

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

বিস্মিল্লাহির রাহমানির রাহীম

$$\begin{array}{r} 613 \text{ (2)} \\ 12 \\ \hline 7 \end{array}$$



উদ্ভাস

একাডেমিক এন্ড এডমিশন
কেয়ার

POLL 1

$$\begin{array}{r} 6 \overline{) 13} \\ \underline{12} \\ 1 \end{array}$$

$$\begin{array}{r} 6 \overline{) 13} \\ \underline{12} \\ 1 \end{array}$$

IF (-11) is divided by 3, then what will be the quotient?

a) -3 ~~x~~

b) 3 ~~x~~

c) -4 ✓

d) 0

e) None of above

$$\begin{array}{r} 3 \overline{) -11} \\ \underline{-9} \\ -2 \end{array}$$

$$-9 > -11$$

$$\begin{array}{r} 6 \overline{) 13} \\ \underline{6} \\ 7 \end{array}$$

$$\begin{array}{r} 3 \overline{) -11} \\ \underline{-12} \\ 1 \end{array}$$

$$\begin{array}{r} 3 \overline{) -11} \\ \underline{-9} \\ -2 \end{array}$$

$$9 > -11$$

POLL 2

IF (-11) is divided by 3, then what will be the remainder?

a)2

b)-2

c)-20

d)1

e)None of above

$$\begin{array}{r} 3 \overline{) -11} \quad (-4) \\ \underline{-12} \\ 1 \end{array}$$

Handwritten red ink showing the division of -11 by 3. The divisor is 3, the dividend is -11. The quotient is -4, and the remainder is 1, which is circled.

What is Remainder?

$$x-2=0 \therefore x=2$$

$$f(x) \rightarrow (x-a) \rightarrow f(a)$$

$$(x-3) \rightarrow f(3)$$

$$x+2=0 \therefore x=-2 \rightarrow f(-2)$$

Suppose, the dividend is $f(x)$, the divisor is $(x-a)$

And with them, the quotient is $h(x)$ and the remainder is r

$$f(x) = (x-a)h(x) + r$$

So, we can say,

$$f(x) = (x-a) \cdot h(x) + r$$

If we replace the value of x with a , we get,

$$f(a) = (a-a) \cdot h(a) + r$$

$$\text{Or, } f(a) = r$$

Which was our remainder.

$$\begin{array}{r} x-2 \overline{) x^2 - 8x + 6} \\ \underline{x^2 - 2x} \\ -6x + 6 \\ \underline{-6x + 12} \\ -6 \end{array}$$

$f(x) = x^2 - 8x + 6$
 $f(2) = r$
 $-6 \rightarrow r$

$$f(x) = x^2 - 8x + 6$$

$$f(2) = (2)^2 - 8 \cdot 2 + 6 = 4 - 16 + 6 = -6$$

Poll Question: 03

01. $f(x) = x^3 - x - 6$

If we divide $f(x)$ by $(x-3)$, what will be the remainder?

(a) 16

(b) 17

☒ (c) 18

(d) 19

✓

$x=3$

$f(3) = 3^3 - 3 - 6$
 $= 27 - 3 - 6$
 $= 18$

$\begin{array}{r} 36 \\ -9 \\ \hline 27 \end{array}$ (18)

The Remainder Theorem

- If you divide a polynomial $f(x)$ by $(x - a)$, then the remainder is $f(a)$.
- If you divide a polynomial $f(x)$ by $(x + a)$, then the remainder is $f(-a)$.
- If the degree of $f(x)$ is positive, and $a \neq 0$, then if you divide $f(x)$ by $(ax+b)$, the remainder will be $f(-b/a)$

$$2x+3=0$$

$$2x=-3$$

$$\therefore x = -\frac{3}{2}$$

$$\begin{array}{r} 6 \overline{) 12} \\ \underline{12} \\ 0 \end{array}$$

$$\begin{array}{r} 6 \overline{) 12} \\ \underline{12} \\ 0 \end{array}$$

Factor Theorem:

$$\begin{array}{ccc} f(x) & (x-a) & f(a) \\ f(x) & (2x+3) & f(-\frac{3}{2}) \end{array}$$

$$f(x) \rightarrow (x-a) \rightarrow f(a)$$

If and only if the value of $f(a)$ is 0, then the polynomial $f(x)$ is divisible by $(x-a)$.

$$\begin{array}{r} f(x) \\ (x+3) \end{array}$$

$$f(-3)=0$$

Then $(x-a)$ is a factor of $f(x)$.

$$\begin{aligned} f(x) &= x^2 - x - 6 \\ f(-2) &= (-2)^2 - (-2) - 6 \\ &= 4 + 2 - 6 = 0 \end{aligned}$$

$$(x+2) \rightarrow \text{factor of } f(x)$$

Poll Question: 4

If $f(x) = 3x^2 - 7x - 6$, then which one will be a factor of $f(x)$?

