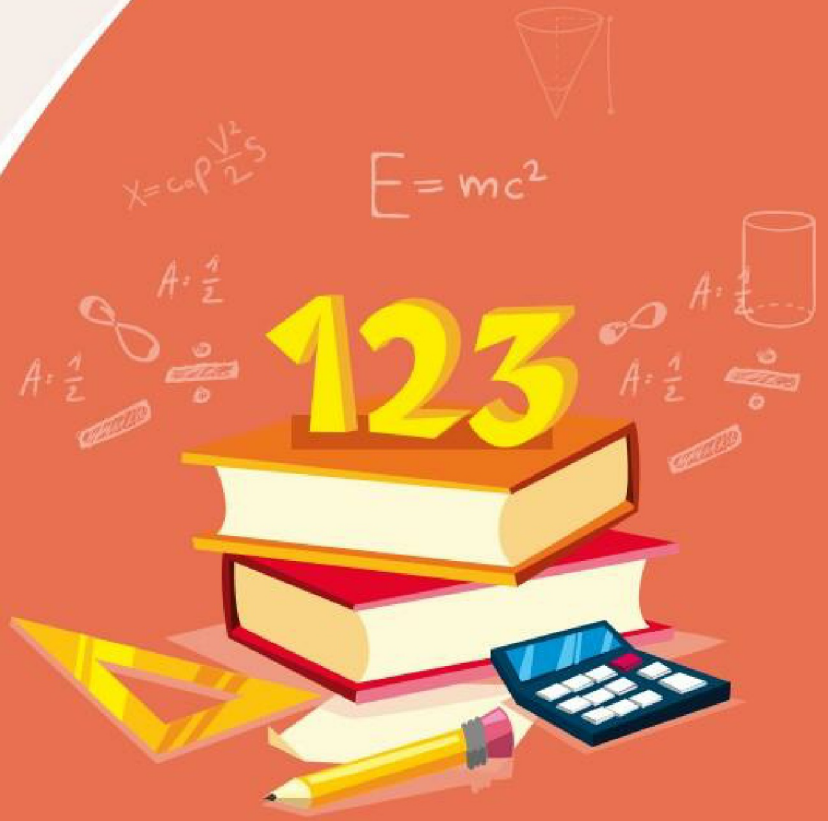
CLASS XI ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : HM-06

Chapter 3 : Straight lines

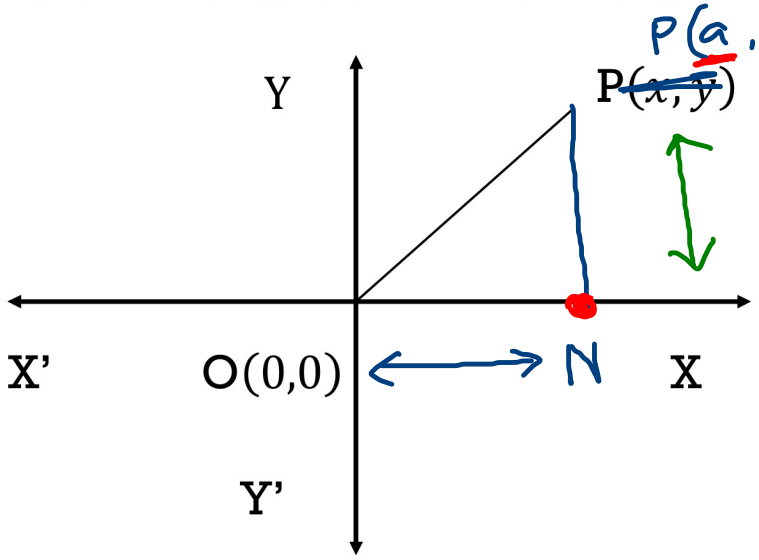


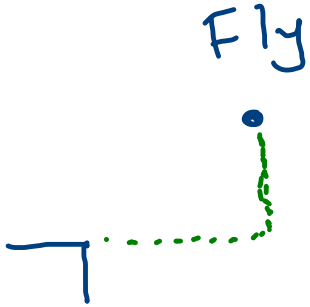
উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

