CLASS 11 ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : HM-07

Chapter 3 : Straight lines

AREA OF TRIANGLE

$\boxed{150\%}$

The co-ordinate of three vertices of a triangle are given, we are to find the area:

$$\Delta ABC = \{ABLM + ACNM - BCNL\}$$

Trapezium

$$= \frac{1}{2}(y_1+y_2)(x_1-x_2) + \frac{1}{2}(y_1+y_3)(\boxed{x_3}-x_1)$$

$$- \frac{1}{2}(y_2+y_3)(\boxed{x_3}-x_2)$$

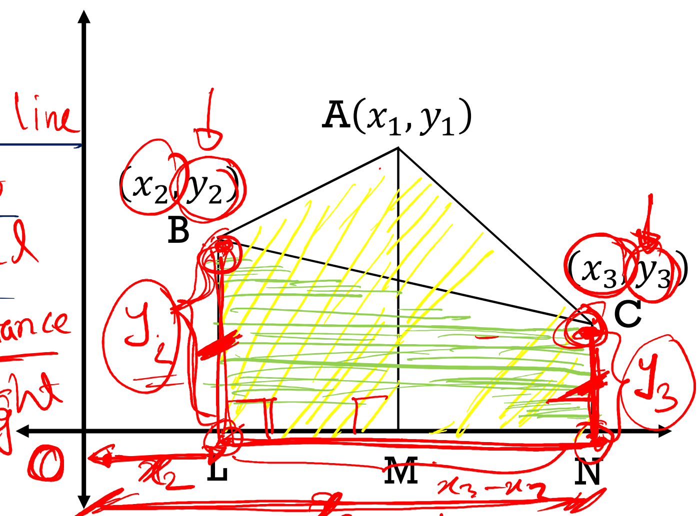
$$= \frac{1}{2}\{x_1(\cancel{y_1}+y_2-\cancel{y_1}-y_3) - x_2(y_1+\cancel{y_2}-\cancel{y_2}-y_3)$$

$$+ x_3(y_1+\cancel{y_3}-y_2-\cancel{y_3})\}$$

Area

$= \frac{1}{2} \times$ parallel line এর length এর sum $\times$ parallel lines এর distance

height



BCNL → BL || CN

$LN = ON - OL$
$= x_3 - x_2$

AREA OF TRIANGLE

$$= \frac{1}{2}\{x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)\}$$

$+a$ $-b$ $+c$

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

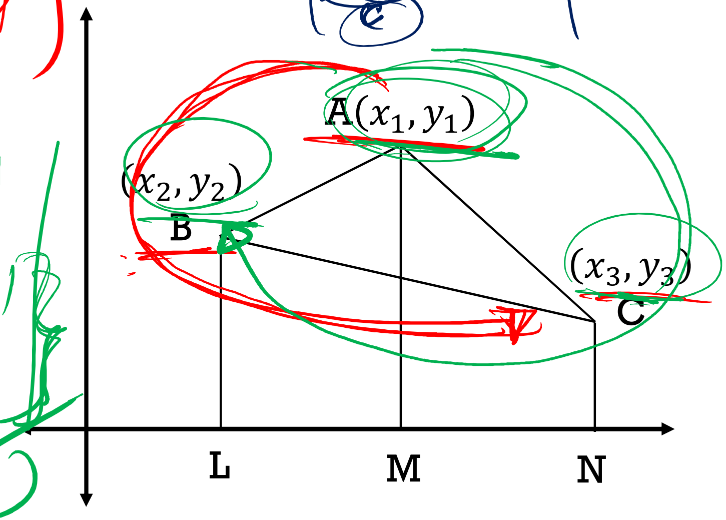
(+ve)

always (1)

1st row

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$$

(-ve)



Area → always (+ve)

The area can be (+)ve or (-)ve based on the sequence of points taken anti-clockwise or clockwise. But the unsigned value will be the answer here.

