



১০ম শ্রেণি একাডেমিক প্রোগ্রাম ২০২০

উচ্চতর গণিত

লেকচার : HM-24

অধ্যায় ৮.৩ : ত্রিকোণমিতি

$$x = \sqrt{\frac{c^2}{c} + c - \frac{b}{2}}$$



বিভিন্ন কোণের ত্রিকোণমিতিক অনুপাত

- সূক্ষ্মকোণ
 - $0 < \theta < \frac{\pi}{2}$
- যেকোনো কোণের জন্য
অনুপাত
 - ৪৫
 - ১১০
 - ২১০
 - ৩৩০

Ratios \ Angles	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\text{cosec } \theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot \theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$(-\theta)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin \theta =$$

$$\frac{y}{r}$$

$$\cos \theta =$$

$$\frac{x}{r}$$

$$\tan \theta =$$

$$\frac{y}{x}$$

$$ON = x$$

$$PN = y$$

$$OP = r = \sqrt{x^2 + y^2}$$

$\triangle OQN - Q$

$$\sin(-\theta) = -\frac{y}{r} = -\frac{y}{r} = -\sin \theta$$

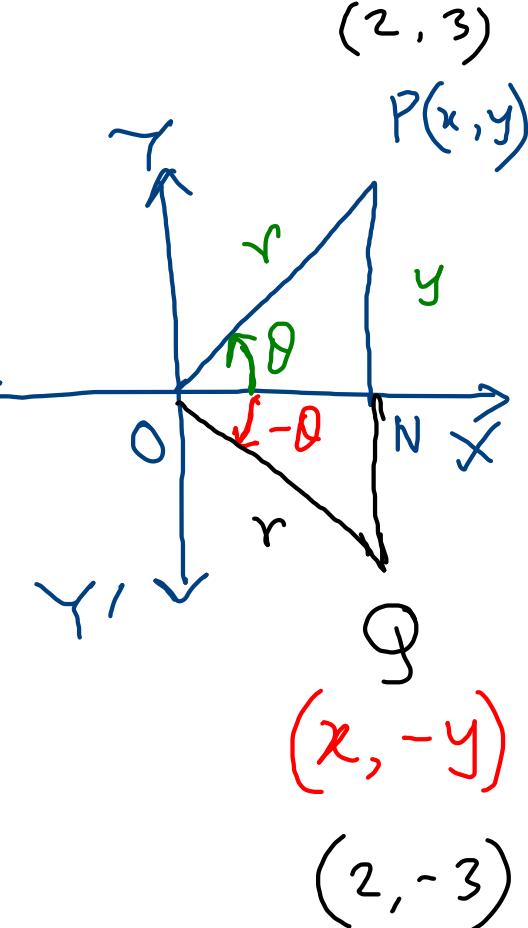
$$OQ = r$$

$$ON = x$$

$$QN = -y$$

$$\cos(-\theta) = \frac{x}{r} = \cos \theta$$

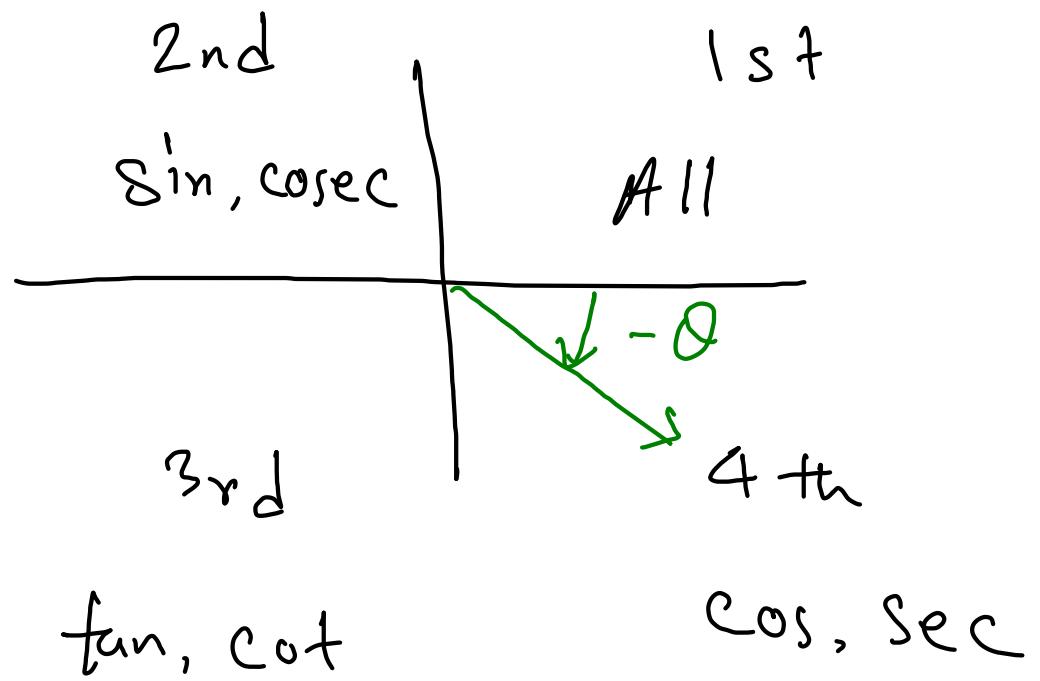
$$\tan(-\theta) = -\frac{y}{x} = -\frac{y}{x} = -\tan \theta$$



