



CLASS 11 ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : HM-08

Chapter 3 : Straight lines

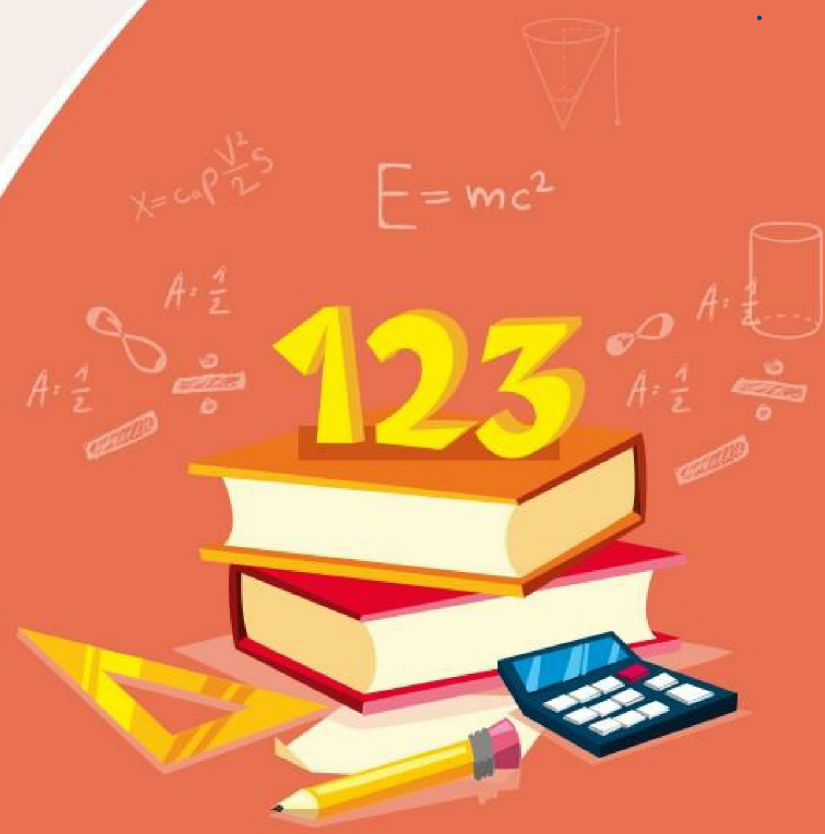


$$x = \sqrt{\frac{c^2}{a} + c} - \frac{b}{2a}$$



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GENERAL FORM OF EQUATION OF STRAIGHT LINE

2 variable
power = 1

$$ax + by + c = 0$$

$a, b \neq 0$ ✓

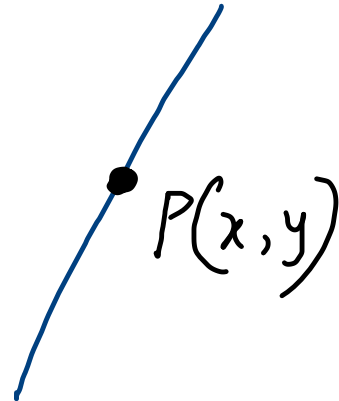
$$ax^1 + by^1 + c = 0$$

var

2 eqn same?

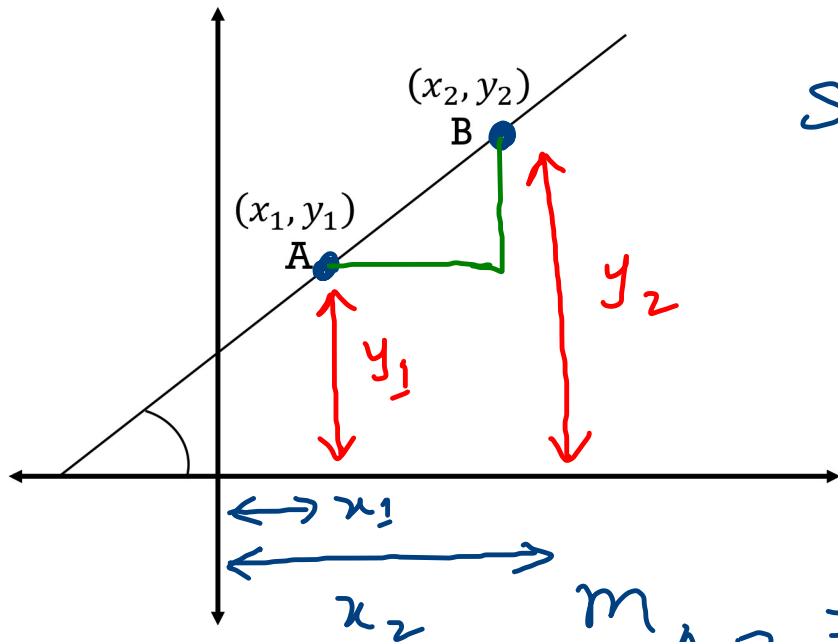
$$2x - 3y = 10 \quad \text{--- (i)}$$
$$6x - 9y = 36 \quad \text{--- (ii)}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$



Always
slope
Same

SLOPE OF A LINE CONNECTING TWO POINTS



slope/gradient, $m = \frac{\text{change of } y}{\text{change of } x}$

$$m = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{y_1 - y_2}{x_1 - x_2}$$

(rise / run)

