VARSITY ‘Ka’ ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-03

Chapter 03 : Straight Line

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

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V.V.I.

Type-1: Change of Coordinate System

(i) Converting co-ordinate of a point from polar to Cartesian:

$\xrightarrow{} (r, \theta)$ $\xrightarrow{} (x, y)$

Unknown

known

Example: If polar coordinate of a point is $(3,\ 90°)$, then what is its Cartesian coordinate? **[JU'14-15]**

$r \quad \theta$

Soln :

$x = 3\cos 90° = 3 \times 0 = 0$

$y = 3\sin 90° = 3 \times 1 = 3$

$c(0,3)$ Ans.

(r,θ) P

(x,y) C

$$\left\{ \begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array} \right\}$$

Type-1: Change of Coordinate System

(ii) Converting co-ordinate of a point from Cartesian to polar:

Known (x, y) → Unknown (r, θ) ??

$r^2 = x^2 + y^2$

$\therefore \boxed{r = \sqrt{x^2 + y^2}}$

$\tan\theta = \frac{y}{x}$

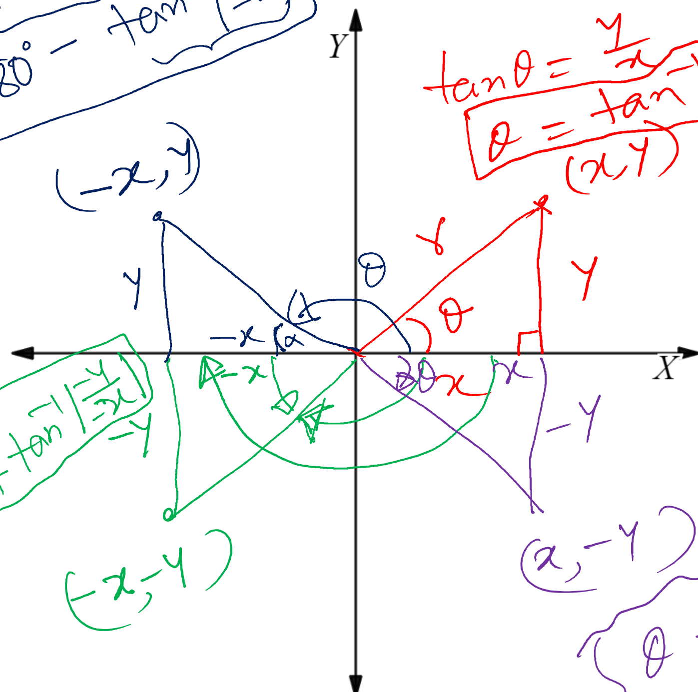
$\boxed{\theta = \tan^{-1}\left|\frac{y}{x}\right|}$ (x, y)

$\theta = 180° - \alpha$

$\boxed{\theta = 180° - \tan^{-1}\left|\frac{y}{-x}\right|}$ $(-x, y)$

$\boxed{\theta = -180° + \tan^{-1}\left|\frac{-y}{-x}\right|}$ $(-x, -y)$

$\boxed{\theta = -\tan^{-1}\left|\frac{-y}{x}\right|}$ $(x, -y)$

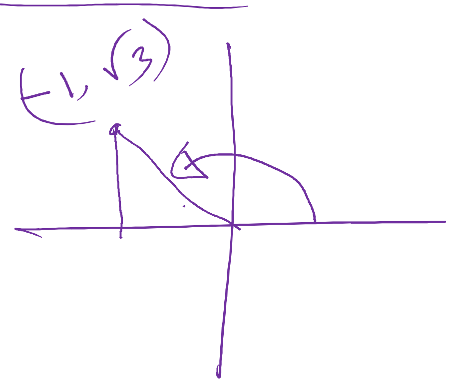


Type-1: Change of Coordinate System

(ii) Converting co-ordinate of a point from Cartesian to polar:

Example: If Cartesian coordinate of a point is $(-1, \sqrt{3})$, then what is its polar coordinate? (r, θ) ??

[JU' 18-19, 11-12, JnU' 12-13]



$$r = \sqrt{(-1)^2 + (\sqrt{3})^2}$$
$$= \sqrt{4}$$
$$= 2$$

$$\theta = 180^\circ - \tan^{-1}\left|\frac{\sqrt{3}}{-1}\right|$$
$$= 180^\circ - \tan^{-1}\sqrt{3}$$
$$= 180^\circ - 60^\circ = 120^\circ = \frac{2\pi}{3}$$

$(r, \theta) = (2, 120^\circ)$ Ans.

Type-1: Change of Coordinate System

(x, y) (r, θ)

(iii) Converting Cartesian equation to polar equation:

- Replace x by $r\cos\theta$ and replace y by $r\sin\theta$
- Change $x^2 + y^2$ with r^2

Example: Transform from Cartesian equation to polar equation: $x^2 + y^2 - 2ax = 0$ **[CU'14-15]**

Soln:

$r^2 - 2ar\cos\theta = 0$

$\Rightarrow r - 2a\cos\theta = 0$

$\Rightarrow \boxed{r = 2a\cos\theta}$ Ans

Type-1: Change of Coordinate System

(iv) Converting polar equation with Cartesian equation:

- Change r^2 with $x^2 + y^2$
- Replace $r\cos\theta$ and $r\sin\theta$ with x and y respectively.