$$\sin(-\theta) = - \sin \theta$$

$$\cos(-\theta) = + \cos \theta$$

$$\tan(-\theta) = - \tan \theta$$



Poll Question 01

$$\cos\left(-\frac{\pi}{3}\right) = ?$$

$$\cos(-\theta) = \cos\theta$$

- (a) $\frac{1}{2}$
- (b) $-\frac{1}{2}$
- (c) $\frac{\sqrt{3}}{2}$
- (d) $-\frac{\sqrt{3}}{2}$

$$\begin{aligned} & \cos \frac{\pi}{3} \\ &= \frac{1}{2} \end{aligned}$$

Poll Question 02

$$\sin\left(-\frac{\pi}{3}\right) = ?$$

- (a) $\frac{1}{2}$
- (b) $-\frac{1}{2}$
- (c) $\frac{\sqrt{3}}{2}$
- (d) $-\frac{\sqrt{3}}{2}$

$$\sin(-\theta) = - \sin\theta$$

$$-\sin\frac{\pi}{3}$$

$$= -\frac{\sqrt{3}}{2}$$

$\left(\frac{\pi}{2} - \theta\right)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin \theta = \frac{y}{r}$$

$$r = \sqrt{x^2 + y^2}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x}$$

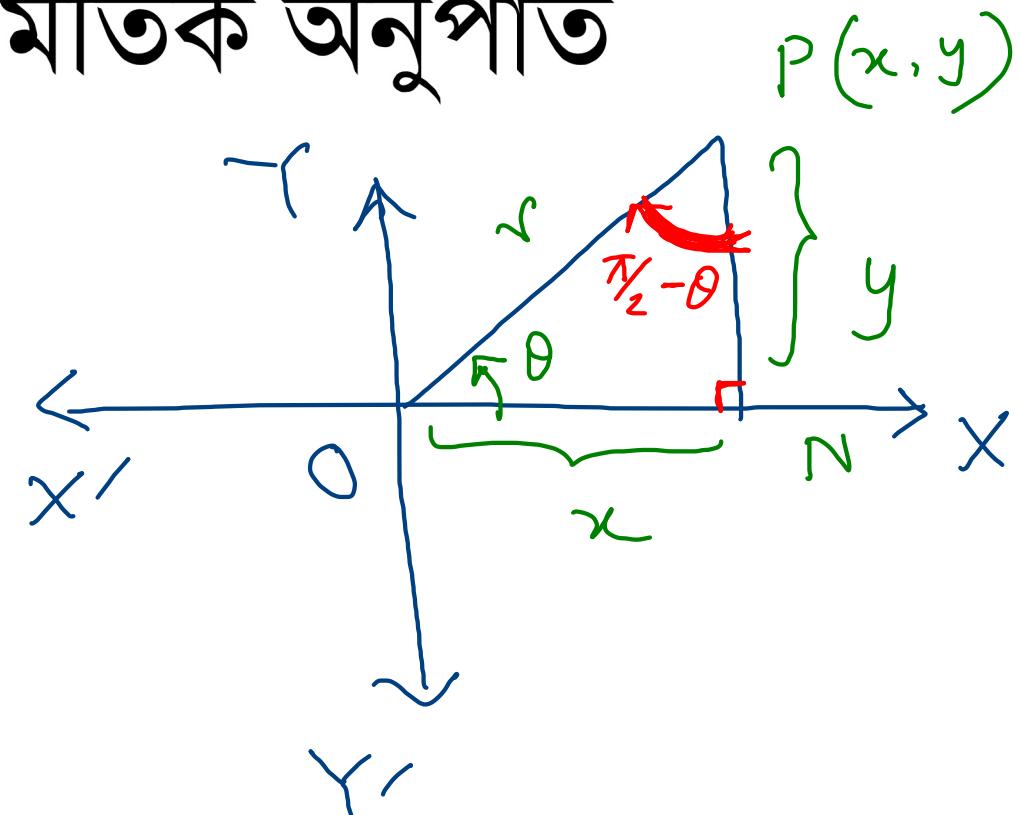
$$\cot \theta = \frac{x}{y}$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{y}{r} = \cos \theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \frac{y}{r} = \sin \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{x}{y} = \cot \theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \frac{y}{x} = \tan \theta$$