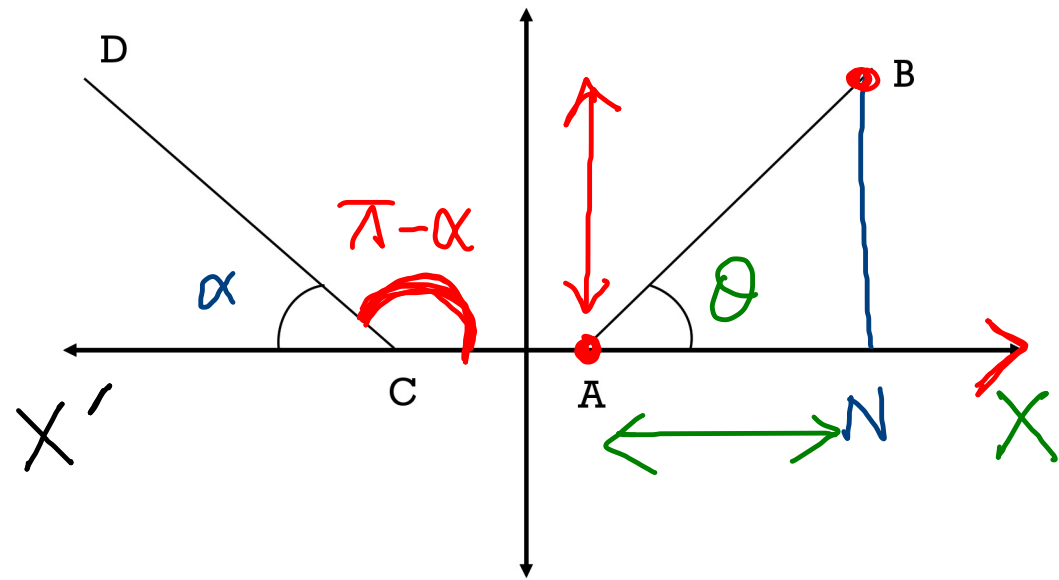
SLOPE OF A STRAIGHT LINE

- The tangent (tan) of the inclination angle of a line with positive x axis is called the slope or gradient of a line (m).

$$m_{AB} = \tan \theta$$

$$m_{CD} = \tan(\angle DCA) \\ = \tan(\pi - \alpha)$$

In $\triangle ABN$, $\tan \theta = \frac{BN}{AN} = \frac{\text{rise}}{\text{run}} = m$



POLL QUESTION-01

□ What's the slope of the line connecting A (x_1, y_1) and B (x_2, y_2)?

(a) -6

(b) 2

~~(c) -2~~

(d) 6

$$m = \frac{\Delta y}{\Delta x}$$

$$= \frac{13 - 7}{-1 - 2}$$

$$= \frac{6}{-3}$$

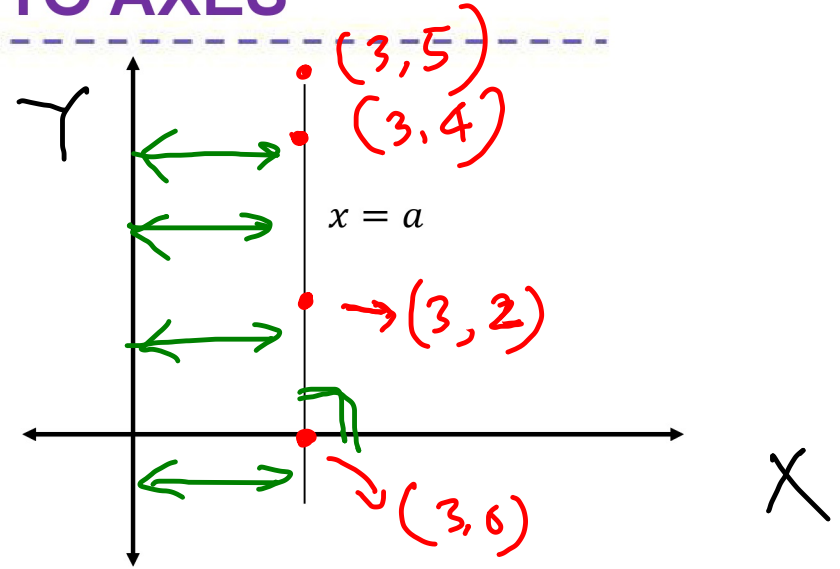
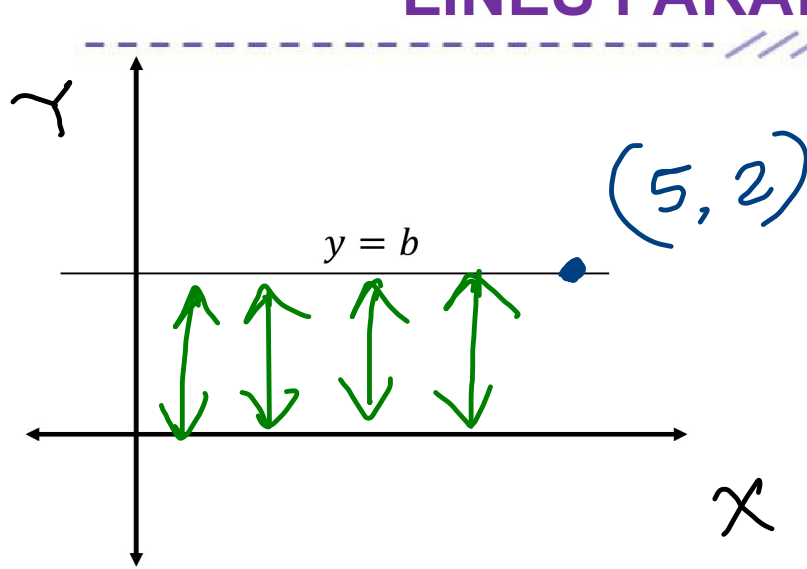
$$= -2$$

