct color or shape?

$$\rightarrow \bigcirc \leftarrow \frac{(10-1)!}{2}$$

Form group on several condition

01. In how many ways a group of 15 people can be made choosing from 40 people?

Case – 1: If Ashik is in the group:

$${}^{39}C_{14}$$

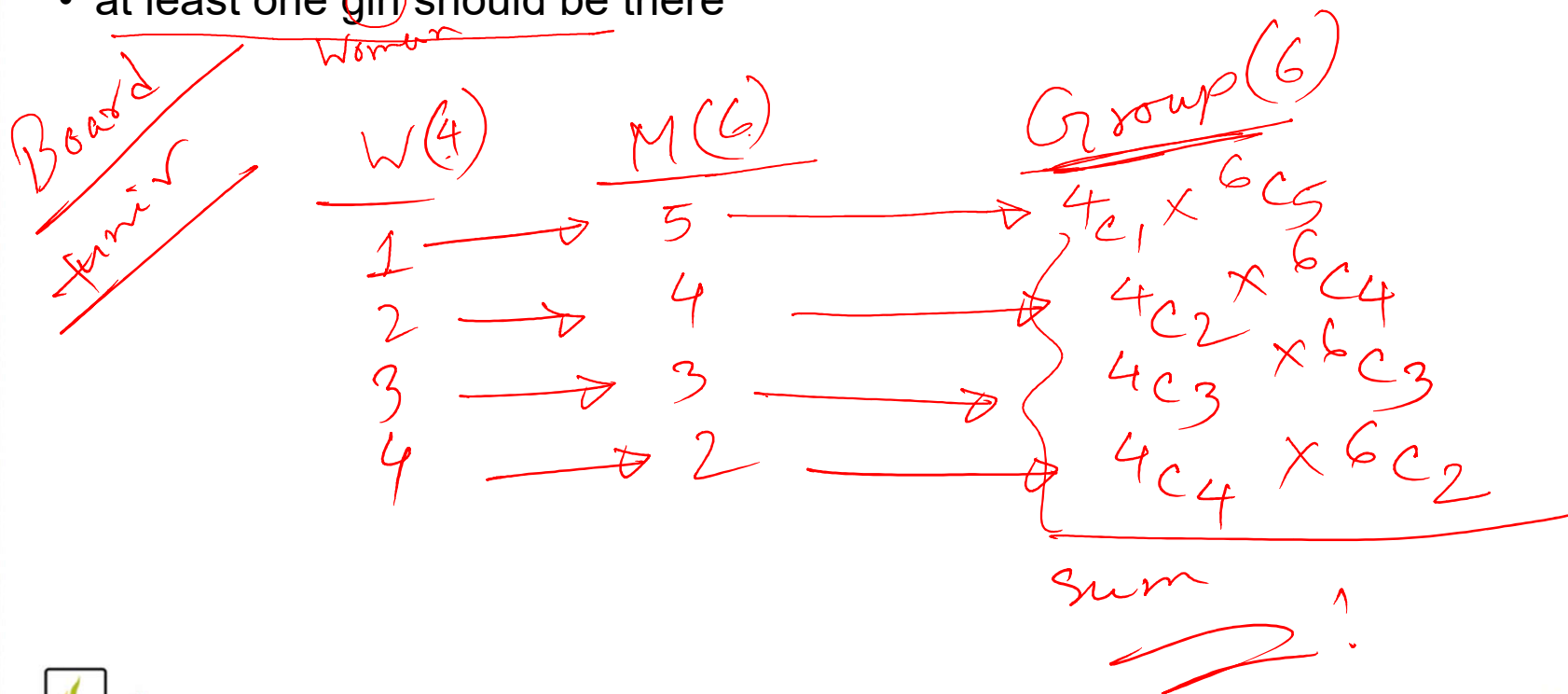
Case – 2: If Ashik is not in the group:

$${}^{39}C_{15}$$

Form group on several condition

From a group of 10 person including 4 women, 6 persons are to be selected. In how many ways can it be done? - such that:

- at least one girl should be there



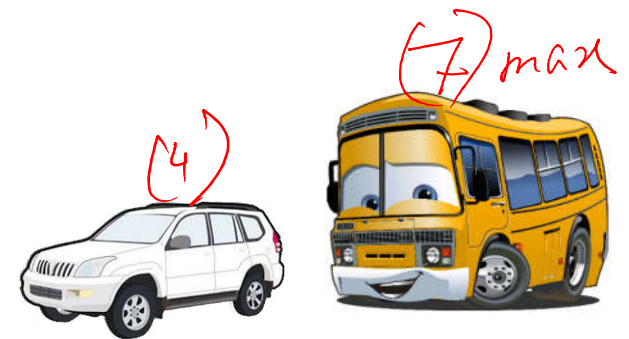
Imp. 4 Everywhere

Form group on several condition

A group of 9 persons can travel by two car, one car cannot accommodate more than 7 persons and the another cannot accommodate more than 4 persons. In how many ways the can travel?

$\frac{\max(7)}{C_1}$	$\frac{\max(4)}{C_2}$	
5	4	$9C_5 \times 4C_4$
6	3	$9C_6 \times 3C_3$
7	2	$9C_7 \times 2C_2$
8		

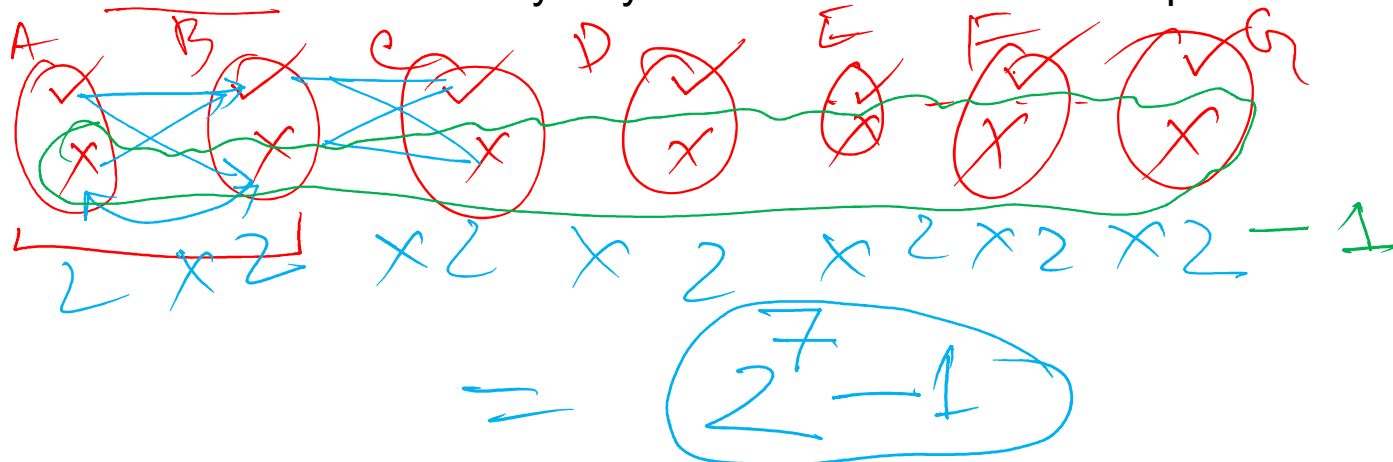
Sum



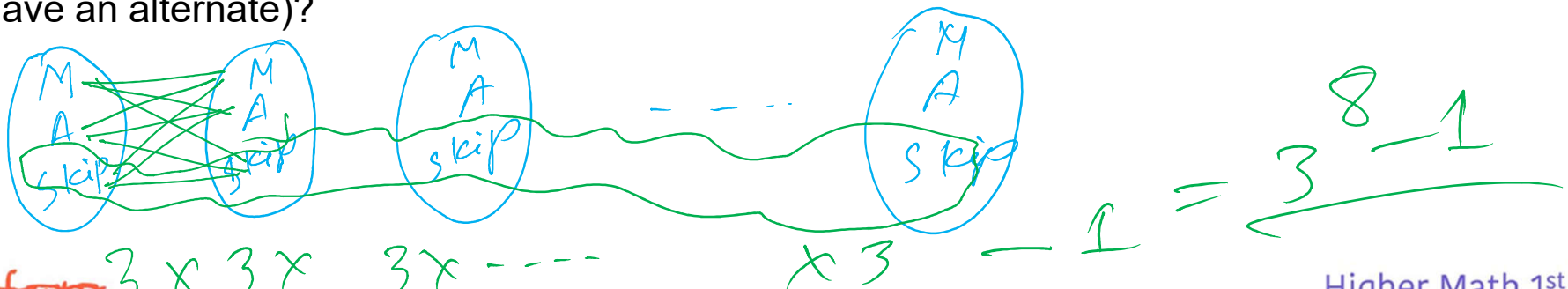
$${}^n C_n = 1$$