Example: Transform from polar equation to Cartesian equation: $r = a\sin\theta$

Soln:

$r = a\sin\theta$

$\Rightarrow r.r = a r\sin\theta$

$\Rightarrow r^2 = ay$

$\Rightarrow x^2 + y^2 = ay$

Ans

Poll Question-01

Transform from polar equation to Cartesian equation: $2r\sin^2\left(\frac{\theta}{2}\right) = 1$ **[DU'18-19]**

(a) $x^2 + y^2 = (1 + x)^2$ ✓

(b) $y^2 = 1 + 2x$ ✓

(c) both a & b ✓

(d) None

$2\sin^2\frac{\theta}{2} = 1 - \cos\theta$

$2r\sin^2\frac{\theta}{2} = 1$

$\Rightarrow r(1-\cos\theta) = 1$

$\Rightarrow r - \underbrace{r\cos\theta} = 1$

$\Rightarrow r - x = 1$

$\Rightarrow r = (1+x)$

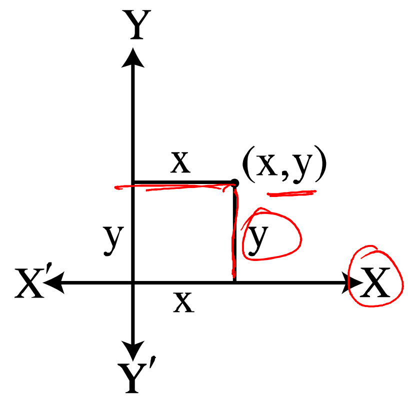
$\Rightarrow r^2 = (1+x)^2 \rightarrow (a)$

$\Rightarrow x^2 + y^2 = 1 + 2x + \cancel{x^2}$

$\Rightarrow \cancel{x^2} + y^2 = 1 + 2x$

$y^2 = 1 + 2x \rightarrow (b)$

Type-2: Problems related to determination of distance

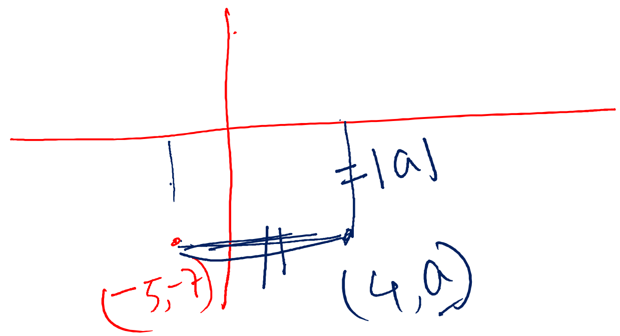


- For (x, y) point i) Distance from x-axis $= |y|$ units
  ii) Distance from y-axis$= |x|$

- Distance between (x_1, y_1) and (x_2, y_2), $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ units

Type-2: Problems related to determination of distance

Example: If distances of point $(4,\ a)$ from x-axis and point $(-5, -7)$ are equal, then what is the value of a? **[JU'18-19, JU'11-12,RU'17-18]**



A/C,

$$|a| = \sqrt{(4+5)^2 + (a+7)^2}$$

$$\Rightarrow \cancel{a^2} = 81 + \cancel{a^2} + 14a + 49$$

$$\Rightarrow 14a = -130$$

$$\therefore a = -\frac{65}{7}$$

Ans.

Poll Question-02

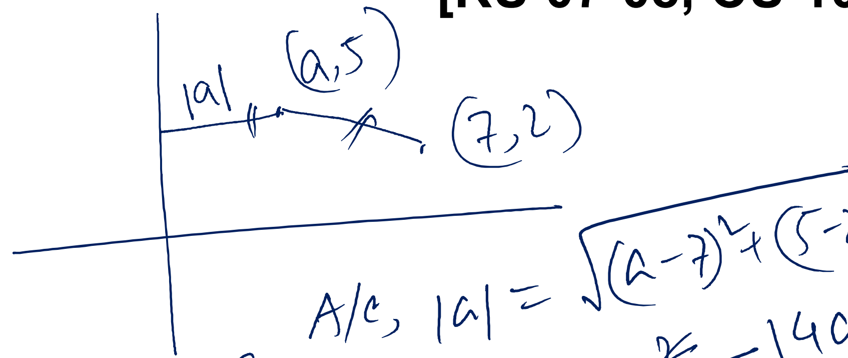
If distances of point $(a,\ 5)$ from y-axis and point $(7,2)$ are equal, then what is the value of a? **[RU'07-08, CU'10-11, 14-15]**

(a) $19/7$

(b) 29/7

(c) 19/9

(d) 29/9



A/c, $|a| = \sqrt{(a-7)^2+(5-2)^2}$

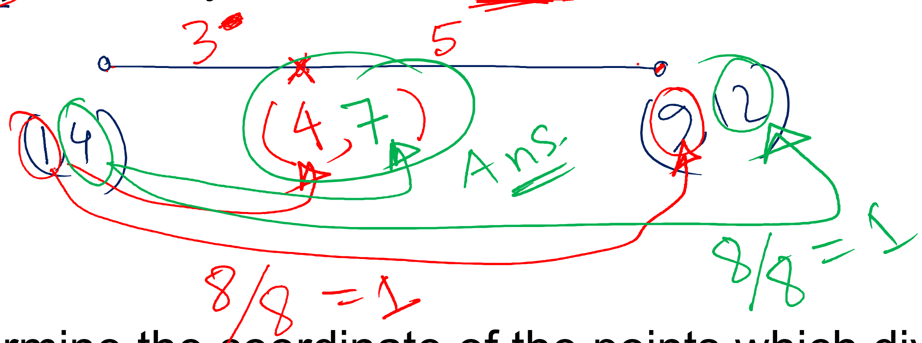
$\Rightarrow \cancel{a^2} = \cancel{a^2} - 14a + 49 + 9$

$\Rightarrow 14a = 58$

$\therefore a = 29/7$

Type-3: Problems related to dividing the line connecting two points at a definite ratio