MATHEMATICAL PROBLEM

4

❖ If A(x, y), B$(1,2)$ and C$(2,1)$ form a triangle of 6 unit square area, show that $x + y = 15$

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} x-1 & y-2 & 1-1 \\ 1-2 & 2-1 & 1-1 \\ 2 & 1 & 1 \end{vmatrix}$$

$\left[\begin{array}{l} r_1' = r_1 - r_2 \\ r_2' = r_2 - r_3 \end{array}\right]$

$$= \frac{1}{2}\begin{vmatrix} x-1 & y-2 & 0 \\ -1 & 1 & 0 \\ 2 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left\{0(\ ) + 0(\ ) + 1\begin{vmatrix} x-1 & y-2 \\ -1 & 1 \end{vmatrix}\right\}$$

$$= \frac{1}{2}\left(x-1-(-y+2)\right) = \frac{1}{2}(x+y-3) = 6$$

$$\Rightarrow x+y-3 = 12 \Rightarrow \boxed{x+y=15}$$

POLL QUESTION-01

- Given, A$(0,0)$, B$(2,1)$ and C$(6,2)$ are three vertices of a triangle, what's the area?

  (a) 2

  (b) -1

  (c) 1

  (d) 6

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & \\ 6 & 2 & \end{vmatrix}$$

row operation লাগবে না

$$+\frac{1}{2}\begin{vmatrix} 2 & 1 \\ 6 & 2 \end{vmatrix} = \frac{1}{2}(4-6)$$

$$= \frac{1}{2}(-2) = -1$$

Area = (+ve) value = 1

MATHEMATICAL PROBLEM

❖ Given, four points are A$(t-4,-2)$, B$(t,t+3)$, C$(2t+1,1)$ and D$(t-3,1)$ and O is origin, find the ratio $\Delta \boldsymbol{OAB} : \Delta \boldsymbol{OCD}$ and show that if $t = 4$ the area of those two triangles will be the same.

(0,0)

ΔOCD

$(0,0), (2t+1,1), (t-3,1)$

$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2t+1 & 1 & 1 \\ t-3 & 1 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 2t+1 & 1 \\ t-3 & 1 \end{vmatrix}$

$= \frac{1}{2}(2t+1-t+3)$

$\boxed{\frac{1}{2}(t+4)}$

$\Delta OAB : (0,0), (t-4,-2), (t,t+3)$

$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ t-4 & -2 & 1 \\ t & t+3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} t-4 & -2 \\ t & t+3 \end{vmatrix}$

$= \frac{1}{2}\{(t-4)(t+3)-(-2t)\}$

$= \frac{1}{2}\{t^2-t-12+2t\}$

$= \frac{1}{2}(t^2+t-12)$

$= \frac{1}{2}(t+4)(t-3)$

$\frac{\Delta OAB}{\Delta OCD} = \frac{\frac{1}{2}(t+4)(t-3)}{\frac{1}{2}(t+4)}$

$= (t-3)$

$t=4$;

$\frac{\Delta OAB}{\Delta OCD} = (4-3) = 1$

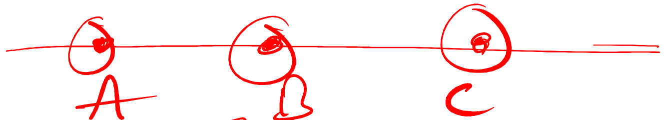
$\Rightarrow \boxed{\Delta OAB = \Delta OCD}$

CONDITIONS OF COLLINEARITY OF THREE POINTS

The area of the triangle ΔABC will be zero which means,

- ❖ $AB + BC = AC$
- ❖ The determinant formed by those three points will be zero.