$$= \frac{7 - 13}{2 - (-1)}$$

$$= \frac{-6}{3}$$

$$= -2$$

LINES PARALLEL TO AXES



Parallel to x axis

Parallel to y axis

$$m = \tan(90^\circ) = \infty$$

$y = b$
 passing through $(5, 2)$
 $2 = b$
 $y = 2$

$$m = \tan(0^\circ) = 0$$

$$x = a$$

POLL QUESTION-02

□ A line, perpendicular to y axis goes through the point (6, 3) what's the equation?

(a) $x = 3$

(b) $y = 6$

(c) $x = 6$

~~(d) $y = 3$~~

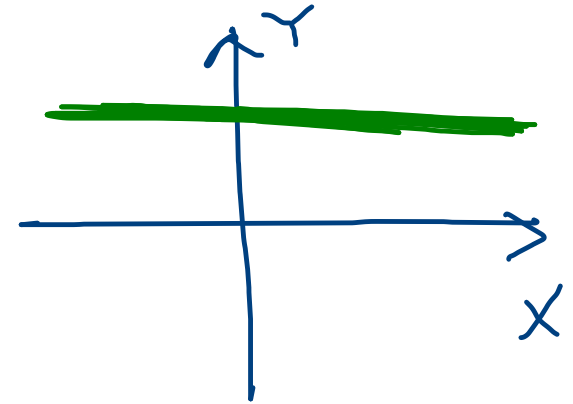
parallel to x axis

$$y = b$$

pass (6, 3)

$$3 = b$$

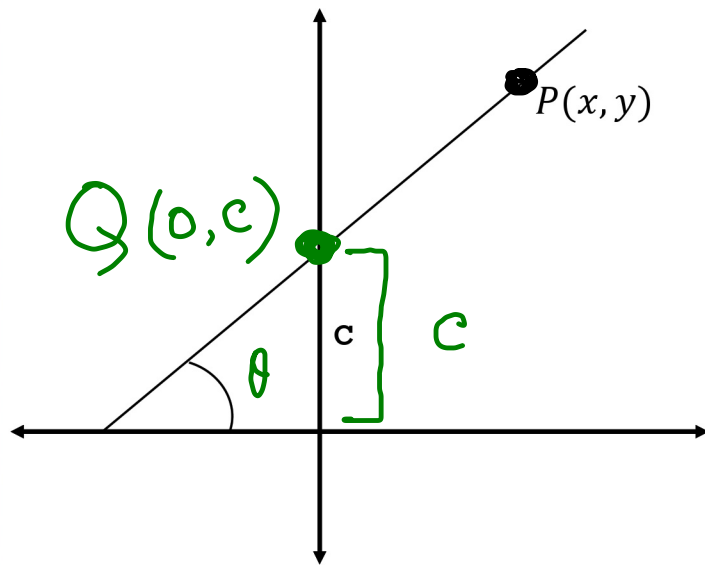
$$y = 3$$



SLOPE-INTERCEPT FORM EQUATION OF A STRAIGHT LINE

❖ m = slope of the line ✓

c = Y intercept



Let, $P(x, y)$ be a point on the st. line

$$m = \frac{y - c}{x}$$

$$mx = y - c$$

$$y = mx + c$$

$$m_{PQ} = \frac{y - c}{x - 0}$$

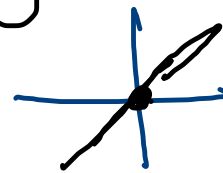
slope = 2

Y intercept = 5

$$y = 2x + 5$$

Passing through the origin, $c = 0$

$$y = mx \quad \checkmark$$



POLL QUESTION-03

$$y = mx + c$$

- The inclination of a line with positive x axis is 135° ; If it passes through the point (2, 3) the equation is-

(a) $y = x + 4$

(b) $y = -2x + 6$

~~(c) $y = -x + 5$~~

(d) $y = -x + 6$

$$m = \tan \theta \\ = \tan (135^\circ) = -1$$

$$y = -x + c \quad \left[m = -1 \right]$$

pass (2, 3)