Example: Determine the coordinate of the points which divide the line connecting points $(1,4)$ and $(9,12)$ internally in the ratio $3:5$. **[DU'~~05-06~~,14-15]**



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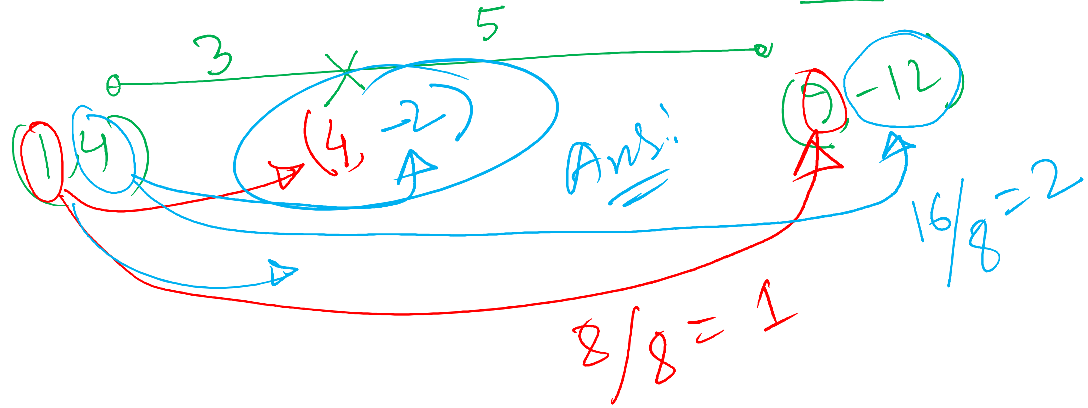
Example: Determine the coordinate of the points which divide the line connecting points $(1,4)$ and $(9,-12)$ internally & externally in the ratio $3:5$. **[DU' 05-06, ~~14-15~~, JU' 18-19]**



Type-3: Problems related to dividing the line connecting two points at a definite ratio

Example: Determine the coordinate of the points which divide the line connecting points $(1,2)$ and $(3,1)$ externally in the ratio $1:2$.

$\boxed{(-1,3)}$ Ans.

Type-3: Problems related to dividing the line connecting two points at a definite ratio

Example: In what ratio x and y axis divide the segment connecting the points (3,2) and (5,-7) ?

x-axis divides the st. line segment connecting the points (x_1, y_1) & (x_2, y_2) $= \frac{y_1}{y_2}$ → −ve ⇒ Int. → +ve ⇒ Ext.

y-axis - - - - - - - - $= \frac{x_1}{x_2}$

Soln:

x-axis: $\frac{m}{n} = \frac{2}{-7}$ −ve

$\therefore m:n = 2:7$ (Int.)

y-axis: $\frac{p}{q} = \frac{3}{5}$ → +ve

$\therefore p:q = 3:5$ (Ext)

Poll Question-03

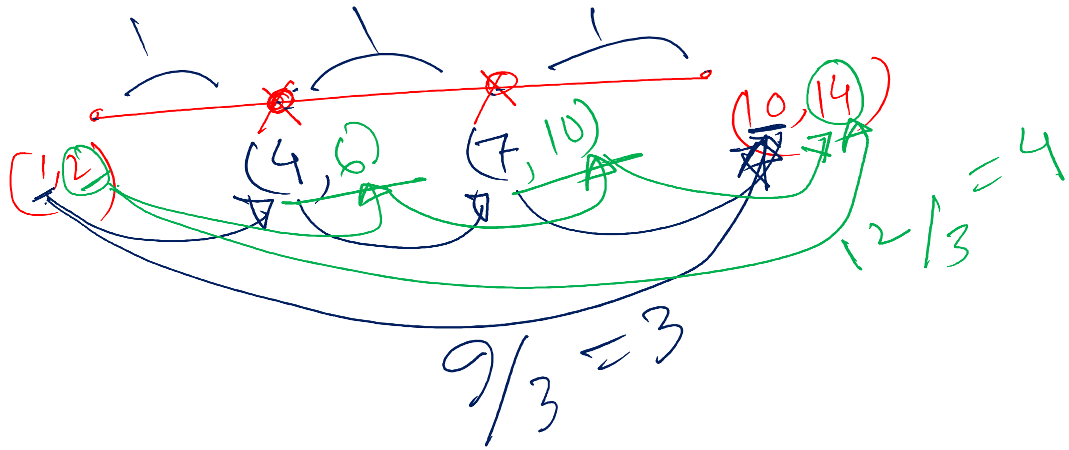
Find the points of trisection of the segment connecting the points (1,2) and (10, 14).