Related To The Selection Of One Or Multiple Objects

01. One has 7 friends. In how many ways he can invite one or multiple friends of him?



02. How many ways are there to choose one or multiple questions from 8 questions (each question have an alternate)?



Geometry related problem.

- How many _____ can be formed using the vertex points of a polygon of 10 arms?

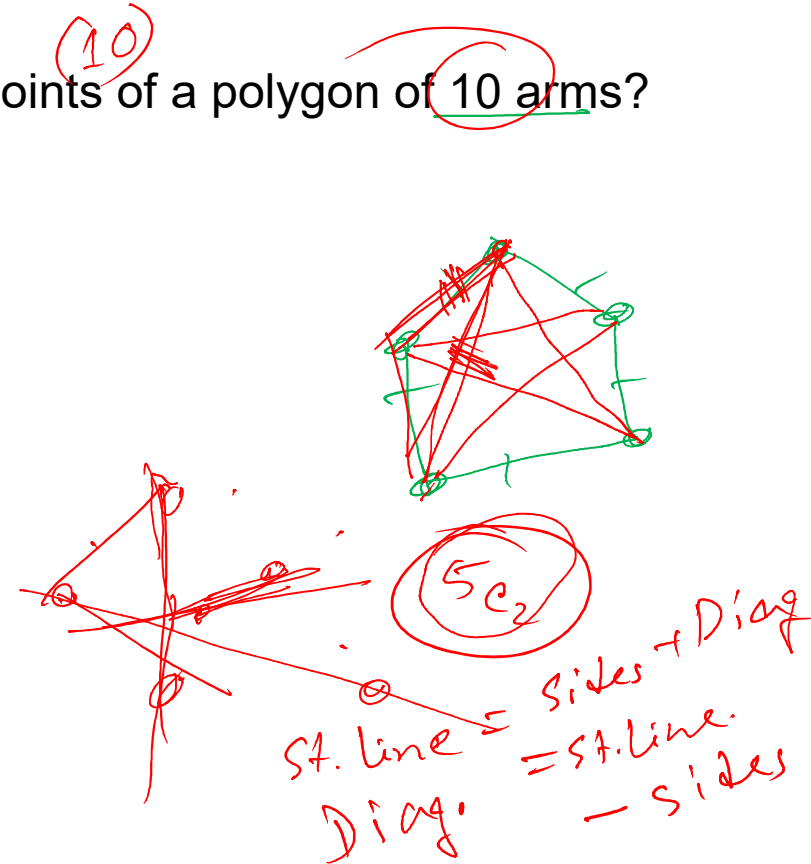
straight line = $^{10}C_2$

Arm = 10

Diagonal = $^{10}C_2 \rightarrow 10$

Triangle = $^{10}C_3$

Quadrilateral = H.W.