$$3 = -2 + c \Rightarrow c = 5$$

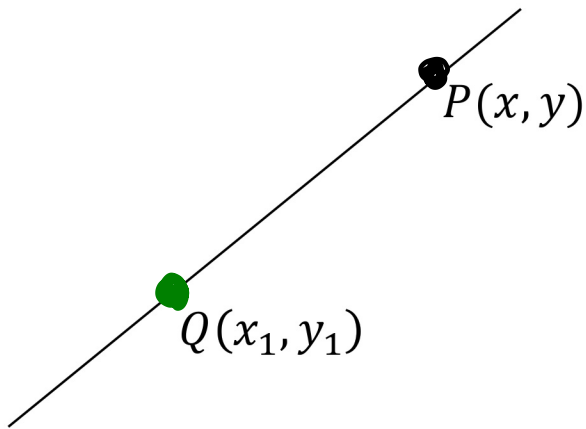
$$y = -x + 5$$

POINT-SLOPE FORM EQUATION OF A STRAIGHT LINE

❖ m = slope of the line ✓

(x_1, y_1) = A fixed point on the line ✓

$$m_{PQ} = \frac{y - y_1}{x - x_1} = \frac{\Delta y}{\Delta x}$$



$$m = \frac{y - y_1}{x - x_1}$$

$$m(x - x_1) = y - y_1$$

slope = 2, (5, 3) pass

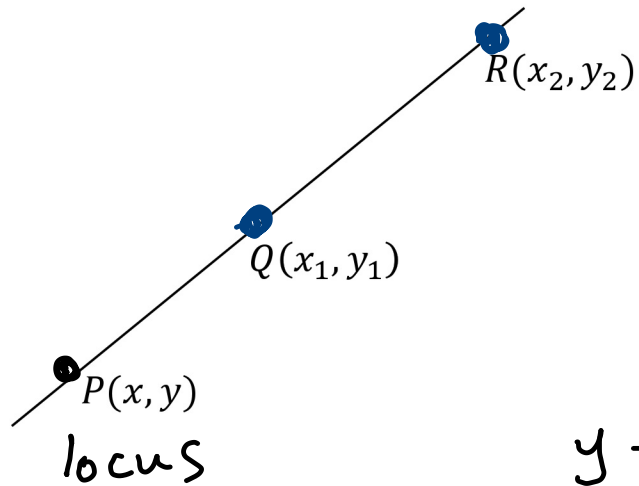
$$2(x - 5) = y - 3$$

$$2x - y - 7 = 0$$

TWO-POINTS FORM EQUATION OF A STRAIGHT LINE

❖ $(x_1, y_1), (x_2, y_2) =$ Two fixed points on the line

$$m_{QR} = \frac{y_1 - y_2}{x_1 - x_2}$$



$$m_{PQ} = \frac{y - y_1}{x - x_1}$$

$$m_{PQ} = m_{QR}$$

$$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

POLL QUESTION-04

- The equation of the line passing through the points $(1, 2)$ and $(3, 4)$ is-

(a) $x + y + 1 = 0$

~~(b) $x - y + 1 = 0$~~ ✓✓

(c) $y - x + 1 = 0$

(d) $y + x - 1 = 0$

$$\begin{array}{ccc} x_1 & y_1 & x_2 & y_2 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ (1, 2) & & (3, 4) & \end{array}$$

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\frac{y - 2}{2 - 4} = \frac{x - 1}{1 - 3}$$

$$\frac{y - 2}{-2} = \frac{x - 1}{-2}$$

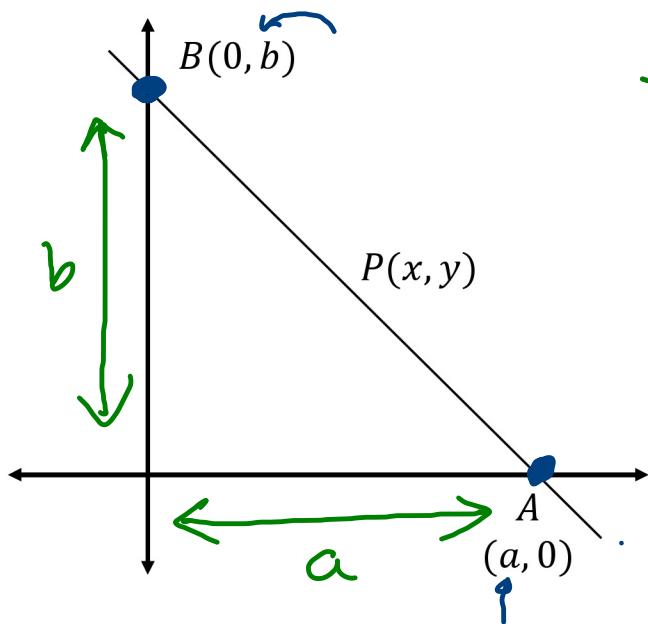
$$x - y + 1 = 0$$

PRACTICE PROBLEM

- (i). A line perpendicular to x axis passes through the point $(3, 4)$. Find the equation of line.
- (ii). Find the equation of line passing through the points $(5, 7)$ and $(0, -4)$.
- (iii). Find the equation of the line of slope 2 which passes through the point $(-1, 2)$.
- (iv). $3x + 7y = 21$ and $2ax - 3by + 6 = 0$ indicates the same line. Find the value of a and b.