(a) $(4, 6)$ & $(7,10)$

(b) $(6,4)$ & $(7,10)$

(c) $(4, 6)$ & $(10,7)$

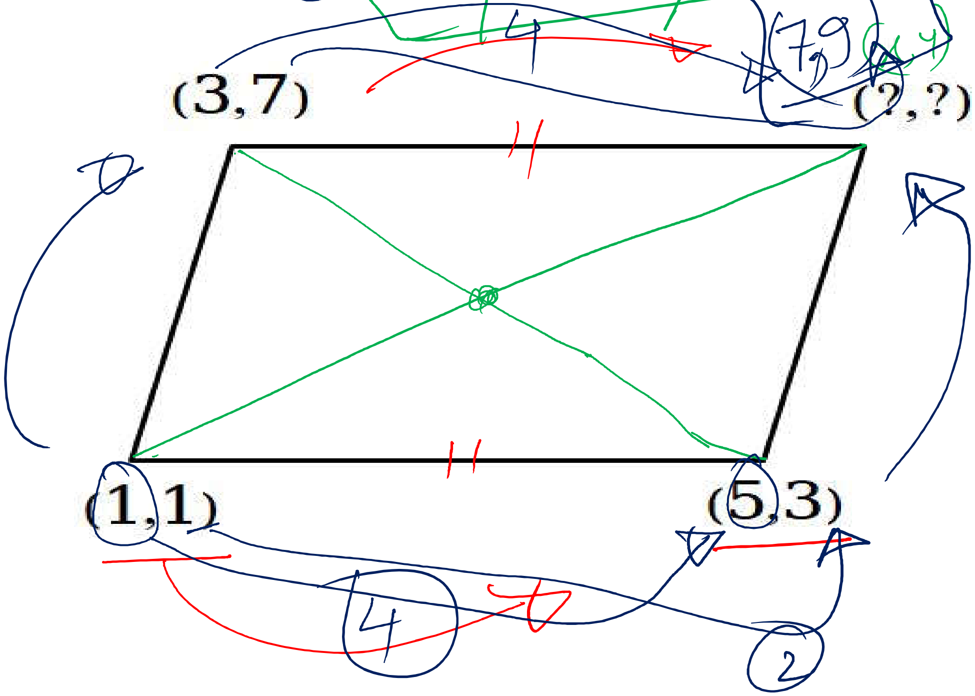
(d) $(6, 4)$ & $(10,7)$



Type-4: Determination of fourth vertex of quadrilateral

Example:

* Pa/ Rect../ Rom/ Sq



Written!

$$\frac{x+1}{2} = \frac{3+5}{2}$$

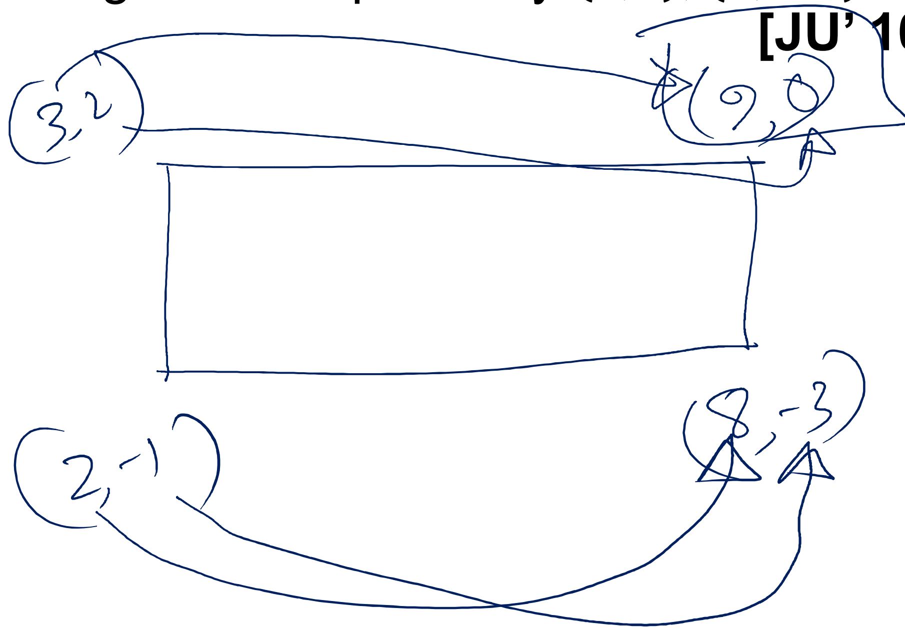
$$\Rightarrow x = 3+5-1$$

$$\frac{y+1}{2} = \frac{7+3}{2}$$

$$\Rightarrow y = 7+3-1$$

Poll Question-04

The three vertices of a rectangle are respectively $(3,2), (2,-1)$ & $(8,-3)$. Find its fourth vertex. **[JU' 10-11, KU' 06-07]**



(a) (0,9)

~~(b)~~ (9,0) ✓

(c) (0,-9)

(d) (-9,0)

Type-5 Area related problem

Example: The vertices of triangle are $(-1,-2)$, $(2,5)$ and $(3,10)$; determine the area of the triangle. **[DU' 14-15, 01-02,03-04, JU' 18-19, 17-18, 11-12, CU' 12-13]**

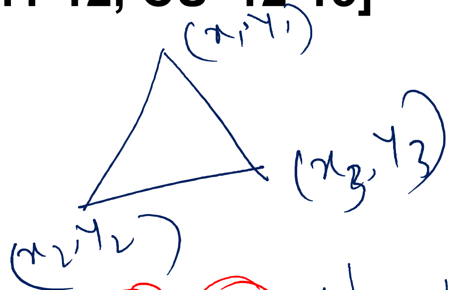
$(-1,-2)$ $(2,5)$ $(3,10)$

$(0,0)$ $(3,7)$ $(4,12)$

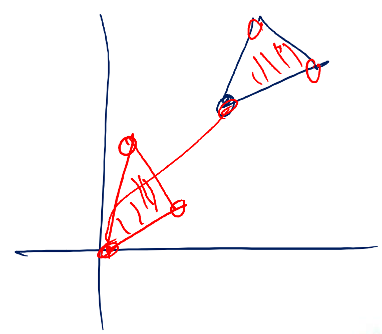
$$\Delta = \frac{1}{2}\left|3\times 12 - 7\times 4\right| \text{ sq. units}$$

$$= \frac{1}{2}\left|36-28\right|$$

$$= 4 \text{ sq units}$$



$$\Delta = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right| \text{ sq unit}$$