$$\sec\left(\frac{\pi}{2} - \theta\right) = \cosec \theta$$

$\left(n \frac{\pi}{2} \pm \theta\right)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

n পূর্ণসংখ্যা

n জোড় হলে

অনুপাত অপরিবর্ত্তিত

n বিজোড় হলে

$\sin \longleftrightarrow \cos$

$\tan \longleftrightarrow \cot$

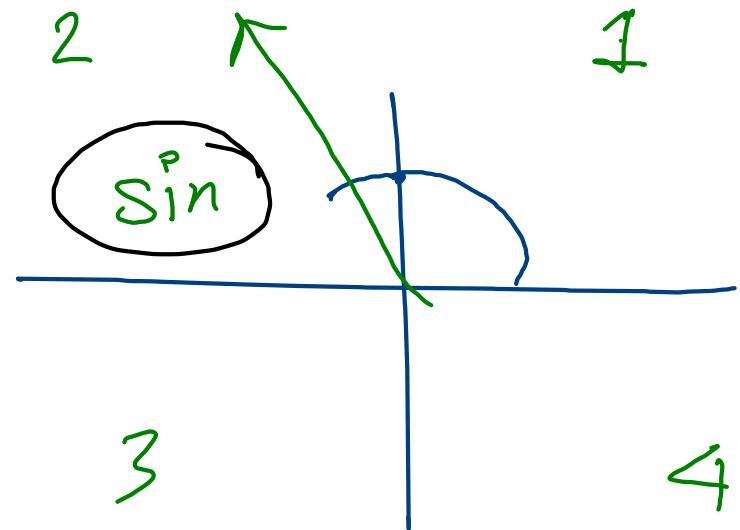
$\sec \longleftrightarrow \csc$

$\left(\frac{\pi}{2} + \theta\right)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin\left(\frac{\pi}{2} + \theta\right) = + \cos\theta$$

$$\cos\left(\frac{\pi}{2} + \theta\right) = - \sin\theta$$

$$\tan\left(\frac{\pi}{2} + \theta\right) = - \cot\theta$$

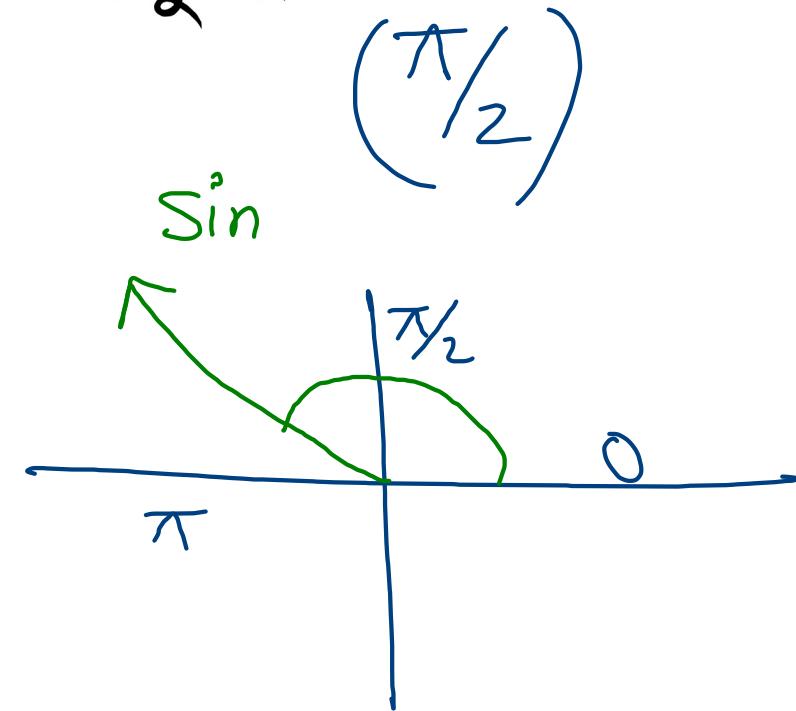


$(\pi - \theta)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin\left(\frac{\pi}{2} - \theta\right) = + \sin\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = - \cos\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = - \tan\theta$$