INTERCEPT FORM EQUATION OF A STRAIGHT LINE

- ❖ $a = x$ -intercept ✓
- $b = y$ -intercept



$A(a, 0)$, $B(0, b)$ pass

$$\frac{y-0}{0-b} = \frac{x-a}{a-0}$$

$$\left[\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2} \right]$$

$$\frac{y}{-b} = \frac{x-a}{a}$$

$$ay = -b(x-a)$$

$$ay = -bx + ab$$

$$ay + bx = ab$$

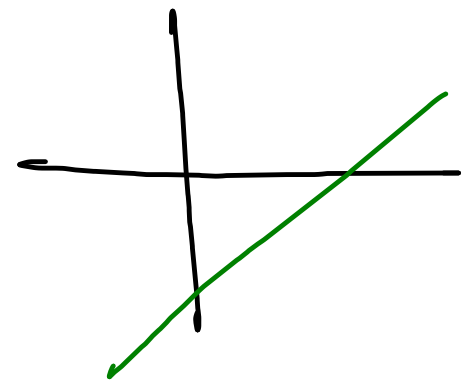
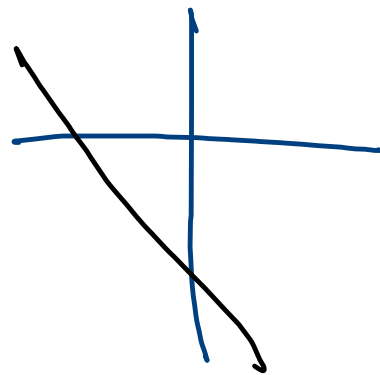
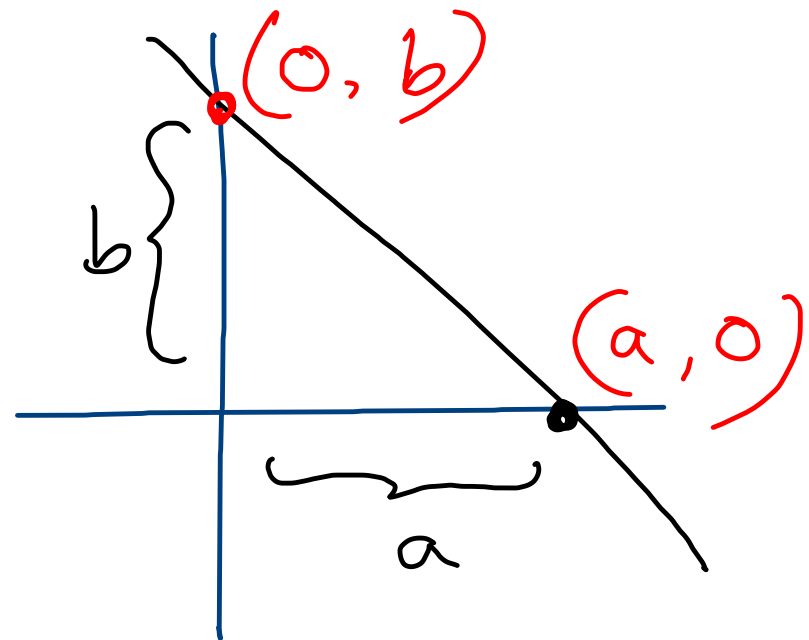
$$bx + ay = ab$$

$$\frac{bx + ay}{ab} = \frac{ab}{ab}$$

$$\frac{bx}{ab} + \frac{ay}{ab} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

\downarrow X-axis intercept
 \downarrow Y-axis intercept



POLL QUESTION-05

$$a + b = ?$$

□ What's the summation of the intercepts of axes of the line $6x + 5y - 30 = 0$?

~~(a) 11~~

(b) 1

(c) -1

(d) -11

$$6x + 5y = 30$$

$$\frac{6x}{30} + \frac{5y}{30} = \frac{30}{30}$$

$$\frac{1x}{5} + \frac{1y}{6} = 1$$

$$a = 5, \quad b = 6$$

$$5 + 6 = 11$$

PERPENDICULAR FORM EQUATION OF A STRAIGHT LINE

❖ $x \cos \alpha + y \sin \alpha = p$

p = The length of the perpendicular line drawn on the straight line from origin

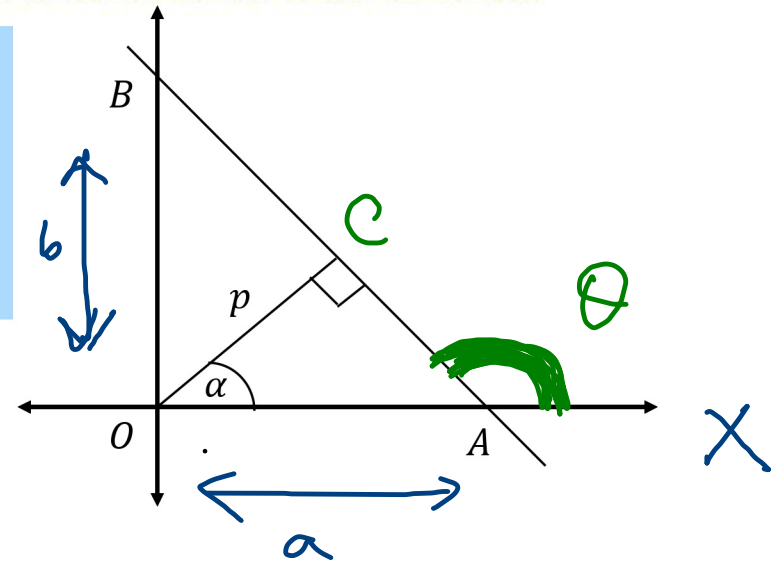
α = The inclination of that perpendicular line p with positive x axis

$$m_{AB} = \tan \theta$$

Eqⁿ of AB :