$AB + BC = AC$



collinear { three points are on the same line

A, B, C
$\rightarrow$ Determinant $= 0$

MATHEMATICAL PROBLEM

❖ Given A$(t+1,1)$, B$(2t+1,3)$ and C$(2t+2,2t)$ are three vertices of a triangle, find the area.
What's the value of t if those points are collinear?

$$\frac{1}{2}\begin{vmatrix} t+1 & 1 & 1 \\ 2t+1 & 3 & 1 \\ 2t+2 & 2t & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} t+1 & 1 & 1 \\ t & 2 & \cancel{0} \\ 1 & 2t-3 & \cancel{0} \end{vmatrix}$$

$$\left[\begin{array}{l} R_3' = R_3 - R_2 \\ R_2' = R_2 - R_1 \end{array}\right]$$

$$= \frac{1}{2}\begin{vmatrix} t & 2 \\ 1 & 2t-3 \end{vmatrix} = \frac{1}{2}(2t^2 - 3t - 2) = \frac{1}{2}(2t^2 - 4t + t - 2)$$

$$= \frac{1}{2}\{2t(t-2) + 1(t-2)\} = \frac{1}{2}(t-2)(2t+1)$$

If collinear, Area $= 0$ $\therefore$ $\frac{1}{2}(t-2)(2t+1) = 0$ $\boxed{\begin{array}{l} t = 2 \\ t = -\frac{1}{2} \end{array}}$

POLL QUESTION-02

- $(k,-1),(6,3)$ and $(0,0)$ are collinear, what's the value of k?

  (a) 4

  (b) -2

  (c) 3

  (d) 2

$$\frac{1}{2}\begin{vmatrix} k & -1 & 1 \\ 6 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$+\frac{1}{2}\begin{vmatrix} k & -1 \\ 6 & 3 \end{vmatrix} = \frac{1}{2}(3k+6) = 0$$

$$3k+6=0$$

$$\boxed{k=-2}$$

PRACTICE PROBLEM

(i). (x, y), $(5,3)$ and $(-2,-4)$ are collinear , show that $x - y - 2 = 0$

(ii). Three vertices of ΔABC are $(-1,2)$, $(2,3)$ and $(3,-4)$. The co-ordinate of a point P is (x, y). Show that $\frac{\Delta PAB}{\Delta ABC} = \frac{x-3y+}{22}$

(iii). $(-1,2)$, $(2,3)$ and $(3,-4)$ forms a triangle of whiche the centroid is G. Show $\Delta ABC = 3\Delta ABG = 3\Delta BCG = 3\Delta CAG$

(iv). Find the area of the triangle formed by A$(2,-1)$, B$(a+1, a-3)$ and C$(a+2, a)$. If the points are collinear what will be the value of a?

LOCUS

অনেকগুলো

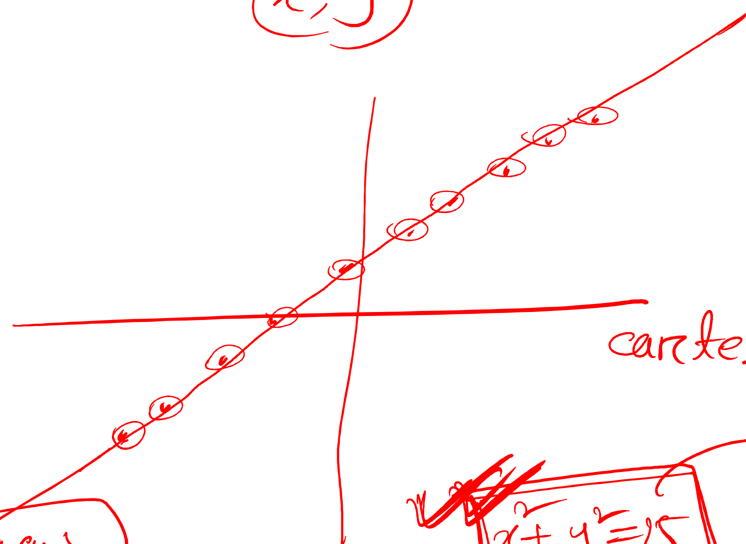
- It's the set of all points whose location/path satisfies or determined by one or more specified conditions.

locus

- The equation formed by variables following the conditions is the equation of locus.