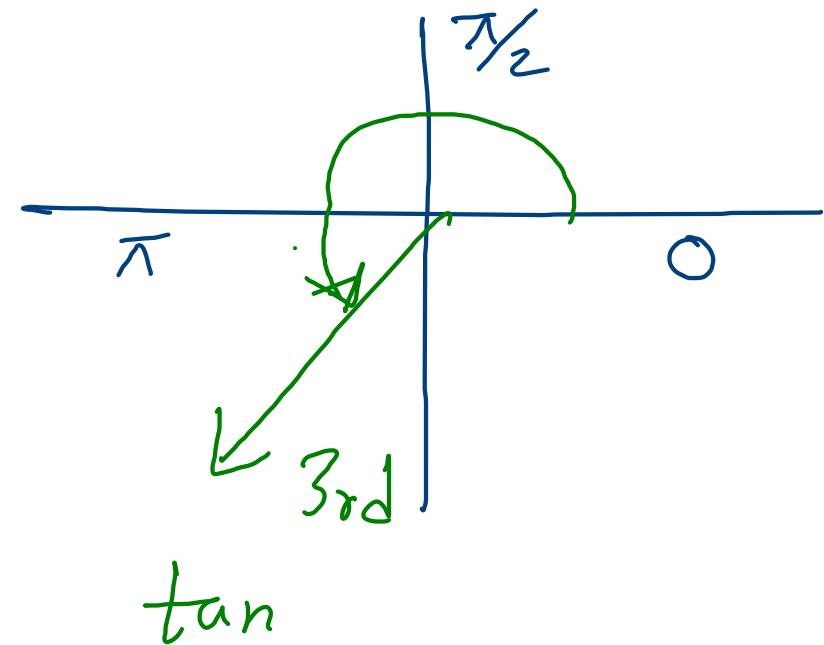


$(\pi + \theta)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin\left(2\frac{\pi}{2} + \theta\right) = -\sin\theta$$

$$\cos\left(2\frac{\pi}{2} + \theta\right) = -\cos\theta$$

$$\tan\left(2\frac{\pi}{2} + \theta\right) = +\tan\theta$$

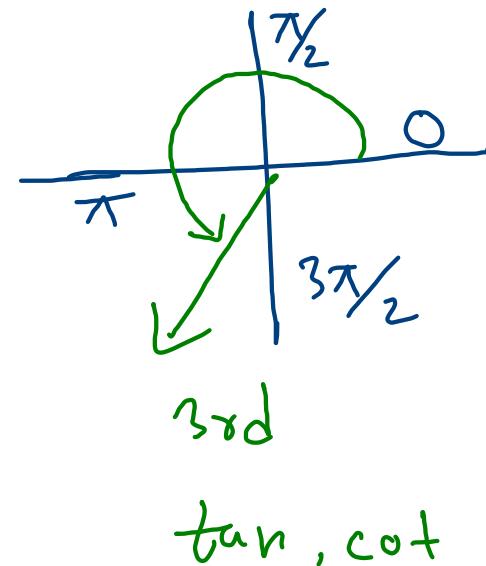


$\left(\frac{3\pi}{2} - \theta\right)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = +\cot\theta$$



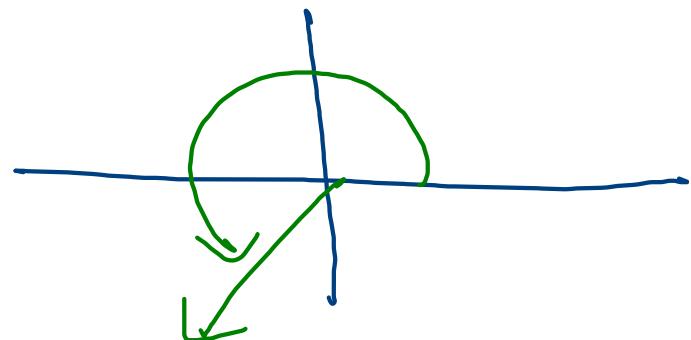
Poll Question 03

$$\tan\left(\frac{3\pi}{2} - \frac{\pi}{3}\right) = ?$$

$$\tan\left(\frac{3\pi}{2} - \theta\right) = \cot \theta$$

- (a) $\frac{1}{\sqrt{3}}$
 (b) $-\frac{1}{\sqrt{3}}$
 (c) $\sqrt{3}$
 (d) $-\sqrt{3}$