$$\frac{x}{OA} + \frac{y}{OB} = 1$$

$$\frac{x}{a} + \frac{y}{b} = 1$$



In $\triangle OAC$

$$\cos \alpha = \frac{OC}{OA} = \frac{p}{OA}$$

$$OA = \frac{p}{\cos \alpha}$$

$\triangle OCB$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{OC}{OB}$$

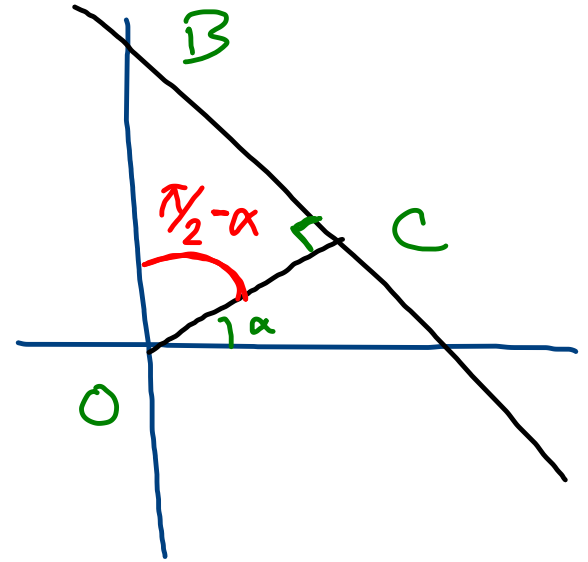
\downarrow
 $\sin\alpha$

$$= \frac{P}{OB}$$

$$OB = \frac{P}{\sin\alpha}$$

$$\frac{x}{OA} + \frac{y}{OB} = 1$$

$$\frac{x}{P/\cos\alpha} + \frac{y}{P/\sin\alpha} = 1$$



$$OA = \frac{P}{\cos\alpha}$$

$$\frac{x \cos\alpha}{P} + \frac{y \sin\alpha}{P} = 1$$

$$x \cos\alpha + y \sin\alpha = P$$

MATHEMATICAL PROBLEM

- ❖ The point $A(h, k)$ lies on the line $6x - y = 1$ and the point $B(k, h)$ on $2x - 5y = 5$ Find the equation of the line AB .

Solⁿ: $A(h, k)$ lies on $6x - y = 1$

$$6h - k = 1 \quad \text{--- (i)}$$

$B(k, h)$ lies on $2x - 5y = 5$

$$2k - 5h = 5 \quad \text{--- (ii)}$$

Solving (i) & (ii), $h = 1$ $k = 5$

$A(1, 5)$ & $B(5, 1)$

$$A(1, 5) \quad B(5, 1)$$

eqⁿ of AB:

$$\frac{y-5}{5-1} = \frac{x-1}{1-5}$$

$$\frac{y-5}{4} = \frac{x-1}{-4}$$

$$y-5 = -(x-1)$$

$$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$$

$$y-5 + (x-1) = 0$$

$$x+y-6 = 0$$

$$x+y=6$$

MATHEMATICAL PROBLEM

- ❖ Find the co-ordinate of the midpoint of the line segment cut by the axes from the straight line $x + 2y + 7 = 0$

$$x + 2y = -7$$

$$\frac{x}{-7} + \frac{2y}{-7} = \frac{-7}{-7}$$

$$\frac{x}{-7} + \frac{y}{-7/2} = 1$$

$$A(-7, 0)$$

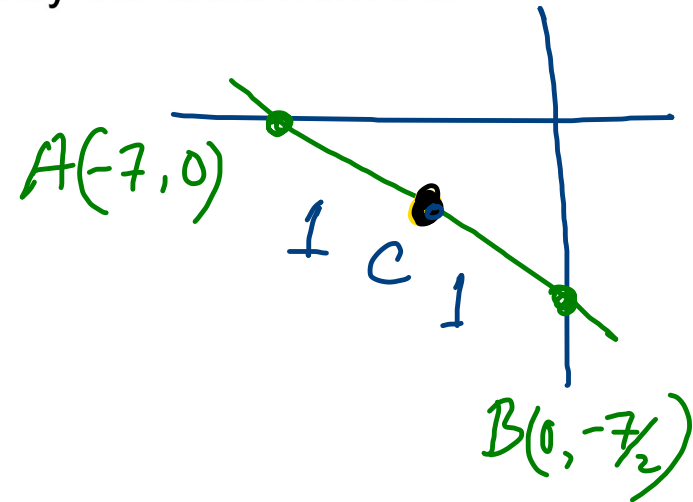
Midpoint

§ B

$$\left(0, -\frac{7}{2}\right)$$

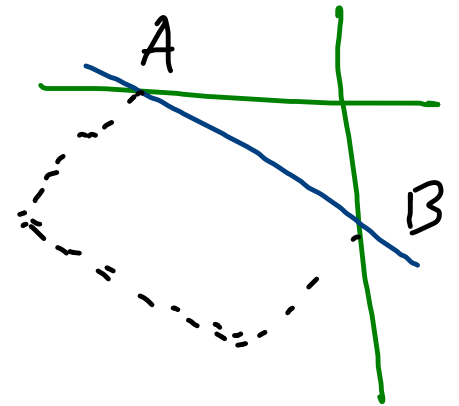
$$\left(\frac{-7+0}{2}, \frac{0+(-7/2)}{2}\right) = \left(-\frac{7}{2}, -\frac{7}{4}\right)$$

$$= \left(-\frac{7}{2}, -\frac{7}{4}\right)$$



If the line segment is a side of a square, what will be its area?

$$A(-7, 0) \quad B(0, -7/2)$$



$$\text{side} = AB$$

$$\text{Area} = AB^2$$

$$= (-7 - 0)^2 + (0 + 7/2)^2$$

$$= 61 \frac{1}{4}$$

MATHEMATICAL PROBLEM

- ❖ A straight line passes through the point (3, 7); It cuts parts of equal value but of opposite sign from the axes. Find the equation of the line.