Type-6: Determination of coordinates of different points of triangle

In the triangle ABC formed by $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$,centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Example: The three vertices of a triangle are respectively (0,0), (0,3) & (4,0). Find its center of mass. **[BAU' 14-15]**

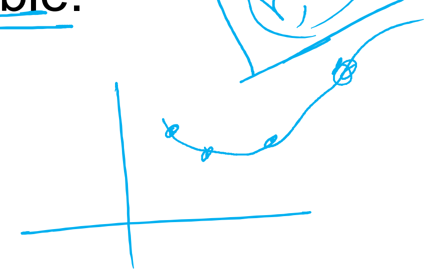
$$C \equiv \left(\frac{0+0+4}{3}, \frac{0+3+0}{3}\right)$$

$$\equiv \left(\frac{4}{3}, 1\right) \text{ Ans}$$

Type-7: Determination of equation of locus

Example: Find the locus of $(2t+1, 3t-1)$ where 't' is variable.

(r, θ)



$$\left\{\begin{array}{l} x = 2t+1 \\ y = 3t-1 \end{array}\right\} \text{Para. eq}^n$$

$$\boxed{\frac{x-1}{2} = \frac{y+1}{3}} \rightarrow \text{st. line}$$

Example: Find the locus of $(a\cos\theta, a\sin\theta)$. → 'θ' variable

$$\left\{\begin{array}{l} x = a\cos\theta \\ y = a\sin\theta \end{array}\right\}$$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 = 1$$

$$\Rightarrow x^2 + y^2 = a^2 \quad \text{eq}^n \text{ of circle.}$$

Poll Queston-05

(x, y)

The distance of a set of points from the x axis is always half its distance from the origin. Find the equation of the locus of such a set of points. **[JU' 09-10]**

(a) $x^2 + y^2 = 4y^2$ ✓
(b) $x^2 = 3y^2$ ✓
(c) both a & b ✓
(d) none

$$|y| = \frac{1}{2}\sqrt{(x-0)^2 + (y-0)^2}$$

$$\Rightarrow y^2 = \frac{1}{4}(x^2 + y^2)$$

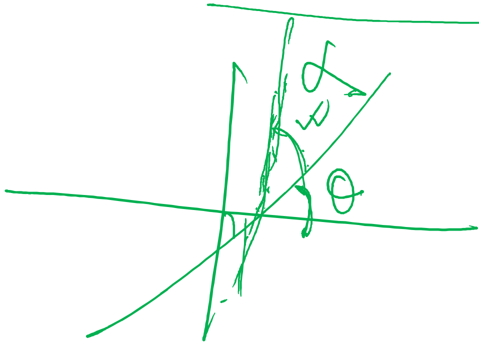
$$\Rightarrow x^2 + y^2 = 4y^2$$

$$\Rightarrow x^2 = 3y^2$$

Type-8: Finding the slop

- Slope of the line formed by joining (x_1, y_1) and (x_2, y_2), $\boxed{m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{\text{Difference of the ordinates}}{\text{Difference of the abscissas}}}$
- If a line segment form angle θ with positive side of x-axis, then slope $m = \tan\theta$
- If $\boxed{ax + by + c = 0}$ is the equation of a straight line, then slope, $m = \frac{-a}{b} = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$

Example: What is the angle Produced by $x - y + 4 = 0$ with the y axis? **[DU' 18-19]**



$x - y + 4 = 0$

$\Rightarrow y = x + 4$

Slope $= 1$

$\tan\theta = 1$

$\boxed{\theta = 45^\circ}$

$y = mx + c$

$\alpha = 90^\circ - \theta$

$= 90^\circ - 45^\circ$

$= 45^\circ$

Type-9: Determination of equation of straight line on different conditions

Case-01: Slope & intercept from y axis are given

A straight line has a slope of m and intercept at y-axis is c, then it's equation is ⇒ $\boxed{y = mx + c}$

Example: The slope of a straight line is 2 & it intercepts 5 unit from positive direction of y axis. Find the equation of the line.

$m = 2 \qquad c = 5$

$y = 2x + 5$ Ans.

Type-9: Determination of equation of straight line on different conditions

Case-02: If point & slope are given.

A straight line has a slope of m and it passes through point (x_1, y_1), then it's equation is ⇒

$$\boxed{(y - y_1) = m\,(x - x_1)}$$

Example: Find the equation of line passing through point (5, 2) and with slope 3.

$$y - 2 = 3(x - 5)$$

Ans.

Type-9: Determination of equation of straight line on different conditions

Case-03: Passing through two definite points.

A straight line passing through of (x_1, y_1) and (x_2, y_2), then equation of straight line is $\Rightarrow \boxed{\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}}$

Example: Find the equation of straight line passing through points (3,4) & (5,2).

$$\frac{x-3}{3-5} = \frac{y-4}{4-2}$$

Ans.

Type-9: Determination of equation of straight line on different conditions

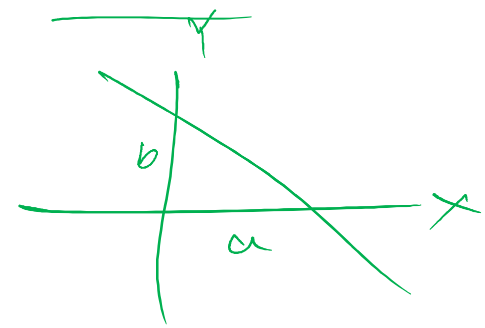
Case-04: The intercept from both axes are given.

If a straight line has intercept of a and b respectively from x-axis and y-axis, then it's equation is $\Rightarrow \boxed{\frac{x}{a} + \frac{y}{b} = 1}$

Example: A straight line intercepts 4 & 3 unit respectively from positive directions of x & y axis. Find the equation of the line.