(x, y)

one or more conditions



cartesian

locus

conditions

apply

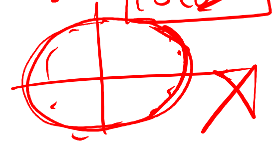
$(x, y) \rightarrow eq^n$

equation of locus

$x^2+y^2=25$

(x, y)

locus



MATHEMATICAL PROBLEM

❖ A (2, 3) and B (-1, 4) are two fixed points. The ratio of the distances of any point of a set from A and B is 2 : 3. Find the equation of the locus.

(x, y) } প্রথমে ধরে নিতে representative ধরতে

condition

P(x, y)

(x, y) (2, 3)

(x, y) (-1, 4)

A(2, 3) fixed

B(-1, 4) fixed

$$\frac{PA}{PB} = \frac{2}{3}$$

PA : PB

2 : 3

$$\Rightarrow \frac{\sqrt{(x-2)^2+(y-3)^2}}{\sqrt{(x+1)^2+(y-4)^2}} = \frac{2}{3}$$

$$\Rightarrow 3\sqrt{(x-2)^2+(y-3)^2} = 2\sqrt{(x+1)^2+(y-4)^2}$$

$$\Rightarrow 9(x^2-4x+4+y^2-6y+9) = 4(x^2+2x+1+y^2-8y+16)$$

$$\Rightarrow 5x^2+5y^2-36x-8x-54y+32y+117-68=0$$

$$\Rightarrow \boxed{5x^2+5y^2-44x-22y+49=0}$$

POLL QUESTION-03

distance $= 4$

- A set of points always maintain 4 unit distance from the point $(2, -1)$. The equation of locus will be-

$P(x, y)$ condition

(a) $x^2 + y^2 + 4x - 2y - 11 = 0$

(b) $x^2 + y^2 - 4x + 2y - 11 = 0$

(c) $x^2 + y^2 - 4x - 2y - 11 = 0$

(d) $x^2 + y^2 + 4x + 2y - 11 = 0$

(x, y) 4 unit $(2, -1)$

$\sqrt{(x-2)^2 + (y+1)^2} = 4$

$\Rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 = 16$

$\Rightarrow \boxed{x^2 + y^2 - 4x + 2y - 11 = 0}$

MATHEMATICAL PROBLEM

❖ A$(-6,-3)$ and B$(6,3)$ are two fixed points. The length of the medians drawn on AB line segment from a set of points is always 5 unit. Find the equation of locus.

(x, y)

condition

$P(x, y)$

length 5 unit

median

A $\quad$ 1 $\quad$ D midpoint $\quad$ 1 $\quad$ B

$(-6,-3)$ $\quad$ $(6,3)$

$1:1$ internally

$D\left(\frac{-6+6}{2}, \frac{-3+3}{2}\right)$

$D(0, 0)$

(x, y) $\quad$ 5 unit $\quad$ $(0, 0)$

$$\sqrt{(x-0)^2+(y-0)^2} = 5$$

$$\boxed{x^2+y^2=25}$$

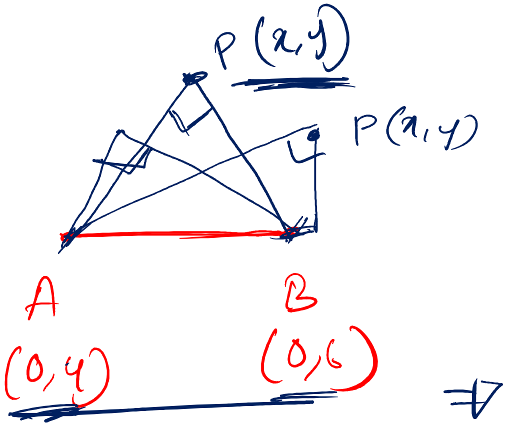
eqn

Ans

MATHEMATICAL PROBLEM

❖ A$(0,4)$ and B$(0,6)$ are two fixed points. There's a set of points at which AB line segment forms a right angle. Find the equation of locus.