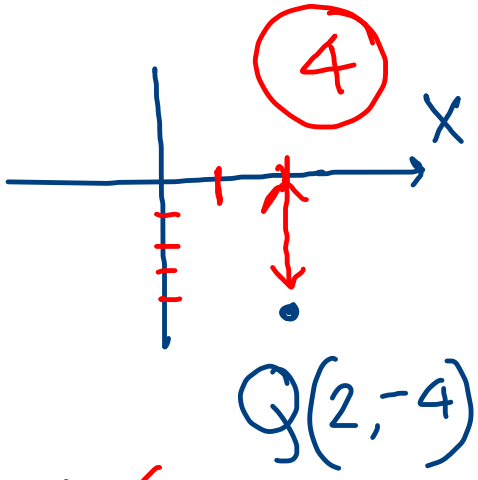
CARTESIAN CO-ORDINATE

ON = x

PN = y







Distance from y axis = |abscissa|

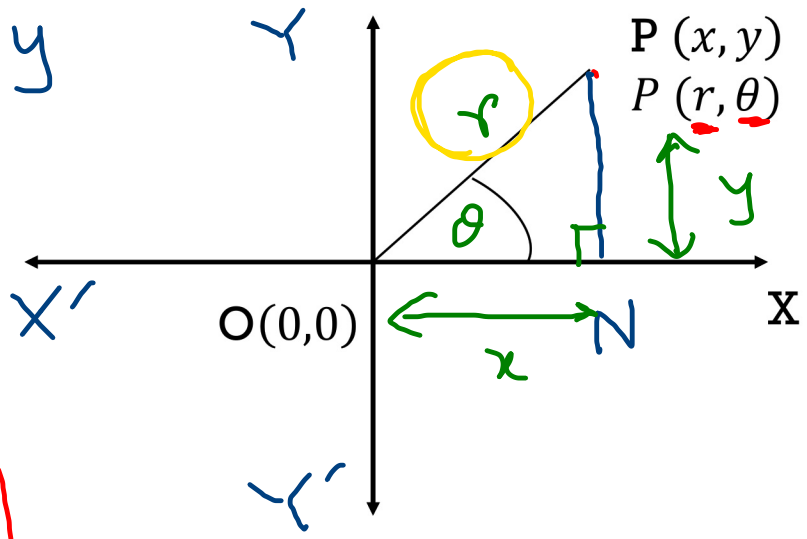
" " x " = |ordinate|

CARTESIAN AND POLAR CO-ORDINATE

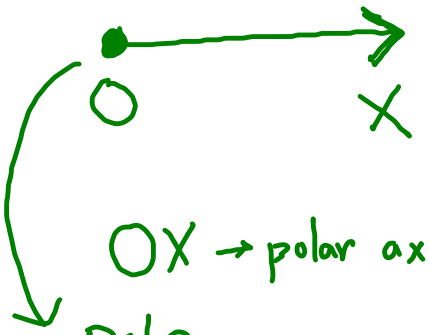
$ON = x$ $PN = y$

$OP = r$

$\angle PON = \theta$



polar



OX → polar axis

Pole

Cartesian → Polar

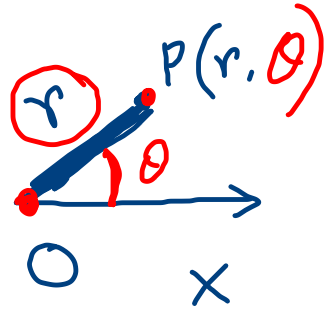
$$r^2 = x^2 + y^2$$

$$\tan\theta = y/x$$

Polar → Cart

$$y = r\sin\theta \leftarrow \sin\theta = y/r$$

$$x = r\cos\theta \leftarrow \cos\theta = x/r$$



POLL QUESTION-01

If $\left(2, \frac{\pi}{3}\right)$ is the polar co-ordinate of a point, what's the Cartesian co-ordinate ?