Let, the eqⁿ be: $\frac{x}{a} + \frac{y}{b} = 1$

A.T.Q., $a = -b$

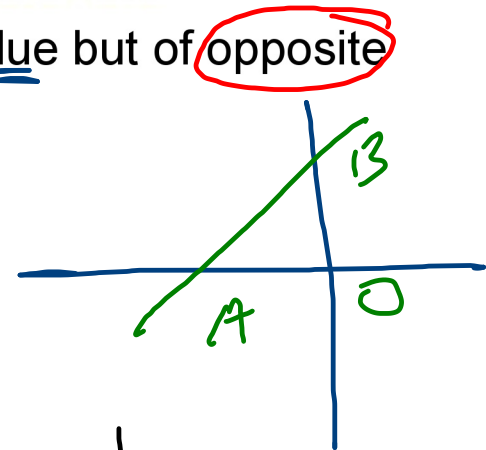
$$\therefore \frac{x}{-b} + \frac{y}{b} = 1$$

$$-x + y = b \quad [\text{multiplying with 'b'}]$$

Passing through (3, 7).

$$-3 + 7 = b \quad \therefore b = 4$$

$$\boxed{-x + y = 4} \quad (\text{Ans})$$



$$\boxed{b = -a}$$

MATHEMATICAL PROBLEM

- ❖ A line forms a triangle of 16 unit square with the axes. The perpendicular line drawn on the straight line from origin inclines at 45° angle with positive x axis. Find the equation of the line.

$$x \cos \alpha + y \sin \alpha = p$$

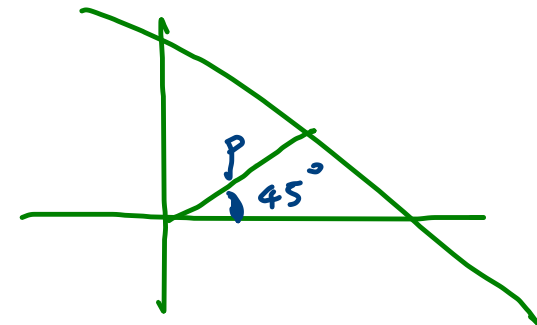
$$x \cos 45 + y \sin 45 = p$$

$$x \frac{1}{\sqrt{2}} + y \frac{1}{\sqrt{2}} = p$$

$$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = p$$

$$\frac{x}{p\sqrt{2}} + \frac{y}{p\sqrt{2}} = 1$$

$$\alpha = 45^\circ$$



$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\left. \begin{array}{l} a = p\sqrt{2} \\ b = p\sqrt{2} \end{array} \right\}$$