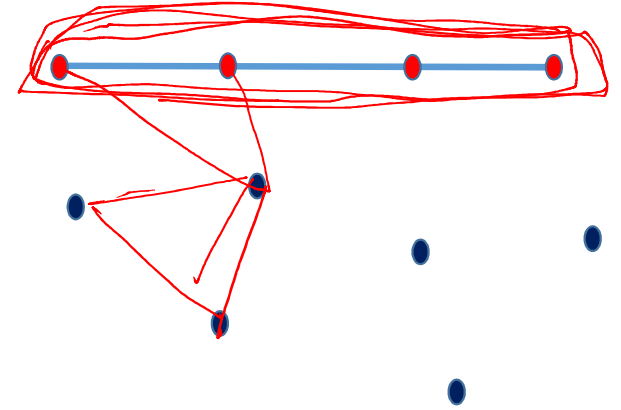


Geometry related problem.

- Find the number of triangles, straight lines that can be formed using 10 points in a plane such that 4 points are collinear?

Tri^o $10C_3 - 4C_3$

St. Lines! $10C_2 - 4C_2 + 1$



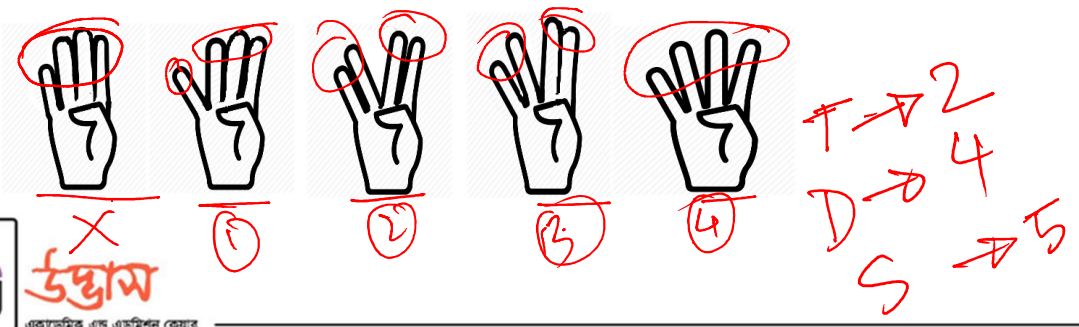
Related To The Combined Problems Of Permutation & Combination

Find the no. of selection that can be formed taking 4 letters from the word "ENGINEERING".

V.V.D.
W

Ways	Examples	Number of Combinations
<u>1I + 1S</u>	✓ E E E R ✓	$2C_1 \times 4C_1$
2D	E E G G	$4C_2$
1D + 2S	E E G R	$4C_1 \times 4C_2$
4S	E N G R	$5C_4$

Sum
~~(E E E) (N N N) (G G) (I) (R)~~



Poll Question 04

Find the different number of selection taking 3 letters each time from the word 'SCHOOL' -

- (a) 10
- (b) 14
- (c) 4
- (d) 15

Handwritten solution:

Letters: S, C, H, O, L

Case 1: Selection with 1 O and 2 other letters (e.g., O, S, C):

$${}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$$

Case 2: Selection with 3 letters from {S, C, H, L}:

$${}^4C_3 = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 4$$

Total number of selections = 6 + 4 = 10

Diagrammatic representation:

- Case 1: A box containing 'O, S, C' with arrows pointing to 'O, S' and 'O, C'.
- Case 2: A box containing 'S, C, H, L' with arrows pointing to 'S, C, H', 'S, C, L', 'S, H, L', and 'C, H, L'.

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ধ্বংস করে

$$X = caP \frac{V^2}{2S}$$

$$X = caP \frac{V^2}{2S}$$

$$E = mc^2$$

$$x = \sqrt{\frac{a^2}{c^2} + c} - \frac{b}{2}$$



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