(a) $(1,1)$

(b) $(\sqrt{3}, 1)$

(c) $(1, \sqrt{3})$

(d) $\left(1, \sqrt{\frac{1}{3}}\right)$

$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$

$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

MATHEMATICAL PROBLEMS

Example : Express this equation in polar system: $x^2 + y^2 - ay = 0$

$r^2 - a\,r\sin\theta = 0$

$r(r - a\sin\theta) = 0$

$r - a\sin\theta = 0$

$r^2 = x^2 + y^2$

$\tan\theta = y/x$

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

Example : Express this equation in Cartesian system: $r(1 + \cos\theta) = 2$

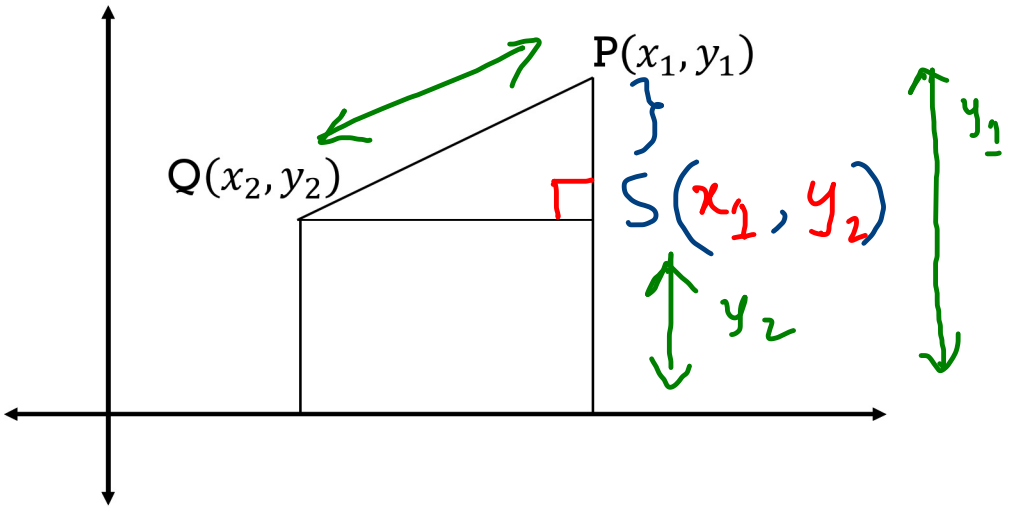
$r + r\cos\theta = 2$

$\sqrt{x^2+y^2} + x = 2$

$\left(\sqrt{x^2+y^2}\right)^2 = (2 - x)^2$

$x^2 + y^2 = 4 - 4x + x^2$

DISTANCE BETWEEN TWO POINTS



$PS = y_1 - y_2$

$QS = x_1 - x_2$

$PQ^2 = QS^2 + PS^2$

$PQ = \sqrt{QS^2 + PS^2}$

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

POLL QUESTION-02

Find the distance between the points $(4,3)$ and $(-2,5)$.

(a) $\sqrt{38}$

(b) 40

(c) $\sqrt{40}$

(d) $\sqrt{66}$

$$= \sqrt{(4+2)^2 + (3-5)^2}$$

$$= \sqrt{6^2 + 2^2}$$

$$= \sqrt{40}$$

MATHEMATICAL PROBLEMS

- Find the co-ordinate of a point of which, the ordinate is twice of the abscissa and which maintain $\sqrt{10}$ unit distance from the point $(4,3)$.

$P(x, 2x)$

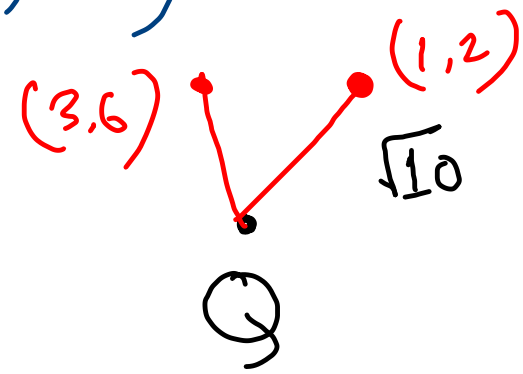
Q

$PQ = \sqrt{10}$

$\sqrt{(x-4)^2+(2x-3)^2} = \sqrt{10}$

. . .