$$+ \cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

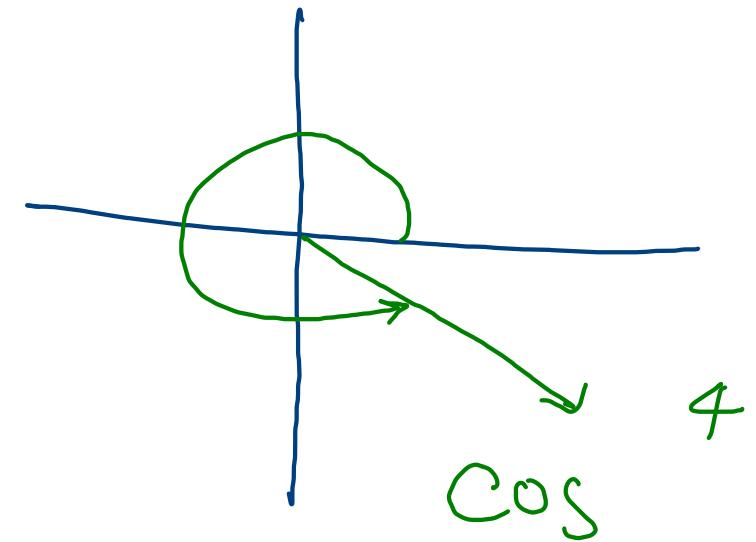


$\left(\frac{3\pi}{2} + \theta\right)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$$

$$\cos\left(\frac{3\pi}{2} + \theta\right) = +\sin\theta$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot\theta$$



Poll Question 04

$$\tan\left(\frac{3\pi}{2} + \frac{\pi}{3}\right) = ?$$

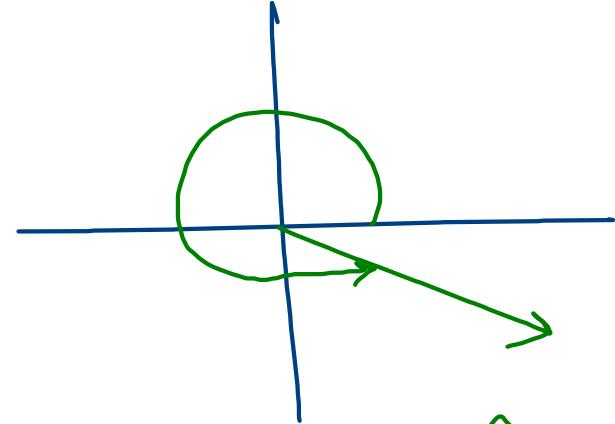
- (a) $\frac{1}{\sqrt{3}}$
- ~~(b) $-\frac{1}{\sqrt{3}}$~~
- (c) $\sqrt{3}$
- (d) $-\sqrt{3}$

$$\tan\left(\frac{3\pi}{2} + \theta\right) =$$

$$= \cot \underline{\theta}$$

$$\cot \frac{\pi}{3}$$

$$= -\frac{1}{\sqrt{3}}$$



4

Cos

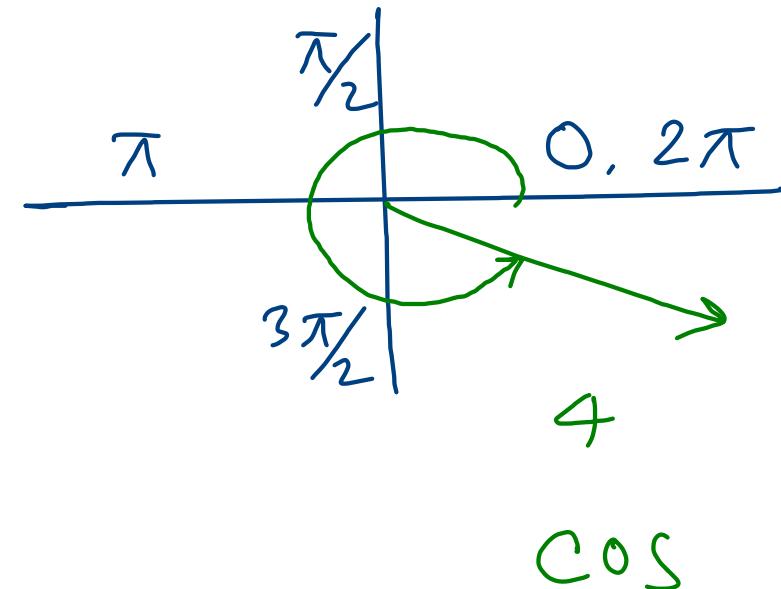
$(2\pi - \theta)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin(2\pi - \theta) = -\sin\theta$$