$a = 4 \quad b = 3$

$$\boxed{\frac{x}{4} + \frac{y}{3} = 1}$$ Ans.



Type-9: Determination of equation of straight line on different conditions

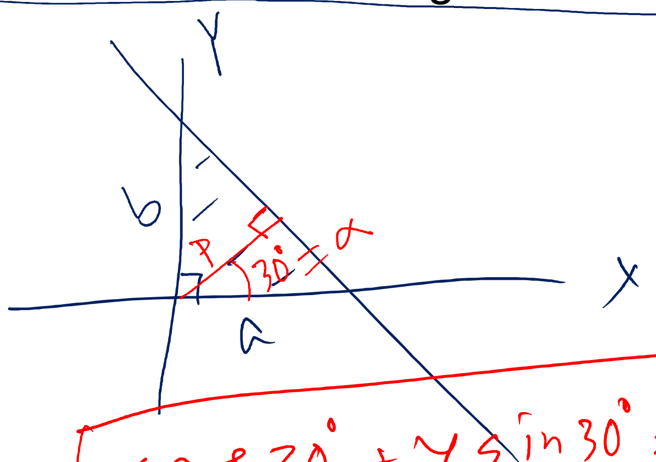
$\frac{x}{a}+\frac{y}{b}=1$

Case-05: Related to the formula $\boxed{x\cos\alpha + y\sin\alpha = p}$

Equation of a line is $x\cos\alpha + y\sin\alpha = p$, where 'p' is the perpendicular distance from the origin to the line and α is the angle made by the perpendicular with positive x-axis.

$\frac{x}{p/\cos\alpha}+\frac{y}{p/\sin\alpha}=1$

Example: A straight line forms a triangle of $\frac{50}{\sqrt{3}}$ sq. Unit with the axes and the perpendicular drawn to it from origin makes $30°$ angle with x-axis find the equation of the line **[RU'18-19]**



$\Delta=\frac{1}{2}ab$

$\frac{50}{\sqrt{3}}=\frac{1}{2}\cdot\frac{p}{\cos\alpha}\cdot\frac{p}{\sin\alpha}$

$\frac{50}{\sqrt{3}}=\frac{p^2}{\sin 2\alpha}=\frac{p^2}{\sin 60°}$

$\Rightarrow \frac{50}{\cancel{\sqrt{3}}}=\frac{p^2}{\cancel{\sqrt{3}}/2}\Rightarrow p^2=25$

$\therefore p=5$

$\boxed{x\cos 30° + y\sin 30° = 5}$

Ans.

Type-10: Equation of parallel line

Example: Find the equation of a line parallel to the line $3x + 4y + 5 = 0$ which passes through the point $(1, 2)$. **[JU' 18-19, RU' 09-10]**

$ax + by + c = 0$

Parallel: $ax + by + k = 0$

$3x + 4y + k = 0$

$(1,2) \rightarrow 3\times1 + 4\times2 + k = 0$

$\Rightarrow k = -(3\times1 + 4\times2)$

Soln:

$3x + 4y - (3\times1 + 4\times2) = 0$

$k =$

Poll Question-06

For what value of α, $(\alpha - 1)x + (\alpha + 1)y - 7 = 0$ line will be parallel to $3x + 5y + 4 = 0$? **[RU'08-09]**

(a) 4
(b) -4
(c) 0
(d) 7

$\alpha - 1 = 3$ $\alpha = 4$
$\alpha + 1 = 5$

$$\left\{\begin{array}{l} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{array}\right\}$$

Parallel : $\boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2}}$

Type-11: Equation of perpendicular line

Example: Find the equation of a line perpendicular to the line $3x + 4y + 5 = 0$ which passes through the point (3, 4). **[RU' 08-09]**

$ax + by + c = 0$

Perpendicular $\rightarrow bx - ay + k = 0$

Sol^n:

Perpendicular

$4x - 3y - (4\times3 - 3\times4) = 0$

$\Rightarrow 4x - 3y = 0$

Ans.

Poll Question-07

For what value of k, $2x + 3y + 5 = 0$ line will be perpendicular to $\boxed{3x + ky + 6 = 0}$?
[RU'08-09,11-12]

$3x - 2y$

(a) 2

(b) 3

(c) -2

(d) -3

Type-12: Angle between two straight lines

- The slope of two straight line is m_1 and m_2. If the angle between them is θ then, $\boxed{\tan\theta = \pm\frac{m_1-m_2}{1+m_1m_2}}$
- Taking +ve value of $\tan\theta$,acute angle is found
- Taking $-$ve value of $\tan\theta$,obtuse angle is found

Example: Determine the acute angle between $\underbrace{2x+3y-1=0}_{m_1=-\frac{2}{3}}$ and $\overbrace{x-2y+3=0}^{m_2=-\frac{1}{-2}=\frac{1}{2}}$.

[DU' 18-19, 08-09, JU' 18-19]

$$\tan\theta = \frac{m_1-m_2}{1+m_1m_2}$$

$$= \frac{-2/3-1/2}{1+\left(-\frac{2}{3}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{-\frac{7}{6}}{\frac{2}{3}} = -\frac{7}{4} \rightarrow \text{obtuse}$$

$$\theta = \tan^{-1}\left(-\frac{7}{4}\right)$$

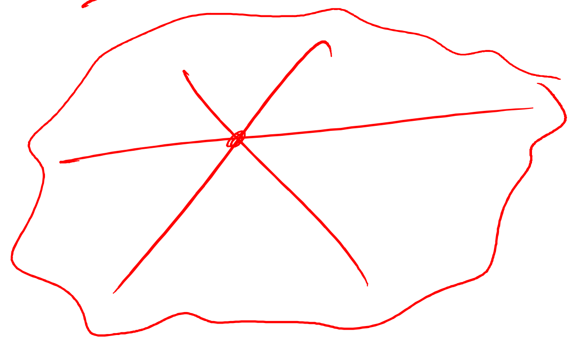
Acute Angle $= 180^\circ - \tan^{-1}\left(-\frac{7}{4}\right)$

Ans:

Type-13: Problems related to concurrence of three lines

Three lines $a_1x + b_1y + c_1 = 0; a_2x + b_2y + c_2 = 0;\ a_3x + b_3y + c_3 = 0$ are said to be concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$



Example: If three lines $2x + by + 4 = 0,\ 4x - y - 2 = 0$ and $3x + y - 1 = 0$ are concurrent then determine the value of b. **[DU' 14-15]**

$$\begin{vmatrix} 2 & b & 4 \\ 4 & -1 & -2 \\ 3 & +1 & -1 \end{vmatrix} = 0$$

$b = ??$ H.W.

Type-14: Perpendicular distance from a point

Perpendicular distance of $ax + by + c = 0$ line from the point (x_1, y_1), $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$ units

Example: Determine the length of Perpendicular on the line $4x - 3y + 1 = 0$ from the point $(-2,1)$. **[JU' 10-11]**

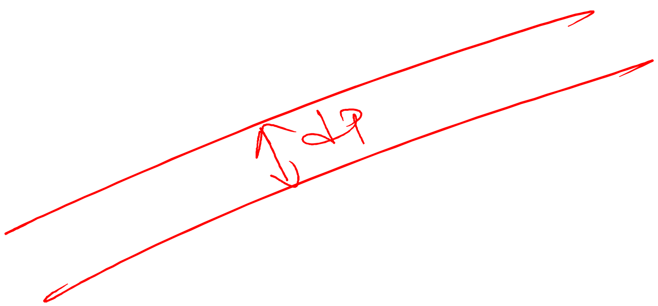
$$d_p = \frac{|4(-2) - 3(1) + 1|}{\sqrt{4^2 + (-3)^2}}$$

$$= \frac{10}{5} = 2 \text{ units}$$

Type-15: Distance between two parallel lines

Distance between two parallel line $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $d_p = \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$ unit

Example: Determine the distance between two parallel lines $5x + 12y + 3 = 0$ and $5x + 12y + 29 = 0$.



$$d_p = \frac{|3 - 29|}{\sqrt{5^2 + 12^2}} \text{ units}$$

$$= \frac{26}{13} \text{ units}$$

$$= 2 \text{ units} \quad \text{Ans.}$$

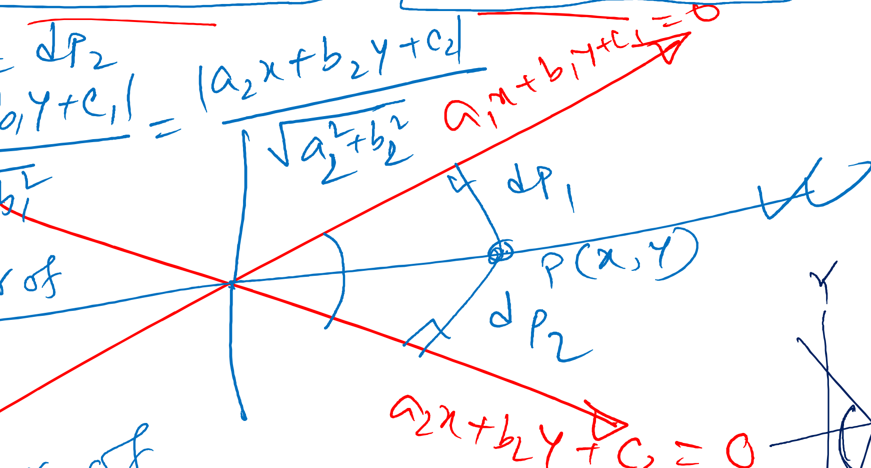
Type-16: Equation of angular bisector

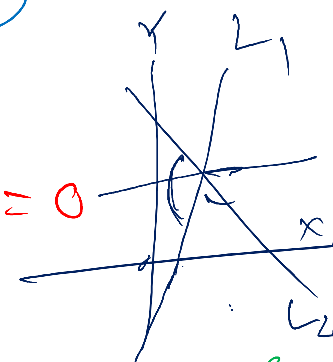
Equation of the bisectors of the angle formed by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

Is, $$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm \frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

$dp_1 = dp_2$

$$\Rightarrow \frac{|a_1x+b_1y+c_1|}{\sqrt{a_1^2+b_1^2}} = \frac{|a_2x+b_2y+c_2|}{\sqrt{a_2^2+b_2^2}}$$





i) $a_1a_2+b_1b_2$ → using it's sign, we will get the bisector of obtuse angle.

→ using it's opposite sign, we will get the bisector of acute angle.

ii) $a_1a_2+b_1b_2=0$ → Perpendicular to each-other.

iii) c_1c_2 → using it's sign; we will get the bisector of the angle which contains the origin.

Type-16: Equation of angular bisector

Board

V.V.I.

Example: Find The angular bisectors of the lines $y = 2x + 1$ and $2y - x = 4$. Also find the bisector of the angle which contains the origin ?