$x = 1, 3$



$P(1, 2)$

$(3, 6)$

MATHEMATICAL PROBLEMS

- Show that, A$(1,2)$, B$(-4,2)$ and C$(-4,7)$ form an isosceles right triangle; determine the area of the triangle.

$$AB = \sqrt{(1+4)^2+(2-2)^2} = 5$$

$$BC = \sqrt{(-4+4)^2+(2-7)^2} = 5$$

$$CA = \sqrt{(-4-1)^2+(2-7)^2} = \sqrt{5^2+5^2} = 5\sqrt{1^2+1^2}$$

$$= 5\sqrt{2}$$

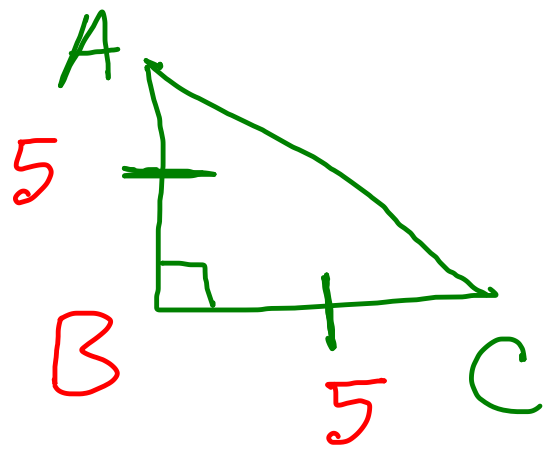
$$AB = BC \neq CA$$

$\therefore$ Isosceles triangle

$AB = 5$ ✓

$BC = 5$ ✓

$AC = 5\sqrt{2}$



$$AC^2 = (5\sqrt{2})^2 = 50$$

$$= AB^2 + BC^2$$

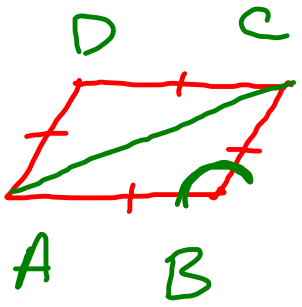
$\therefore$ $\triangle ABC$ is a right angle triangle

$$\triangle ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 5 \times 5$$

MATHEMATICAL PROBLEMS

- Show that,, A(1,1), B(−4,13), C(8,8) and D(13, −4) form a rhombus.