$$\cos(2\pi - \theta) = +\cos\theta$$

$$\tan(2\pi - \theta) = -\tan\theta$$

$$2\pi = 4(\frac{\pi}{2})$$

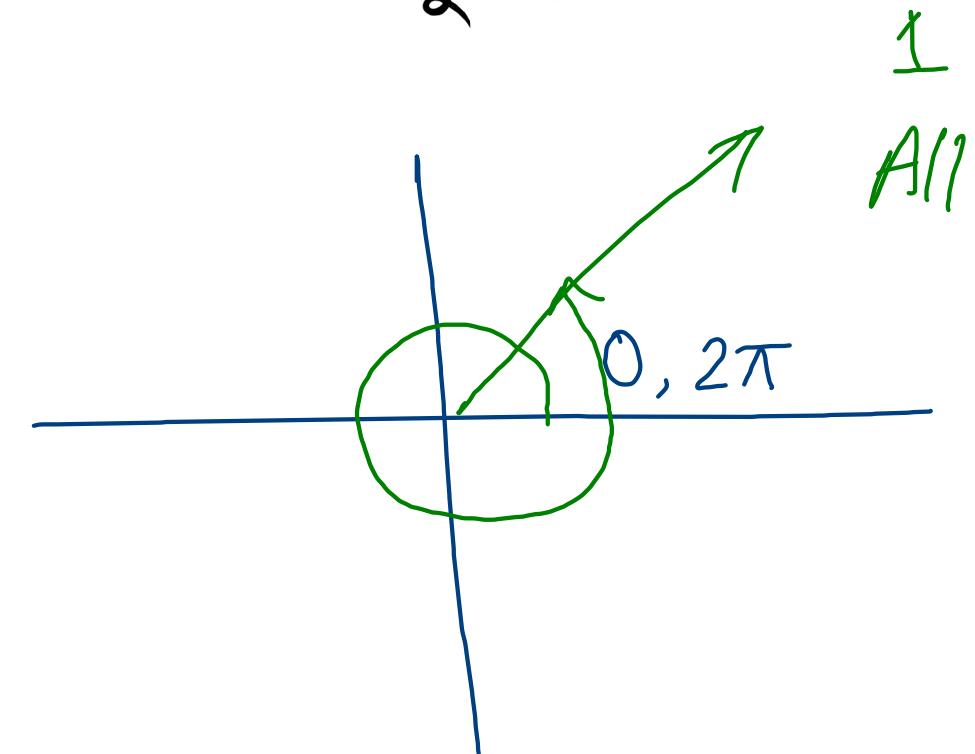


$(2\pi + \theta)$ কোণের জন্য ত্রিকোণমিতিক অনুপাত

$$\sin(2\pi + \theta) = + \sin \theta$$

$$\cos(2\pi + \theta) = + \cos \theta$$

$$\tan(2\pi + \theta) = + \tan \theta$$



মান নির্ণয় কর

$$\sin\left(\frac{31\pi}{6}\right) = ?$$

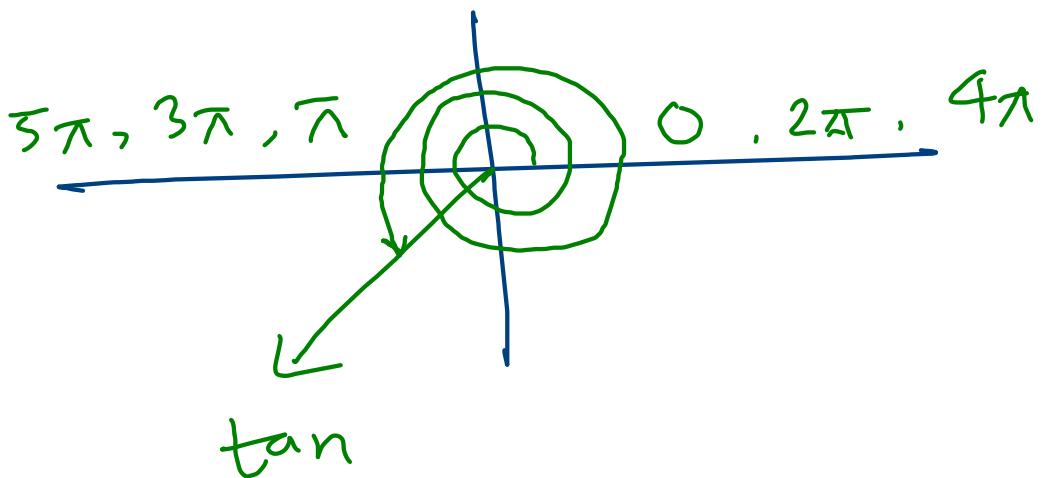
$$= \sin\left(5\pi + \frac{\pi}{6}\right)$$

$$= -\sin\frac{\pi}{6}$$

$$= -\frac{1}{2}$$

$$31 = 5 \times 6 + 1$$

$$5\pi = 10\left(\frac{\pi}{2}\right)$$



মান নির্ণয় কর

$$\cot\left(\theta - \frac{9\pi}{2}\right) = ?$$

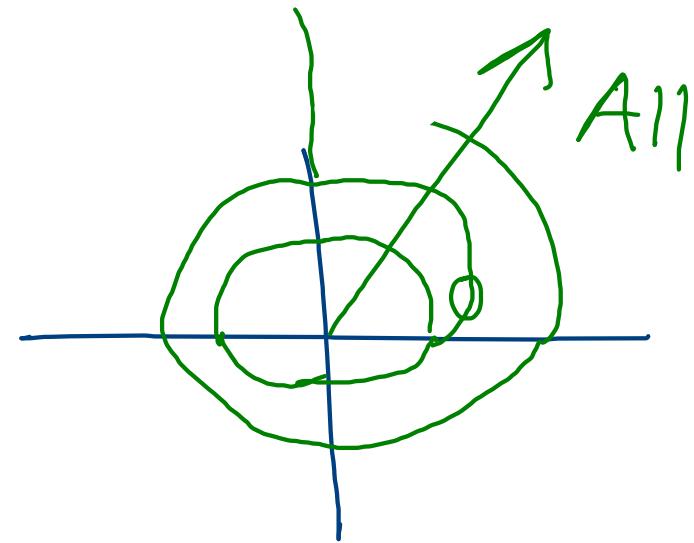
$$= \cot \left\{ -\left(\frac{9\pi}{2} - \theta\right) \right\}$$

$$= -\cot \left(\frac{9\pi}{2} - \theta \right)$$

$$= -\tan \theta$$

$$= -\tan \theta$$

$$\cos(-\theta) = \cos \theta$$



Poll Question 05

$$\tan\left(\frac{11\pi}{6}\right) = ?$$

- (a) $\frac{1}{\sqrt{3}}$
- (b) $-\frac{1}{\sqrt{3}}$
- (c) $\sqrt{3}$
- (d) $-\sqrt{3}$



একাডেমিক এন্ড এডমিশন কেয়ার

$\tan \theta = \frac{5}{12}$ এবং $\cos \theta$ খণ্ডাত্মক হলে, প্রমাণ কর যে, $\frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta} = \frac{51}{26}$

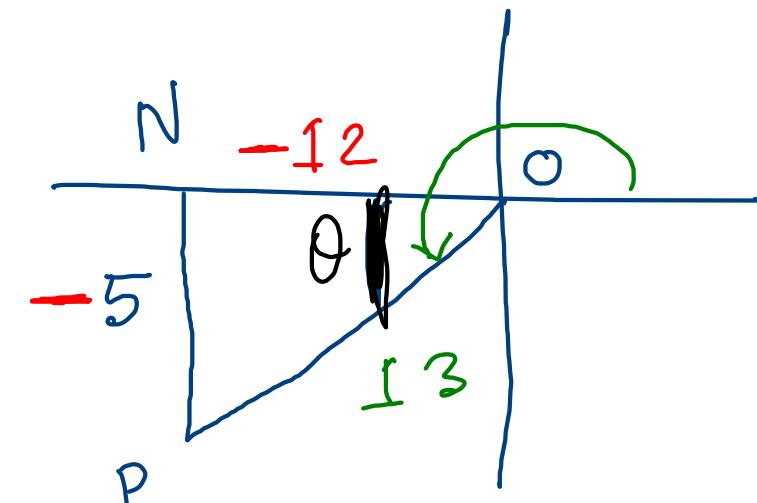
$$OP = \sqrt{12^2 + 5^2} = 13$$

$$\sin \theta = \frac{-5}{13}$$

$$\cos \theta = \frac{-12}{13}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{-13}{12}$$

$$\tan \theta = \frac{\text{পর্যাপ্ত}}{\text{তড়োপি}}$$



$$\text{L.H.S.} = \frac{\sin \theta + \cos(-\theta)}{\sec(-\theta) + \tan \theta}$$

$$= \frac{\sin \theta + \cos \theta}{\sec \theta + \tan \theta}$$

$$= \frac{-\frac{5}{13} + \frac{-12}{13}}{-\frac{13}{12} + \frac{5}{12}}$$

$0 < \theta < \frac{\pi}{2}$ হলে $\sin \theta + \cos \theta = \sqrt{2}$ সমীকরণটি সমাধান কর।

$$\sin \theta = \sqrt{2} - \cos \theta$$

$$\text{সু, } \sin^2 \theta = (\sqrt{2} - \cos \theta)^2$$

$$\sqrt{2} \cos \theta - 1 = 0$$

$$\text{সু, } \underline{\sin^2 \theta} = (\sqrt{2})^2 - 2\sqrt{2} \cos \theta + \underline{\cos^2 \theta}$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$\text{সু, } 1 - \cos^2 \theta = 2 - 2\sqrt{2} \cos \theta + \cos^2 \theta$$