→ acute/obtuse

$2x - y + 1 = 0$ $\quad$ $x - 2y + 4 = 0$

i) Acute:

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \boxed{-} \frac{x - 2y + 4}{\sqrt{1^2 + (-2)^2}}$$

ii) Obtuse:

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \boxed{+} \frac{x - 2y + 4}{\sqrt{1^2 + (-2)^2}}$$

is the bisector of the angle which contains the origin.

$c_1 c_2 = (+1)(+4)$

$= +4$

$a_1 a_2 + b_1 b_2 = 2 \times 1 + (-1)(-2)$

$= +4 \rightarrow +ve$

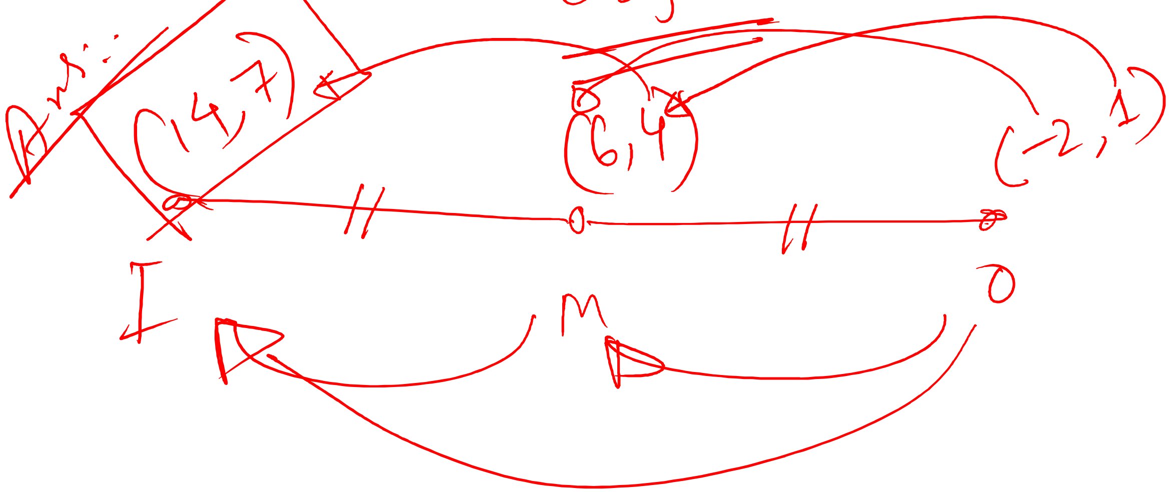
Type-17: Problems related to determining image

Case 1: Image of a point with respect to another point

Example: Find the image of the point (-2,1) with respect to the point (6,4) .

object

Mirror



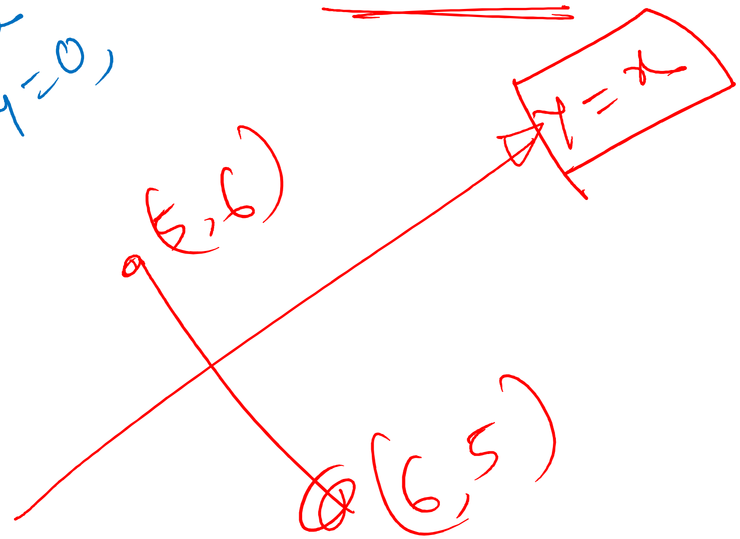
Type-17: Problems related to determining image

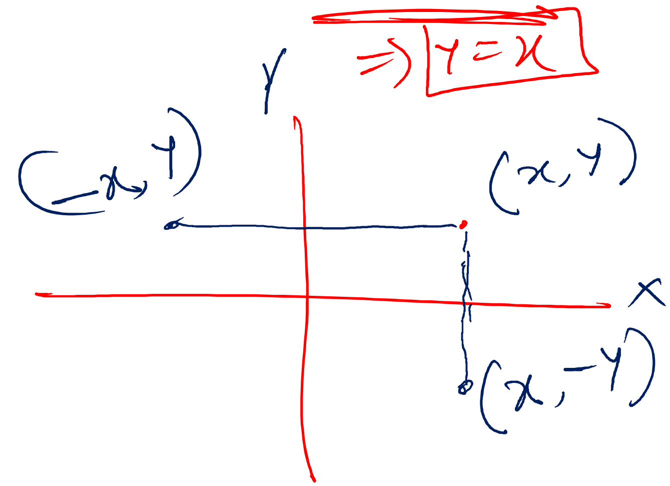
Case 2: Image of a point with respect to a straight line

x - axis
y - axis
y = x → line

Example: Find the image of the point (5,6) with respect to the straight line x – y =0

Image of (x_1, y_1) with respect to the line $y = x$ or $x - y = 0$, is (y_1, x_1)





Type-17: Problems related to determining image

Case 3: Image with respect to x or y axis

Example: Find the image of $(3, -2)$

(a) With respect to the x axis $\longrightarrow (3, +2)$

(b) With respect to the y axis $\longrightarrow (-3, -2)$

না বুঝে
মুখস্থ করার
অভ্যাস প্রতিভাকে
ধ্বংস করে