$AB = \sqrt{(1+4)^2 + (1-13)^2} = \sqrt{5^2+12^2} = 13$

$BC = \sqrt{(-4-8)^2+(13-8)^2} = 13$

$CD = = 13$

$DA = = 13$

$\therefore AB = BC = CD = DA$ → rhombus, square

$AC^2 = (1-8)^2 + (1-8)^2$

$= 7^2 + 7^2 \neq AB^2 + BC^2$

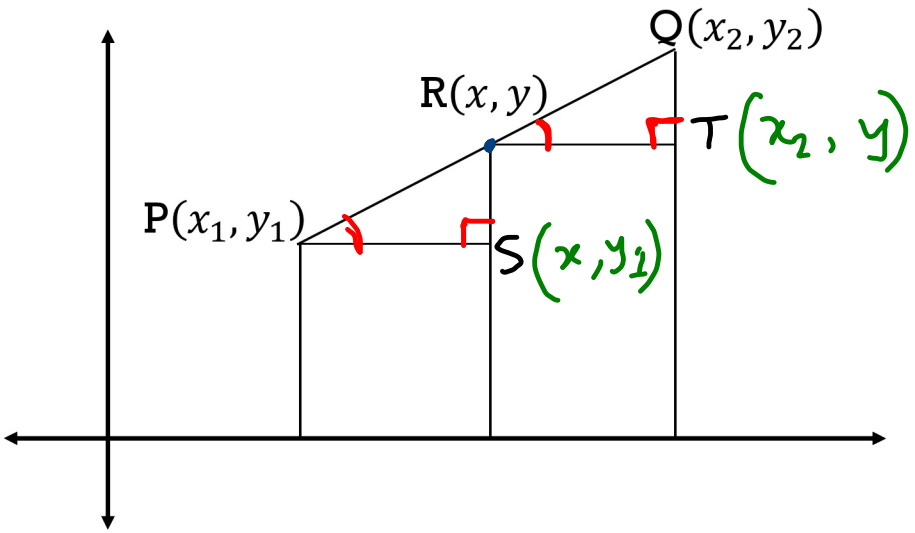
ACTIVITY

(i). Show that, $(4,-1)$, $(2,1)$ and $(1,2)$ are collinear points.

(ii). The radius of a circle is 10 unit and its' centre is $(11,2)$. If the bisecting point of particular chord of here is $(2,-1)$, then what's the length of that chord?

(iii). $(a,5)$ is equidistant from Y axis and the point $(7,2)$. What's the value of a?

SECTION FORMULA: INTERNAL DIVISION OF LINE



PQ line is internally divided in R point, ratio $m : n$

$$\frac{PR}{QR} = \frac{m}{n}$$

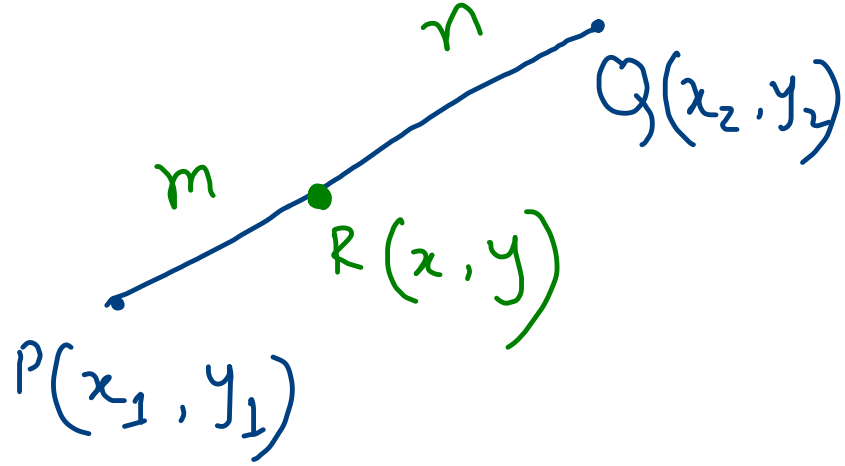
$$\rightarrow PS = \sqrt{(x - x_1)^2 + (y_1 - y_1)^2} = x - x_1$$

$$\rightarrow RT = x_2 - x$$

ΔPRS, ΔRQT similar

$$\frac{PS}{RT} = \frac{PR}{QR} = \frac{m}{n}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m}{n}$$



$$n(x - x_1) = m(x_2 - x)$$

$$nx - nx_1 = mx_2 - mx$$

$$nx + mx = mx_2 + nx_1$$

$$(m+n)x = mx_2 + nx_1$$

$$\boxed{x = \frac{mx_2 + nx_1}{m+n}}$$

$$\boxed{y = \frac{my_2 + ny_1}{m+n}}$$

POLL QUESTION-03

What's the point at which the line segment joining $(2,0)$ and $(7,5)$ is internally divided in $2:3$ ratio?