$$= \cos \frac{\pi}{4}$$

$$2\cos^2 \theta - 2\sqrt{2} \cos \theta + 1 = 0$$

$$(\sqrt{2} \cos \theta - 1)^2 = 0$$

$$\theta = \cancel{\frac{\pi}{4}}$$

$0 < \theta < 2\pi$ হলে $\sin^2 \theta - \cos^2 \theta = \underline{\cos \theta}$ সমীকরণটি সমাধান কর।

$$1 - \cos^2 \theta - \cos^2 \theta = \cos \theta$$

$$\text{বা, } 2\cos^2 \theta + \cos \theta - 1 = 0$$

$$\text{বা, } 2\cos^2 \theta + 2\cos \theta - \cos \theta - 1 = 0$$

$$\text{বা, } 2\cos \theta (\cos \theta + 1) - 1(\cos \theta + 1) = 0$$

$$\text{বা, } (\cos \theta + 1)(2\cos \theta - 1) = 0$$

$$\text{হ্যাঁ } \cos \theta + 1 = 0 \quad \text{বা, } 2\cos \theta - 1 = 0$$

$$\cos \theta = -\frac{1}{2} \quad = \cos \pi$$

$$\cos \theta = \frac{1}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$= \cos \frac{\pi}{3}$$

$$= \cos \left(2\pi - \frac{\pi}{3}\right)$$

$\tan \theta = -\sqrt{3}$ এবং $\frac{\pi}{2} < \theta < 2\pi$ হলে θ এর মান কত?

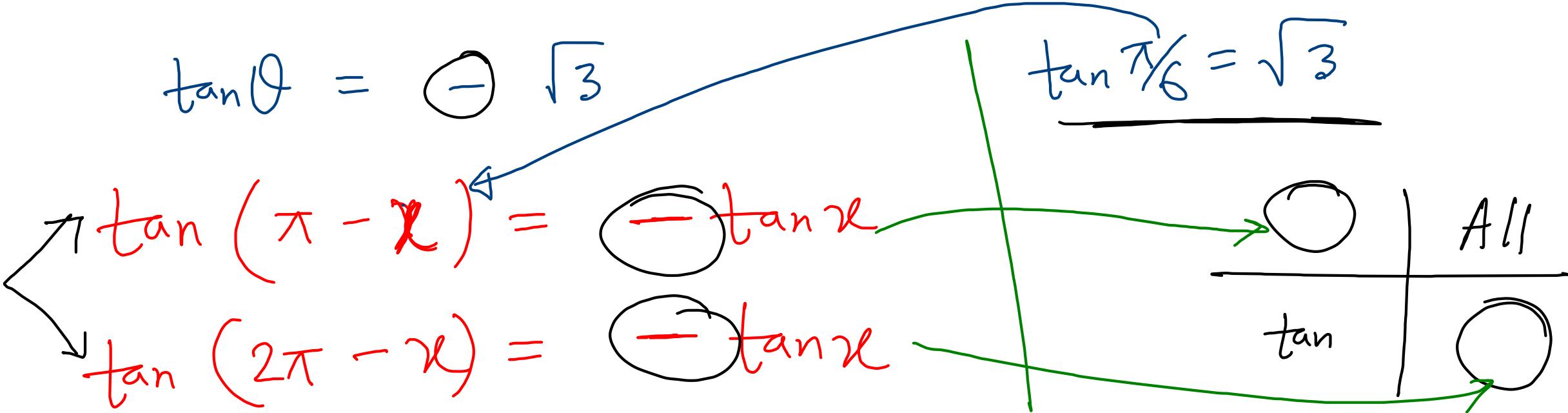
$$\frac{\pi}{2} < \theta < 2\pi$$

$$\tan \theta = -\sqrt{3}$$

$$\tan \frac{\pi}{6} = \sqrt{3}$$

$$\tan(\pi - x) = -\tan x$$

$$\tan(2\pi - x) = -\tan x$$



$$\theta = \pi - \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$$

Home Task

- উদাহরণ ভালভাবে প্র্যাকটিস করা
- উদাহরণের সাথে মিলিয়ে অনুশীলনীর অংক নিজে একবার চেষ্টা করা
(পরবর্তী ক্লাস বুর্ঝতে সুবিধা হবে এতে)

না বুঝে
মুখস্ত করার
অভ্যাস প্রতিভাকে
ধ্বংস করে



উদ্বাশ

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