$$a = b$$

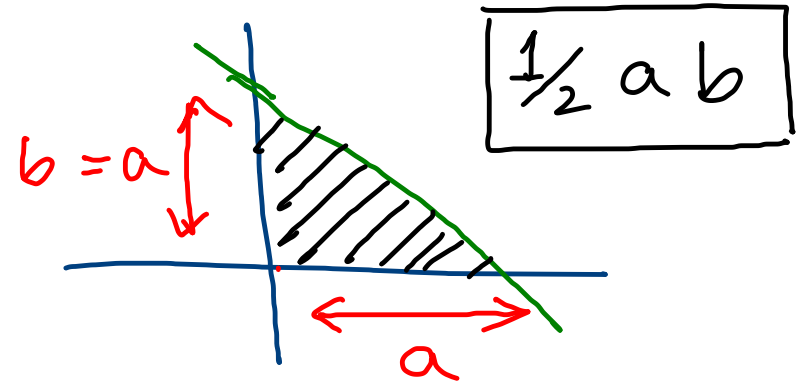
A.C.Q. $\frac{1}{2}ab = 16$

$$ab = 32$$

$$a \cdot a = 32 \quad [\because b = a]$$

$$a^2 = 32$$

$$a = \pm \sqrt{32}$$



$$a = \pm 4\sqrt{2}$$

$$a = 4\sqrt{2} \quad b = 4\sqrt{2}$$

$$a = -4\sqrt{2} \quad b = -4\sqrt{2}$$

X

GRAPHICAL REPRESENTATION OF LINE

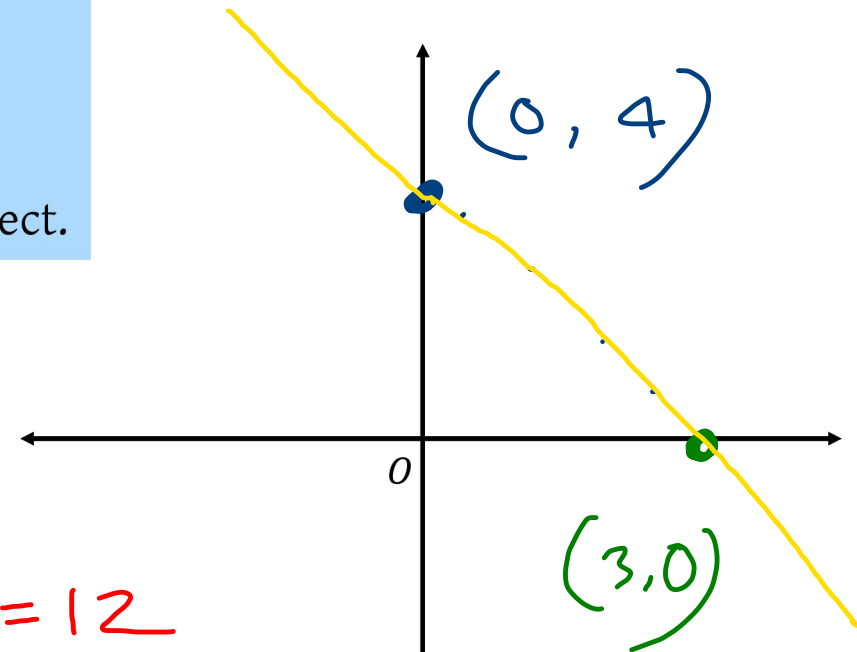
- Take the equation.
- Find some points (x, y) following the equation.
- Place the points (x, y) in Cartesian plane and connect.

$$4x + 3y = 12$$

$$3y = 12 - 4x$$

$$y = \frac{1}{3}(12 - 4x)$$

x				
y				

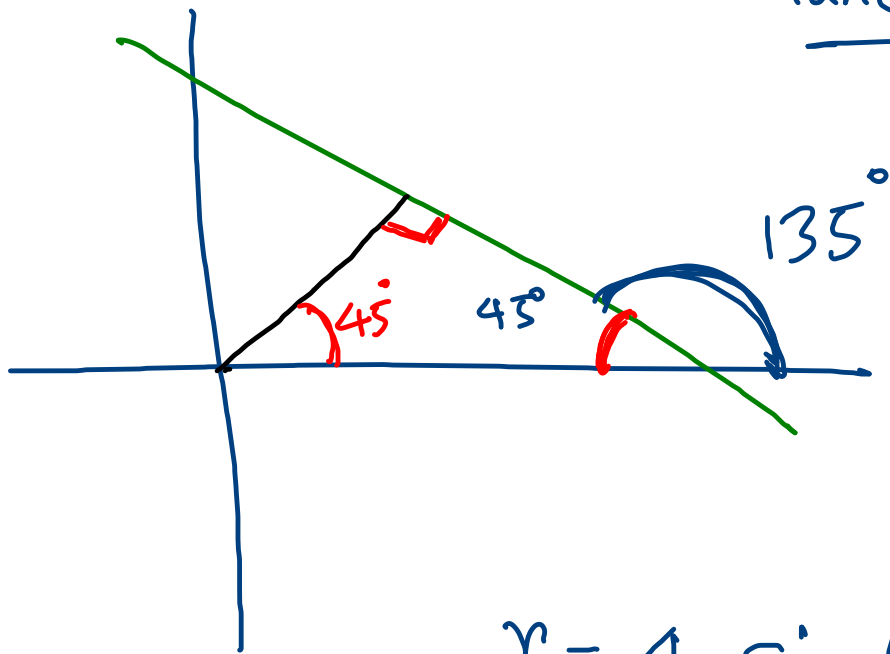


$$4x + 3y = 12$$

$$\frac{4x}{12} + \frac{3y}{12} = \frac{12}{12}$$

$$\frac{x}{3} + \frac{y}{4} = 1$$

Hand raise



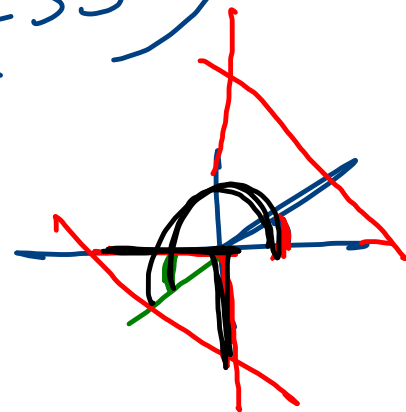
$$r = 4 \sin \theta$$

$$r^2 = 4 r \sin \theta$$

$$x^2 + y^2 = 4y$$

$$m = \tan(135^\circ)$$

=



$$r \sin \theta = y$$

$$r(1 + \cos \theta) = 2$$

$$r + r \cos \theta = 2$$

$$\sqrt{x^2 + y^2} + x = 2$$

PRACTICE PROBLEM

- (i). The line segment cut by the axes from a straight line gets divided internally in 5: 3 ratio at the point $(-4, 3)$. Find the equation of the line.
- (ii). A straight line passes through the point $(-2, 6)$ and the multiplication of the line segment cut by the axes from the line is 6; Find the equation of the line.
- (iii). $12x + 5y - 6 = 0$ and $x\cos\alpha + y\sin\alpha = p$ indicates the same line. Find the value of p .
- (iv). The line segment cut by the axes from a straight line gets divided internally in 2: 3 ratio at the point $(6, 2)$. Find the equation of the line.

না বুঝে
মুখস্থ করার
অভ্যাস প্রতিভাকে
ধ্বংস করে

$$X = c \cdot P \frac{V^2}{2S}$$

$$X = c \cdot P \frac{V^2}{2S}$$

$$E = mc^2$$

$$x = \sqrt{\frac{a^2}{c^2} + c} - \frac{b}{2}$$



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