(a) $(4,2)$

(b) $(2,4)$

(c) $(3,3)$

(d) $(2,5)$

$2:3$ $\quad m:n$

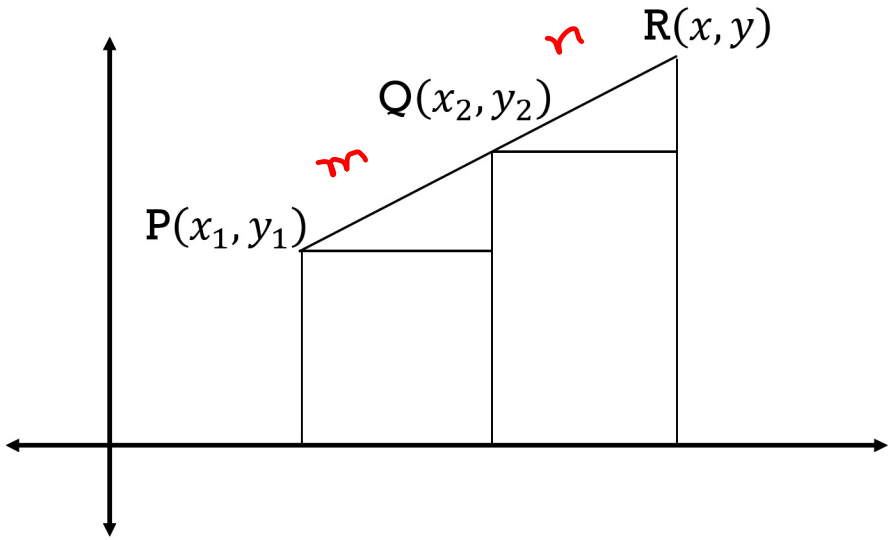
P $\quad$ Q

$R(x,y)$

$$x = \frac{2\times 7 + 3\times 2}{2+3} = \frac{20}{5} = 4$$

$$y = \frac{2\times 5 + 3\times 0}{2+3} = \frac{10}{5} = 2$$

SECTION FORMULA: EXTERNAL DIVISION OF LINE



$$\frac{PR}{QR} = \frac{m}{n}$$

$$x = \frac{mx_2 - nx_1}{m - n}$$

$$y = \frac{my_2 - ny_1}{m - n}$$

POLL QUESTION-04

$2:3$

What's the point at which the line segment joining $(3,4)$ and $(5,9)$ is externally divided in $2:3$ ratio?

(a) $(8,10)$

(b) $(5,4)$

(c) $(-1,-6)$

(d) $(-2,-5)$

$$x = \frac{mx_2 - nx_1}{m-n}$$

$$x = \frac{2\times5 - 3\times3}{2-3} = \frac{1}{-1} = -1$$

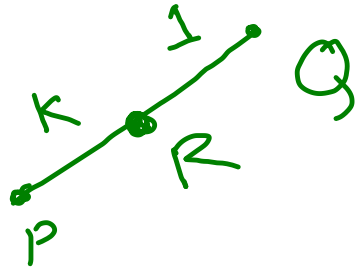
$$y = \frac{2\times9 - 3\times4}{2-3} = \frac{6}{-1} = -6$$

MATHEMATICAL PROBLEMS

$m:n = \frac{m}{n} = \frac{k}{1}$, $m/n = k = k/1$

- What's the ratio in which the line segment joining $(1,2)$ and $(6,7)$ is internally divided at the point $(3,4)$?

P (1,2) k, Q (6,7) 1, R (3,4)