(a) $(x + 1)$

(b) $(x - 1)$

(c) $(x - 2)$

☒ (d) $(x - 3)$

$$\begin{aligned} f(-1) &= 3(-1)^2 - 7(-1) - 6 \\ &= 3 + 7 - 6 \\ &= 4 \end{aligned}$$

$$\begin{aligned} f(3) &= 3 \cdot 3^2 - 7 \cdot 3 - 6 \\ &= 27 - 21 - 6 \\ &= 0 \end{aligned}$$

$$3x^2 - 7x - 6$$

Time Consumption

- **Smart Way:** calculator

1. Write the total function $f(x)$ on your scientific calculator.
2. Make this function an equation by adding ' $=0$ ' at last part (you can get $=$ sign by pressing 'Alpha' and then 'CALC' button).
3. Click 'Shift' and then 'CALC'
4. Repeat point 3 again
5. BOOM! You got a value of a

$$\underline{x^2 - x - 6}$$



$$x^2 - x - 6 = 0 \quad \boxed{=}$$

x

?

EXAMPLES

Example : 1 $3a^3 + 2a + 5$

Example : 2 $x^3 + 2x^2 - 5x - 6$

Example : 3 $4a^4 + 12a^3 + 7a^2 - 3a - 2$

Example : 4 $a^3 - 7a^2b + 7a^2 - b^3$

Example : 5 $x^6 - x^5 + x^4 - x^3 + x^2 - x$

EXAMPLE

• **Example-1 :** $3a^3 + 2a + 5$

$$f(a) = 3a^3 + 2a + 5$$

$$f(-1) = 3(-1)^3 + 2(-1) + 5$$

$$f(-1) = -3 - 2 + 5 = 0$$

$(a+1)$, factor of $f(a)$

$$\begin{array}{r} 2^{2020} \\ 2^{2019} \overline{) 2^{2020}} \\ \underline{2^{2019}} \\ 2^{2019} \\ \underline{2^{2019}} \\ 0 \end{array}$$

$$\begin{array}{r} 2^{2020} - 2^{2019} \\ \underline{2^{2019}(2-1)} \\ 2^{2019}(1) \\ \underline{2^{2019}} \\ 0 \end{array}$$

$$\begin{aligned} & \underline{3a^3 + 2a + 5} \\ &= 3a^3 + 3a - 3a - 3a + 5a + 5 \\ &= 3a^2(\underline{a+1}) - 3a(\underline{a+1}) + 5(\underline{a+1}) \\ &= (a+1)(3a^2 - 3a + 5) \end{aligned}$$

$$2^{2020} - 2^{2019} = ?$$

- (a) 2 (d) 2^{2020}
 (b) 0 (e) 2^{2019}
 (c) 1 (f) none of the above

GRAPHICAL EXPLANATION:

Example- 1: $3a^3 + 2a + 5 = f(a)$

$$f(-1) = 0$$

$$(a+1)$$

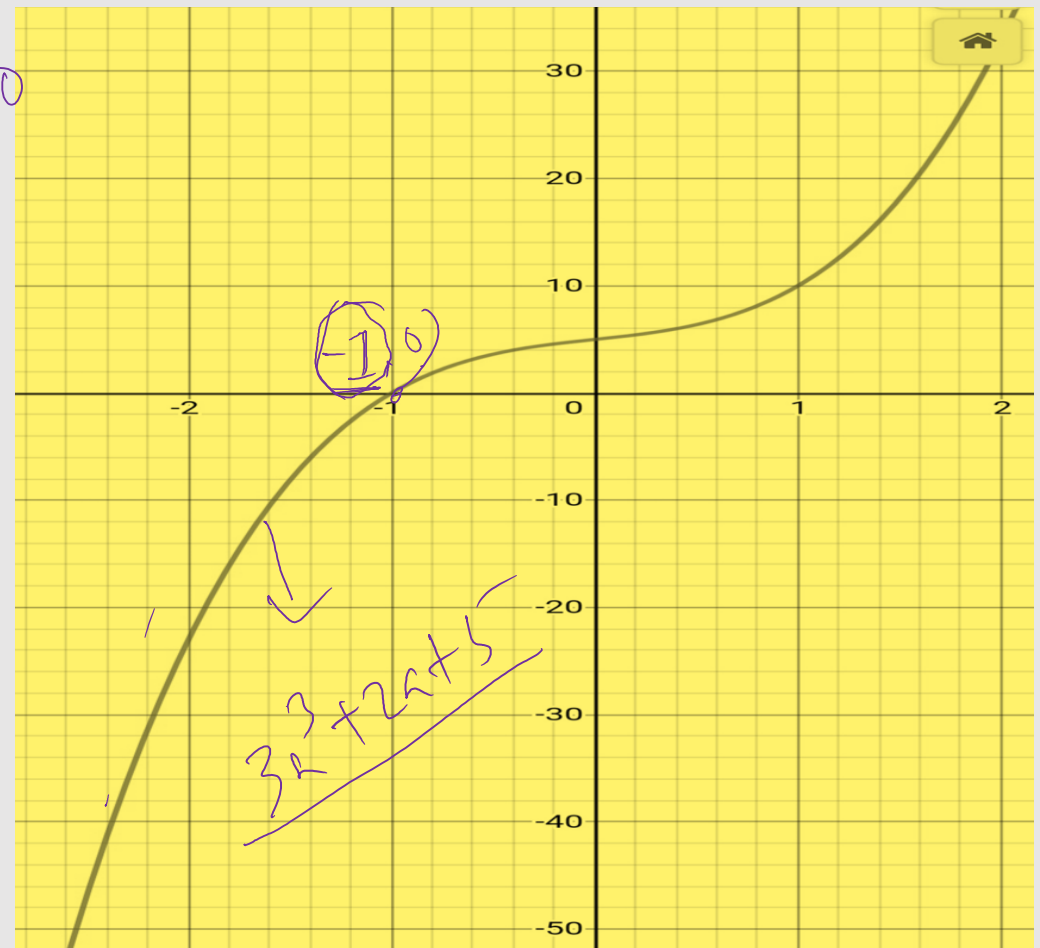
$$a = -1$$

$$(-1, 0)$$

$$a = -1$$
$$(a+1)$$

$$a = -1$$

$$(a+1)$$



EXAMPLES

• Example : 2

$$x^3 + 2x^2 - 5x - 6 = f(x)$$

$$\begin{aligned} f(2) &= 2^3 + 2 \cdot 2^2 - 5 \cdot 2 - 6 \\ &= 8 + 8 - 10 - 6 \\ &= \underline{\underline{0}} \end{aligned}$$

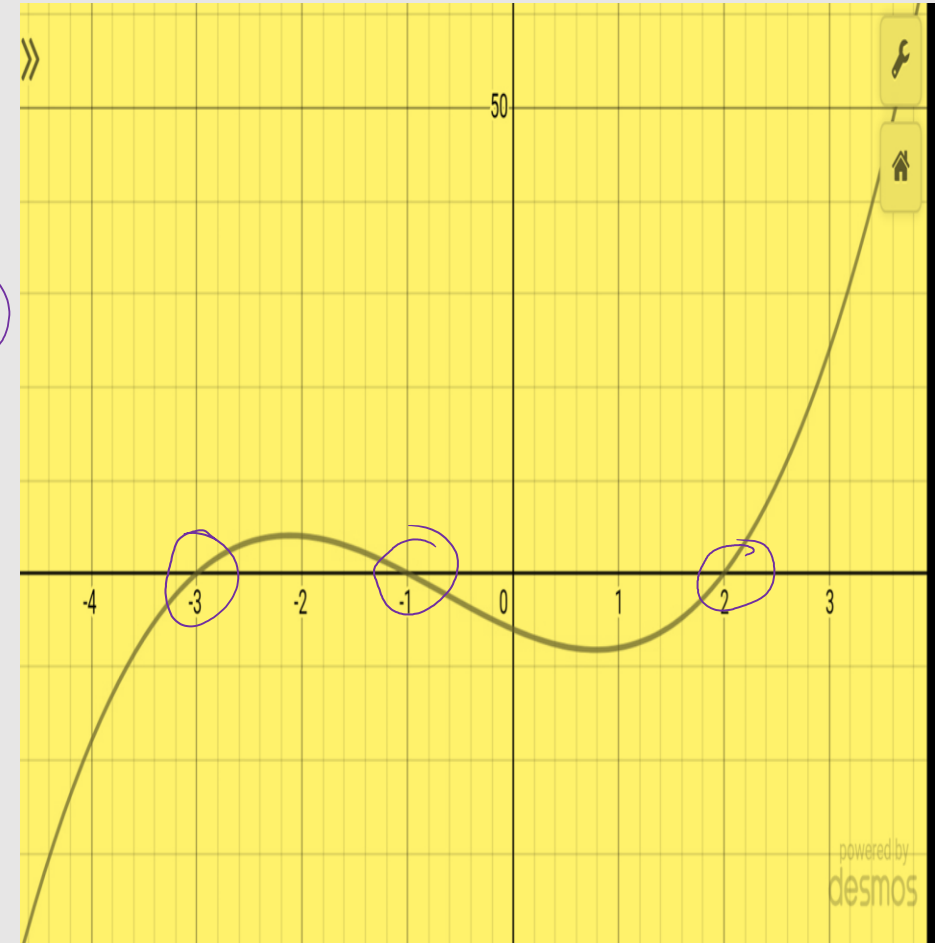
$(x-2)$ is a factor of $f(x)$

$$\begin{aligned} & x^3 + 2x^2 - 5x - 6 \\ &= x^3 - 2x^2 + 4x^2 - 8x + 3x - 6 \\ &= x^2(x-2) + 4x(x-2) + 3(x-2) \\ &= (x-2)(x^2 + 4x + 3) \\ &= (x-2)(x^2 + 3x + x + 3) \\ &= (x-2)\{x(x+3) + 1(x+3)\} \\ &= (x-2)(x+3)(x+1) \end{aligned}$$

GRAPHICAL EXPLANATION:

Example : 2 $x^3 + 2x^2 - 5x - 6$

$$\begin{aligned} x+3 &= 0 & x-2 &= 0 \\ x &= -3 & x &= 2 \\ x+1 &= 0 & & \\ x &= -1 & & \end{aligned}$$



Poll 5

$$(2x+1)(3x+2)(3x-1)$$

- The abscissas of the points where the graph of $f(x)=18x^3 + 15x^2 - x - 2$ will intersect are-

a) $-0.5, \frac{-2}{3}, \frac{-1}{3}$

b) $\frac{1}{2}, \frac{2}{3}, \frac{1}{3}$

c) $\frac{-1}{2}, \frac{-2}{3}, \frac{1}{3}$

d) $\frac{1}{2}, \frac{2}{3}, \frac{-1}{3}$

EXAMPLE

• Example : 3 $4a^4 + 12a^3 + 7a^2 - 3a - 2 = f(a)$

$$f(-2) = 4(-2)^4 + 12(-2)^3 + 7(-2)^2 - 3(-2) - 2$$

$= 0$
 $(a+1)$ is a factor of $f(a)$

$$\begin{aligned} & 4a^4 + 12a^3 + 7a^2 - 3a - 2 \\ &= 4a^4 + 4a^3 + 8a^3 + 8a^2 - a^2 - a - 2a - 2 \\ &= 4a^3(a+1) + 8a^2(a+1) - a(a+1) - 2(a+1) \\ &= (a+1)(4a^3 + 8a^2 - a - 2) \\ &= (a+1)(4a^3 + 8a^2 - a - 2) \\ &= (a+1)\{4a^2(a+2) - 1(a+2)\} \\ &= (a+1)(a+2)(4a^2 - 1) \\ &= (a+1)(a+2)\{2a^2 - 1\} \\ &= (a+1)(a+2)(2a+1)(2a-1) \end{aligned}$$

EXAMPLE

Example : 4 $\underline{a}^3 - 7\underline{a}^2\underline{b} + 7\underline{a}\underline{b}^2 - \underline{b}^3 = f(a)$

$$f(a) = a^3 - 7a^2b + 7ab^2 - b^3$$

$$f(b) = b^3 - 7b^2b + 7b^2b - b^3$$

$$= 0$$

$(a-b)^0$ is a factor of $f(a)$

$$101 - 102 = 1$$

$$a^3 - 7a^2b + 7ab^2 - b^3$$

$$= a^3 - a^2b - 6a^2b + 6ab^2 + ab^2 - b^3$$

$$= a^2(a-b) - 6ab(a-b) + b^2(a-b)$$

$$= (a-b)(a^2 - 6ab + b^2)$$

EXAMPLE:

Example : 5 $\underline{x^6} - \underline{x^5} + \underline{x^4} - \underline{x^3} + \underline{x^2} - \underline{x} = f(x) \rightarrow (x-1) \text{ is a factor of } f(x)$

$$= x(x^5 - x^4 + x^3 - x^2 + x - 1)$$

$$= x(x^5 - x^4 + x^3 - x^2 + x - 1)$$

$$= x \{ x^4(x-1) + x^3(x-1) + x^2(x-1) + x(x-1) + 1(x-1) \}$$

$$= x \{ (x-1)(x^4 + x^3 + x^2 + x + 1) \}$$

$$= x(x-1) \{ (x^2)^2 + 2 \cdot x^2 \cdot 1 + 1^2 - x^2 \}$$

$$= x(x-1) \{ (x^2 + 1)^2 - x^2 \}$$

$$= x(x-1)(x^2 + x + 1)(x^2 - x + 1)$$

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

ঔদ্যাম-উন্মেষ শিক্ষা
পরিবার

$$101 - 10^2 = 1$$

Thank You