90° $\quad P(x, y)$



At P point, AB forms $\angle APB = 90^\circ$

$\Delta APB \Rightarrow$ right angled triangle

pythagorus :- $PA^2 + PB^2 = AB^2$

$$\Rightarrow \left(\sqrt{(x-0)^2+(y-4)^2}\right)^2 + \left(\sqrt{(x-0)^2+(y-6)^2}\right)^2 = \left(\sqrt{(0-0)^2+(4-6)^2}\right)^2$$

$$\Rightarrow x^2 + y^2 - 8y + 16 + x^2 + y^2 - 12y + 36 = 4$$

$$\Rightarrow 2x^2 + 2y^2 - 20y + (52-4) = 0 \Rightarrow \boxed{x^2 + y^2 - 10y + 24 = 0}$$

$48/2 = 24$ $\quad x \quad y \quad (x, y) \Rightarrow$

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

MATHEMATICAL PROBLEM

Example: Find the equation of locus of points equidistant from the point $(-2,5)$ and x axis.

(x,y)

d_1 d_2

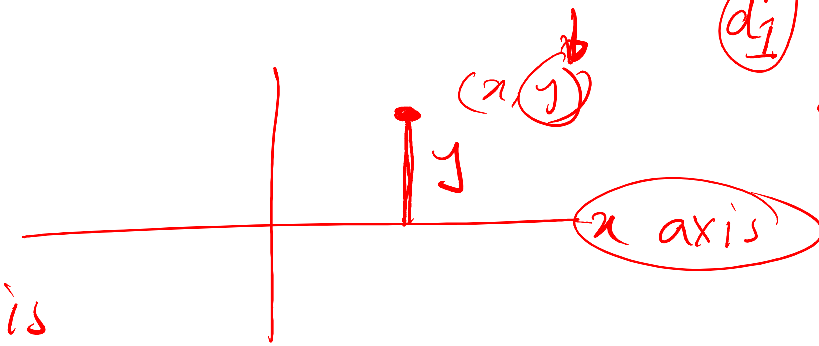
$d_1 = d_2$

$(-2,5)$ and x axis

equidistant

$P(x,y)$

$(-2,5)$ — $\boxed{P(x,y)}$ — x-axis



$$\sqrt{(x+2)^2+(y-5)^2} = y \Rightarrow x^2+4x+4+\cancel{y^2}-10y+25 = \cancel{y^2}$$

$$\Rightarrow \boxed{x^2+4x-10y+29=0}$$

(Ans.)

POLL QUESTION-04

- There's a set of points of which the square of distance from x axis is equal to the distance from y axis. What's the equation of locus?

$4x$ (x, y)

(a) $x^2 = 4y$

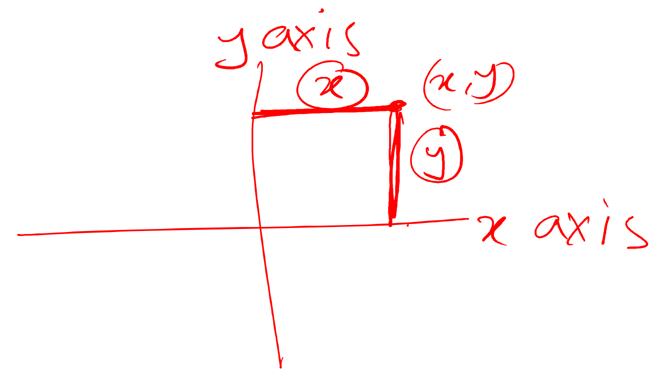
(b) $y^2 = 4x$

(c) $x^2 = 4y^2$

(d) $y = 2x$

$\boxed{y^2 = x}$

$y^2 = 4x$



PRACTICE PROBLEM

(i). Find the equation of locus of points for which the sum of the distance from the points A$(3,0)$ and B$(-3,0)$ is always 10 unit.

(ii). The distance of a set of points from A$(2,0)$ is three times of the distance from the line $x = 0$. Find the equation of locus.

(iii). The ratio of the distance of a set of points from the origin (0,0) and $(-5,0)$ is $3:4$, find the equation of locus.

না বুঝে
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ধ্বংস করে