ratio = k : 1

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$3 = \frac{6k + 1.1}{k+1}$$

$$3(k+1) = 6k + 1$$

$$3k + 3 = 6k + 1$$

$$k = 2/3$$

ratio = k : 1

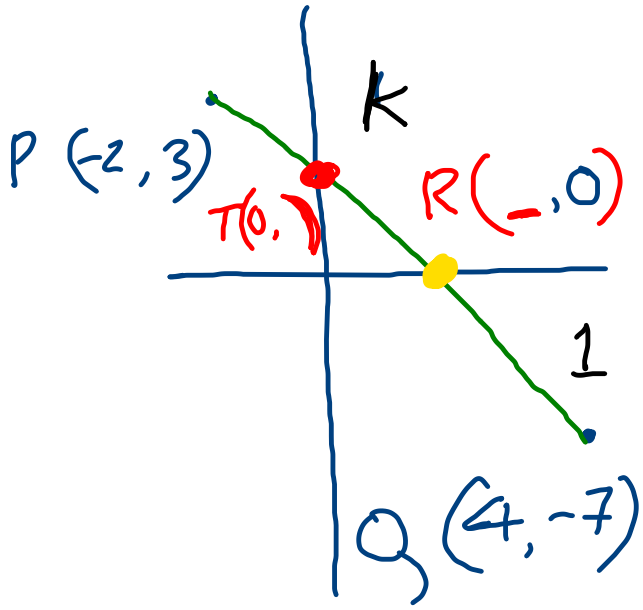
$= 2/3 : 1$

$= 2 : 3$

MATHEMATICAL PROBLEMS

- What's the ratio in which the line segment joining $(-2,3)$ and $(4,-7)$ is divided by x & y axis?

X axis @ R point internally divided at ratio = k : 1



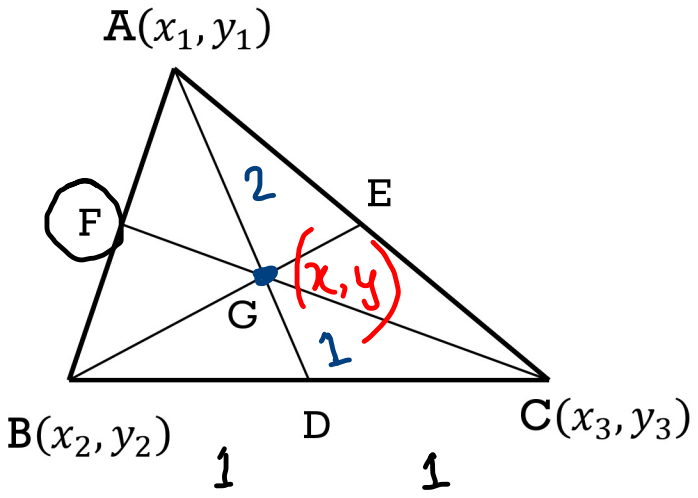
$$0 = \frac{k(-7) + 1 \cdot 3}{k+1}$$

$$0 = -7k + 3$$

$$\therefore k = 3/7$$

$$\boxed{3:7}$$

THE CENTROID OF A TRIANGLE



$\frac{BD}{CD} = \frac{1}{1} = \frac{1}{1}$

$\frac{PR}{QR} = \frac{m}{n}$

BC is internally divided at point D with ratio 1 : 1

Mid point

$D\left(\frac{1.x_2 + 1.x_3}{1+1}, \frac{1.y_2 + 1.y_3}{1+1}\right)$

$D\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

$\frac{AG}{GD} = \frac{2}{1}$

AD internally divided @ G with ratio = 2 : 1

$x = \frac{2\left(\frac{x_2+x_3}{2}\right) + 1.x_1}{2+1} = \frac{x_1+x_2+x_3}{3}$

$y = \frac{y_1+y_2+y_3}{3}$

POLL QUESTION-05

If two of the three vertices of a triangle are $(2,7), (6,1)$ and the centroid is $(6,4)$, what's the third vertex?

(a) $(8,10)$

(b) $(10,4)$

(c) $(4,10)$

(d) $(10,6)$

$$6 = \frac{2+6+x}{3} \Rightarrow x = 10$$

$$4 = \frac{7+1+y}{3} \Rightarrow y = 4$$

ACTIVITY

(i). ABCD is a square and three of its' vertices are A$(8,8)$, B$(9,-5)$ and C$(-4,-6)$; Find the co-ordinate of D and the area of the square.

(ii). ABCD is a rectangle and three of its' vertices are A$(3,2)$, B$(2,-1)$ and C$(8,-3)$; Find the co-ordinate of D and the area of the rectangle.

(iii). What are the intersecting points of x & y axis with the line segment connecting $(2,-4)$ and $(-3,6)$.

(iv). If the origin point be the middle point of the line segment connecting (x,y) and $(r\,cos\theta, r\,sin\theta)$, prove that $x^2+y^2=r^2$.

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে