Class 11 Academic Program 2020

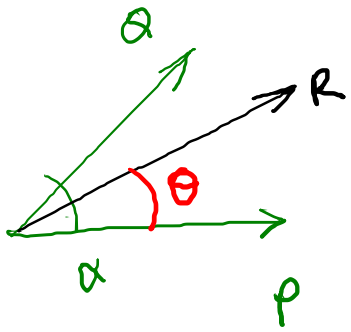
PHYSICS 1ST PAPER

Lecture : 02

Chapter 99 : Vector

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

$\bar{P} + \bar{Q} = \bar{R}$ → magnitude (R)
→ direction (θ)



$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\theta = \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Vectors → Resultant

unique

$*\ 5 + 3 = 8$

$\vec{R} = \bar{P} + \bar{Q}$

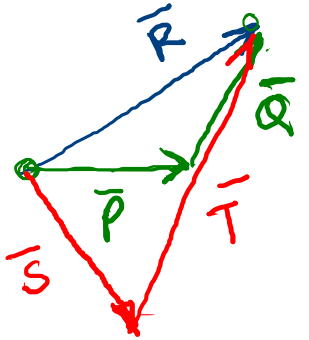
Components of $\vec{R}$

→ not unique

$8 = 5 + 3$

$= 6 + 2$

$= 10 + (-2)$



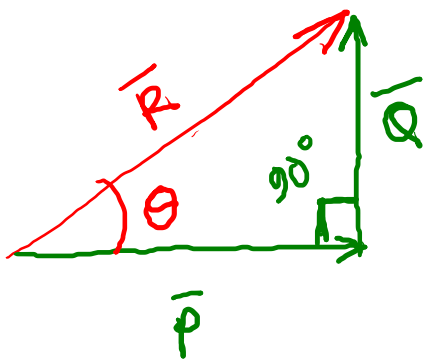
$\bar{P} + \bar{Q} = \bar{R}$

$\bar{S} + \bar{T} = \bar{R}$

Perpendicular Components

$$\bar{R} = \bar{P} + \bar{Q}$$

$$\bar{P} \perp \bar{Q}$$



Given $\vec{R}$ → R, θ

Find: P = ?

Q = ?

WHITEBOARD

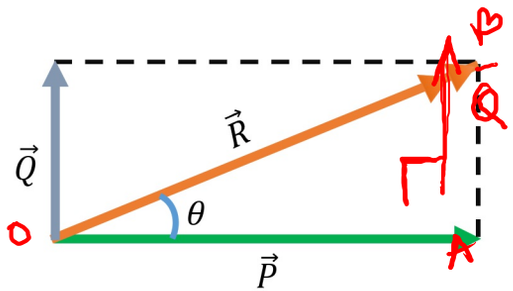
Topics

- Vector Components
- Perpendicular Components
- Theorem of Resolved Parts
- Resolution of vector in 3-D Co-ordinates
- Addition and Subtraction of Resolved Normal Components

Perpendicular Components:

Given: R, θ

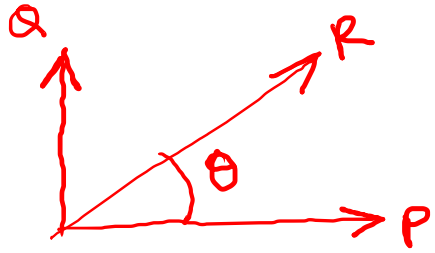
Find: P, Q



ΔOAB

$$\cos\theta = \frac{P}{R} \quad \Rightarrow \quad P = R\cos\theta$$

$$\sin\theta = \frac{Q}{R} \quad \Rightarrow \quad Q = R\sin\theta$$

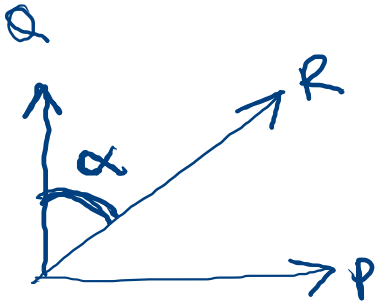


$P = R\cos\theta = R\cos(90^\circ - \alpha)$

$Q = R\sin\theta = R\sin(90^\circ - \alpha)$

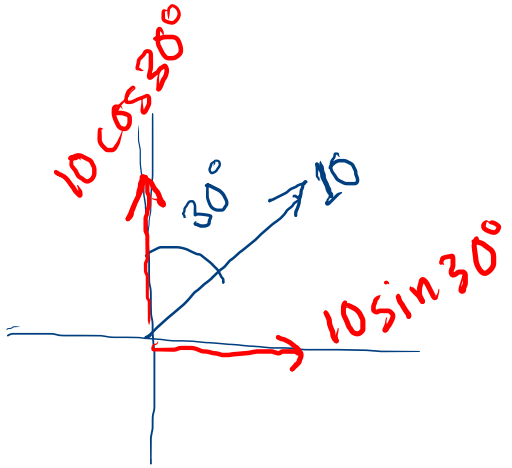
$\theta + \alpha = 90^\circ$

$\theta = (90^\circ - \alpha)$



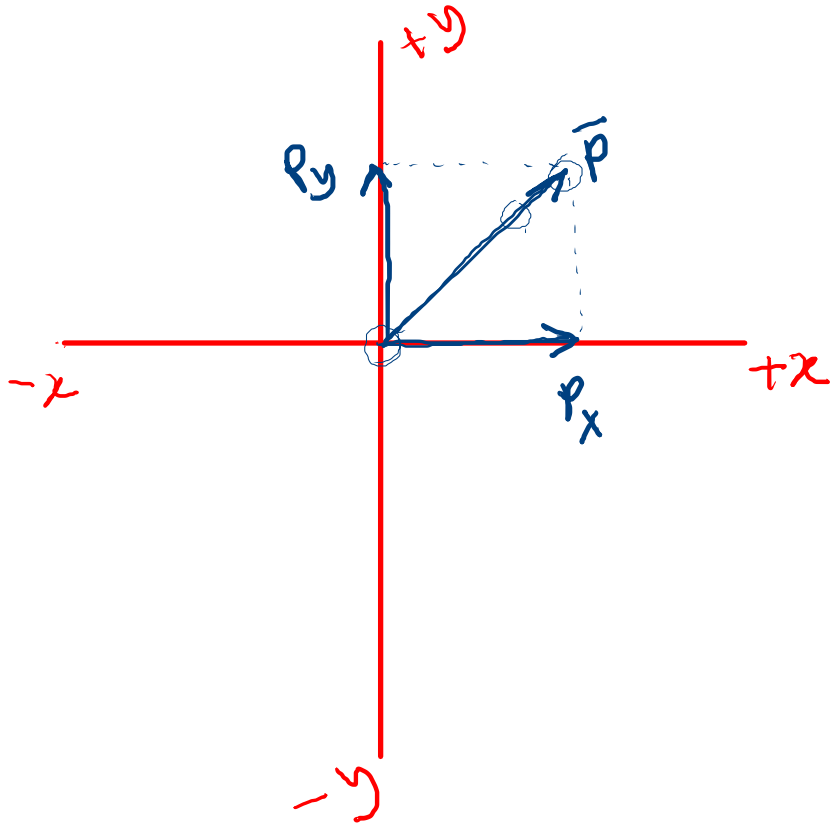
$P = R\sin\alpha$

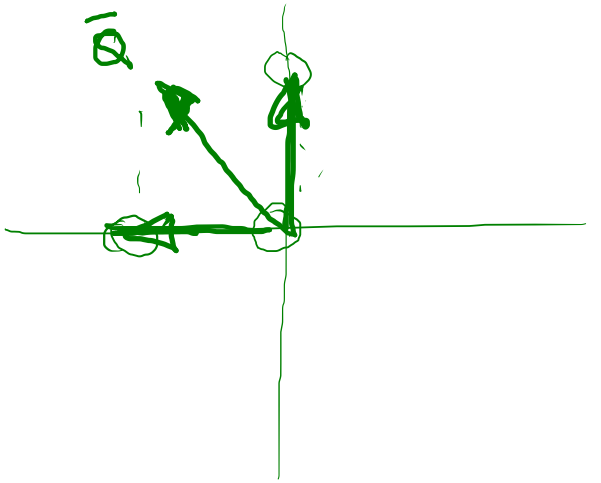
$Q = R\cos\alpha$

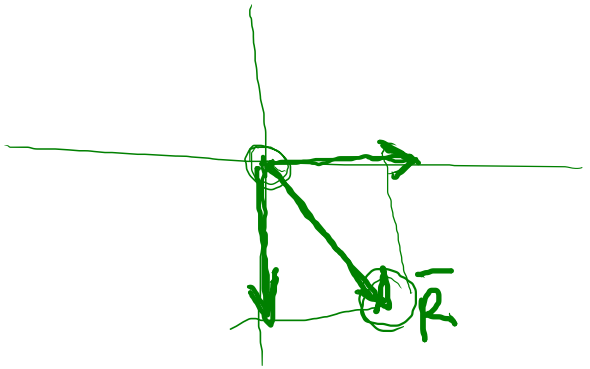


$\cos(90^\circ - \theta) = \sin\theta$

$\sin(90^\circ - \theta) = \cos\theta$


+y
p_y
$\bar{p}$
-x
p_x
+x
-y


$\bar{Q}$


$\bar{R}$

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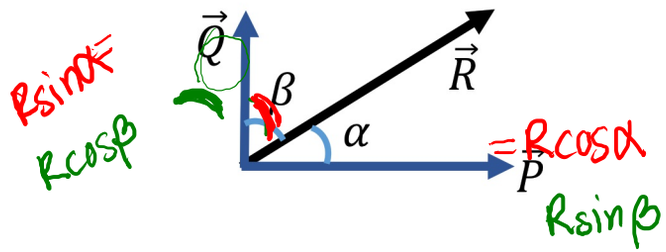
Poll Question 01

$\boldsymbol{\alpha + \beta = 90°}$ Which one is correct?

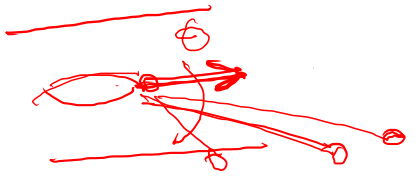


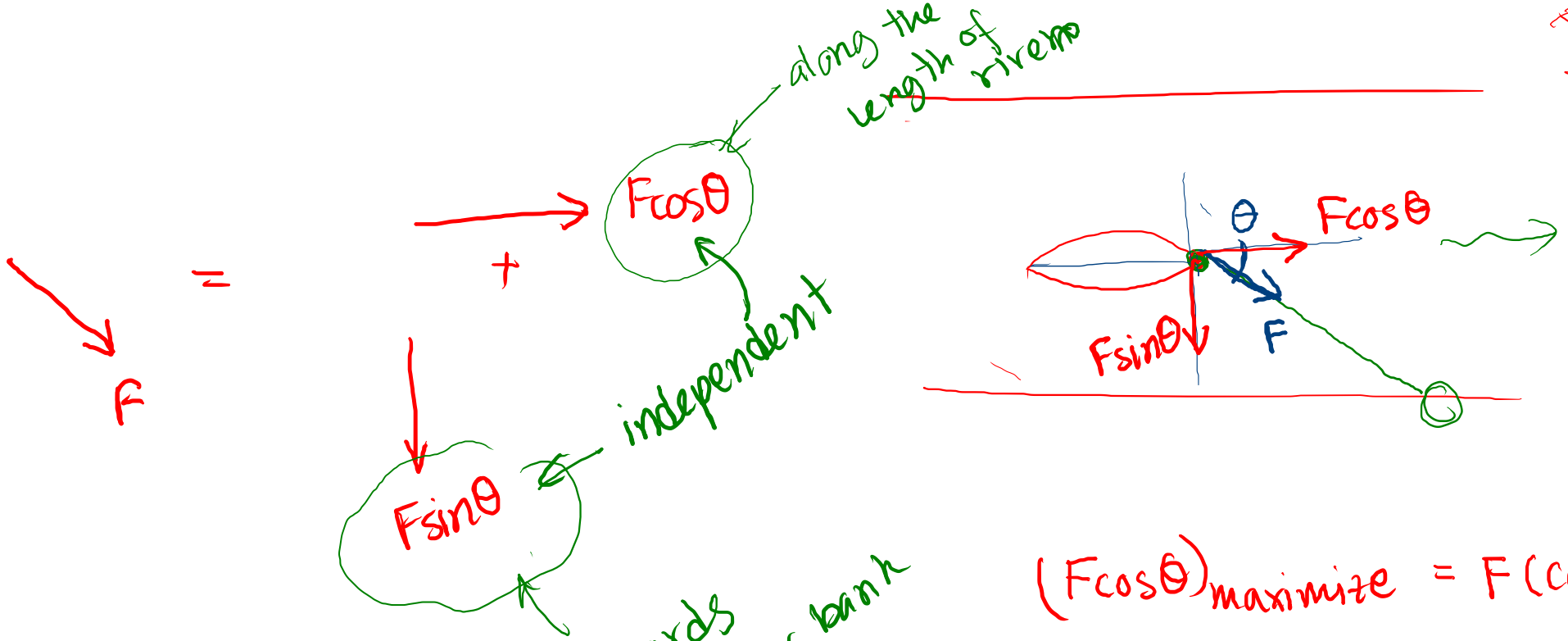
(a) $Q = R\cos\beta$

(b) P= $R\sin\beta$

(c) $Q = R\sin\alpha$

(d) All

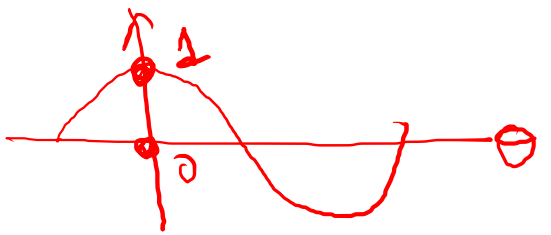




$(F\cos\theta)_{\text{maximize}} = F(\cos\theta)_{\max}$

$\cos\theta \simeq 1$

$\theta = 0^\circ$



*Why perpendicular components? → they are "independent"

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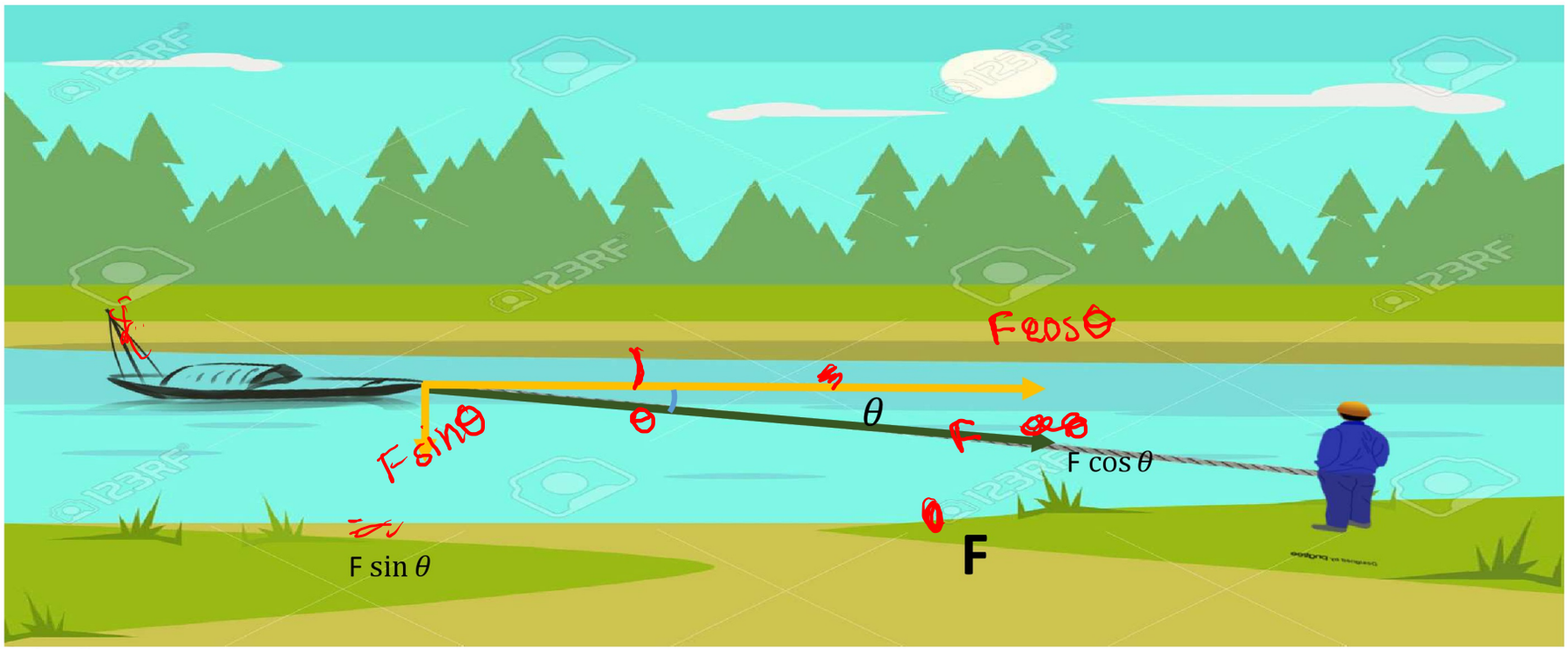
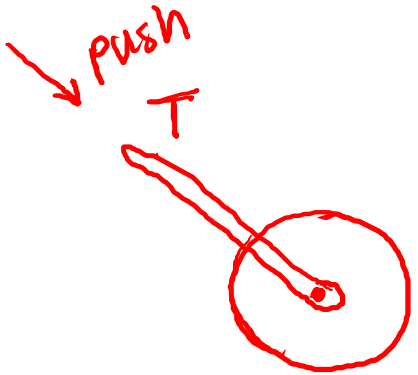
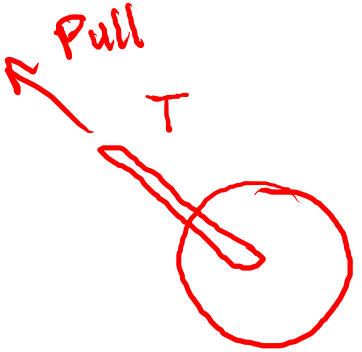
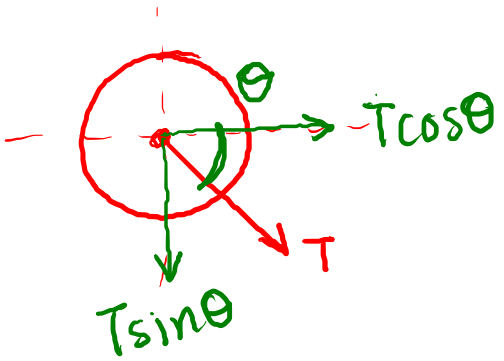
Pulling the Rope of a Boat



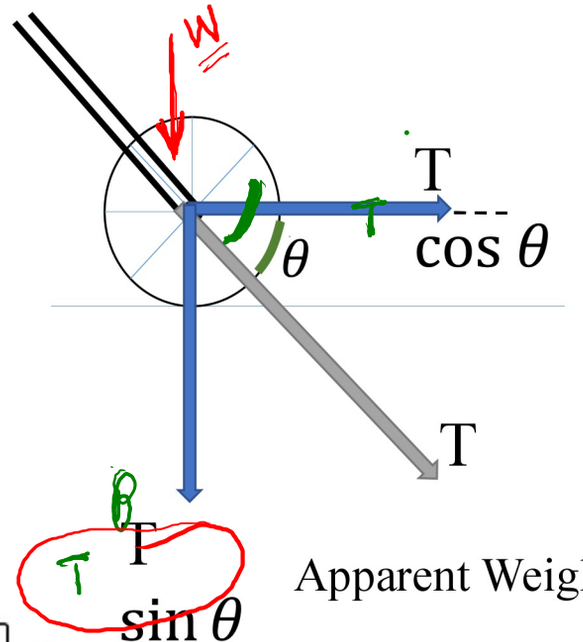

push
T


Pull
T


θ
Tcosθ
T
Tsinθ

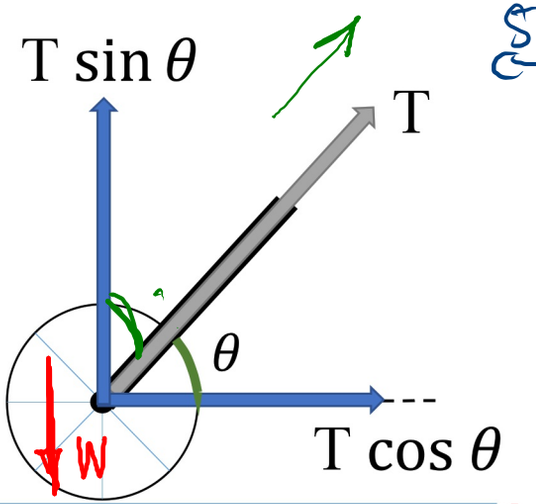
Lawn Roller : Pull and Push

Push

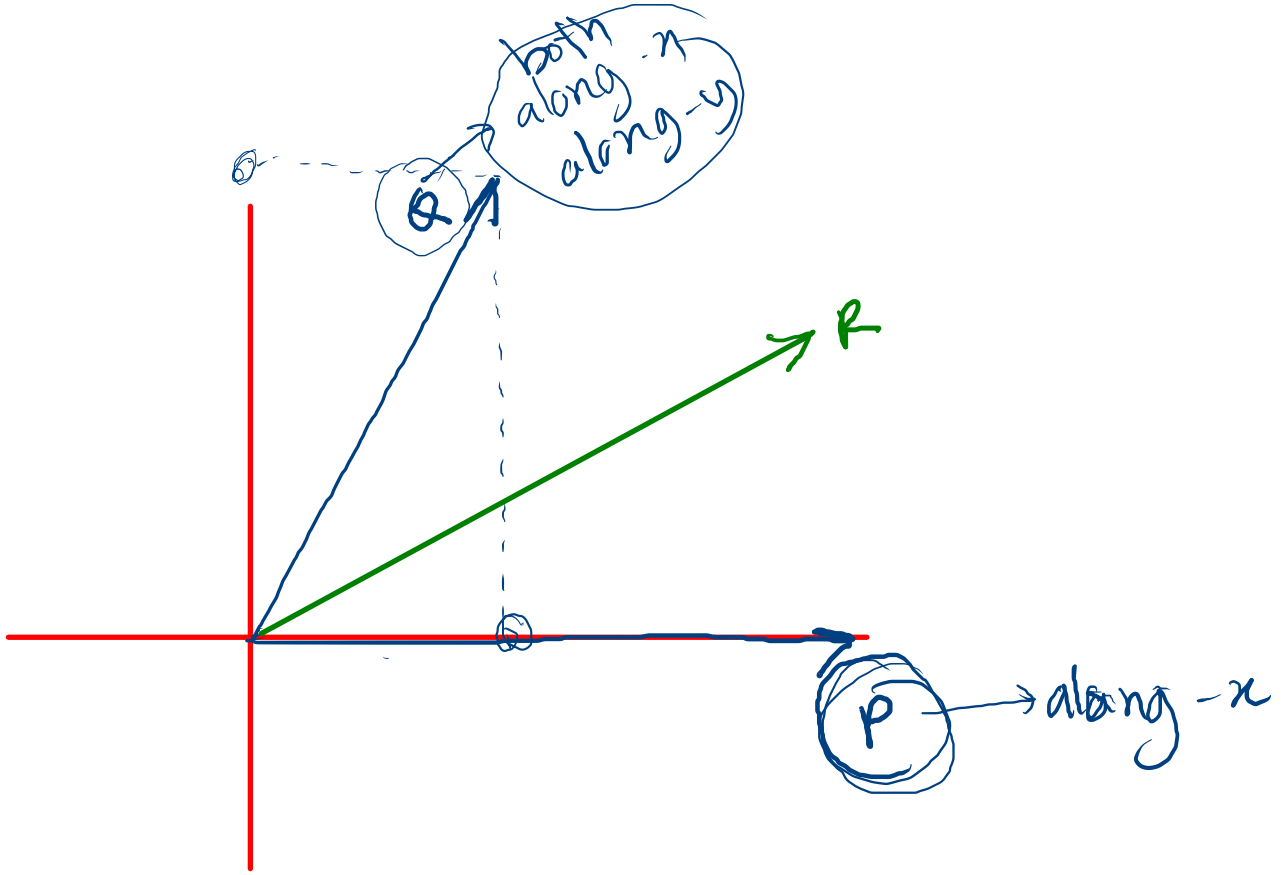


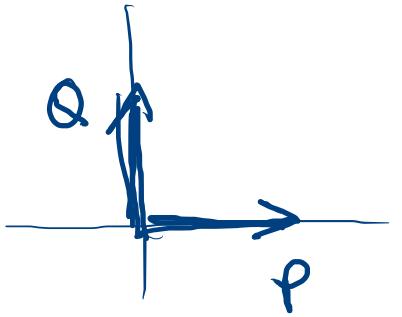
Apparent Weight =

Pull



Apparent Weight =

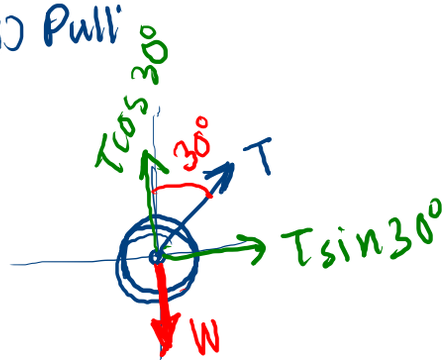

both along -x along -y
Q
R
P
along -x


Q
P

Practice Problem

A lawn roller of 20 kg is being (i) Pulled (ii) Pushed at an angle of 30° with vertical. What will be the apparent weight? Force = 100N.

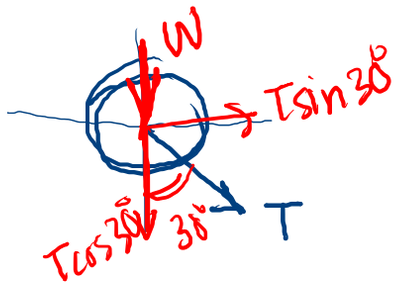
(i) Pull



$W - T\cos 30^\circ$

$= 20 \times 9.8 - 100\cos 30^\circ$

(ii) Pushing



$W + T\cos 30^\circ$

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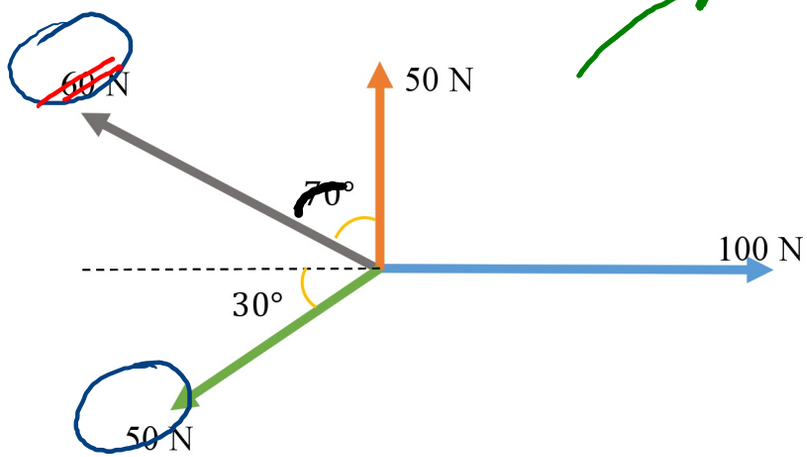
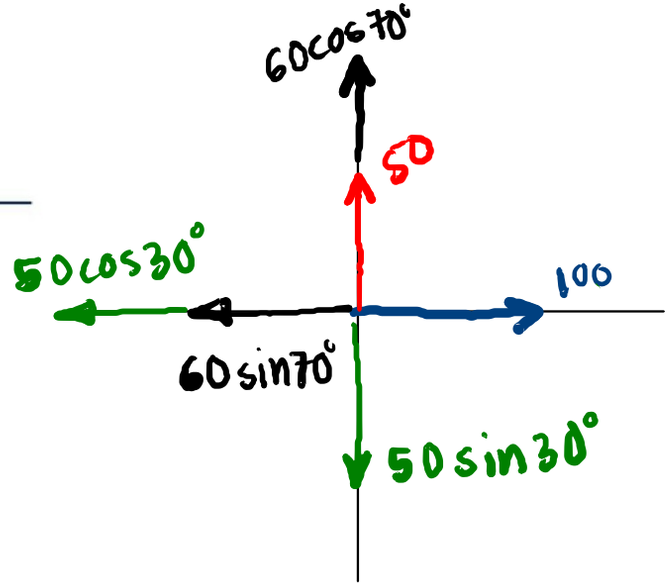
Practice Problem





$R_x = 100 - 60\sin 70^\circ - 50\cos 30^\circ$

$= 0.317$

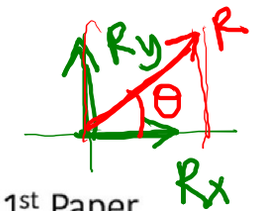
$R_y = 50 + 60\cos 70^\circ - 50\sin 30^\circ$

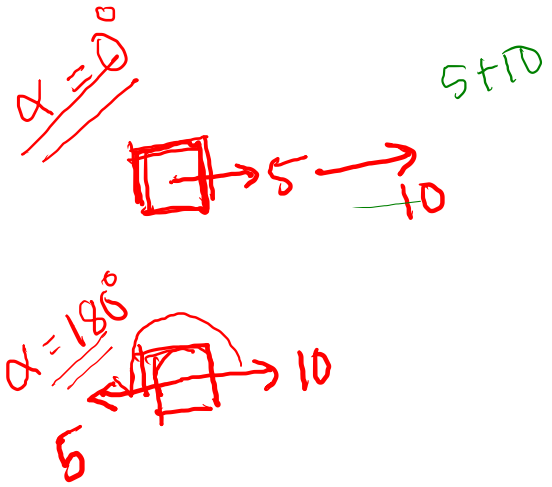
$= 45.52$

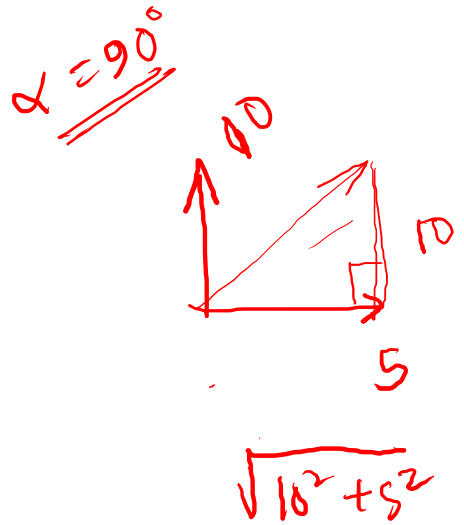
Find the Resultant for the vectors shown in the figure.

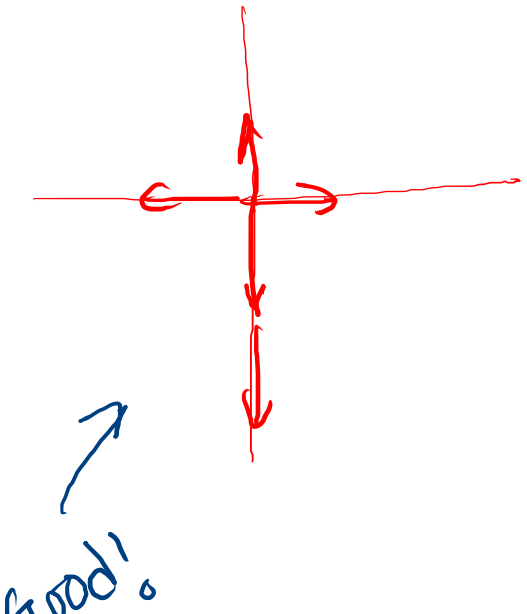
$$R = \sqrt{R_x^2 + R_y^2}$$

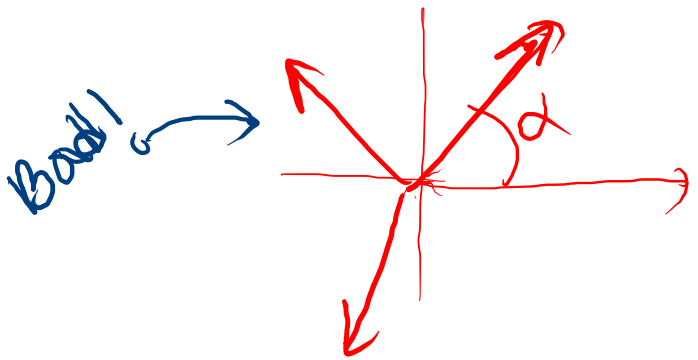
$$\theta = \tan^{-1}\frac{R_y}{R_x}$$




α = 0°
5
10
5+10
α = 180°
10
5


α = 90°
10
10
5
√10² + 5²


Good!


Bad!
α

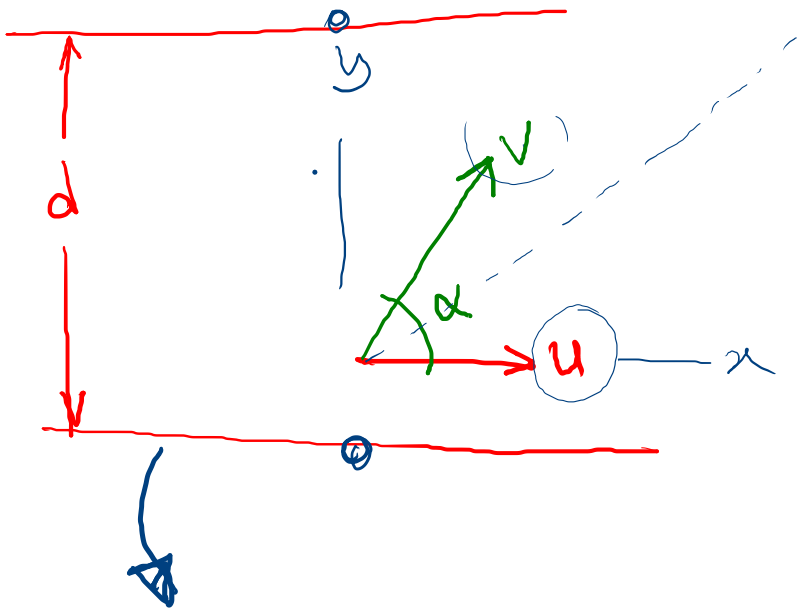
River - Boat

current speed u

Boat speed v

$\alpha = ?$

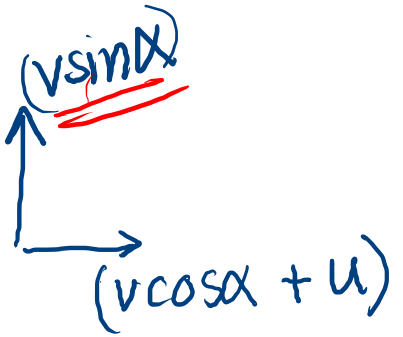
Target: to cross the river
↳ d distance along y



Last-class ✱ minimum distance distance

Condⁿ : $u + v\cos\alpha = 0$

$$\cos\alpha = -\frac{u}{v}$$



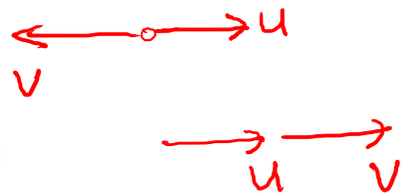
* time = ?

distance = velocity x time

y-axis

$d = v\sin\alpha \times t$

$t = \frac{d}{v\sin\alpha}$



Practice Problem

Velocity towards the tide and velocity opposite to tide of the boat are respectively $18ms^{-1}$ and $6ms^{-1}$. Which way the boatman have to start to cross the river in minimum time ? At which point will the boat reach the opposite side?

width of the river is 1200 m

minimize time

$u + v = 18$
$v - u = 6$

$u = 6 m/s$
$v = 12 m/s$

* Minimum time

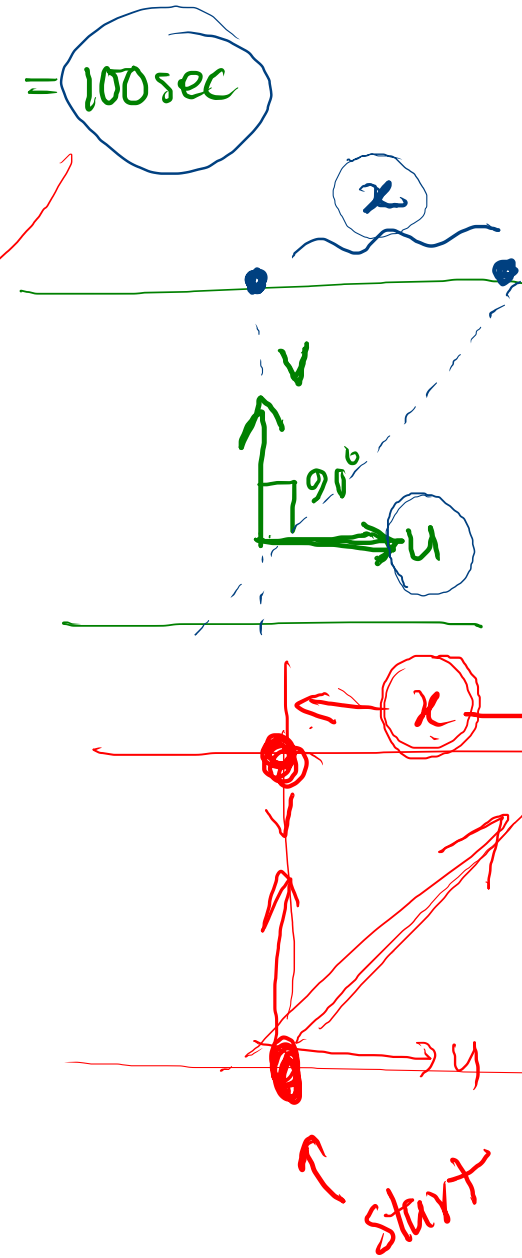
$$t_{min} = \frac{d}{v(\sin\alpha)_{max}} \Rightarrow t_{min} = \frac{d}{V} = \frac{1200}{12} = 100\,sec$$

$$(\sin\alpha)_{max} = 1$$

$$\alpha = 90^\circ$$

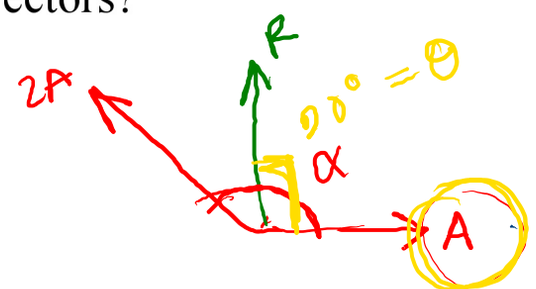
$$x = u \times t$$

$$= 6 \times 100 = 600m$$



Practice Problem

Of two vectors one is double in magnitude of the other one. If their resultant is perpendicular to one of them, what is the angle between the vectors?

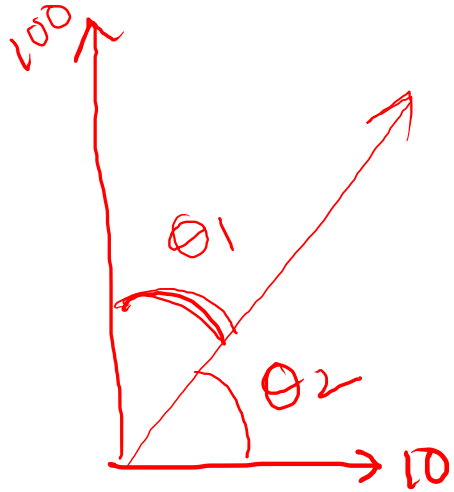


$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

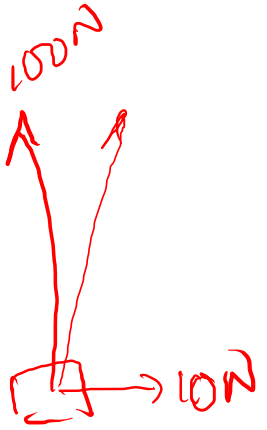
$$\frac{P}{Q} = \tan 90^\circ = \frac{2A\sin\alpha}{A + 2A\cos\alpha}$$

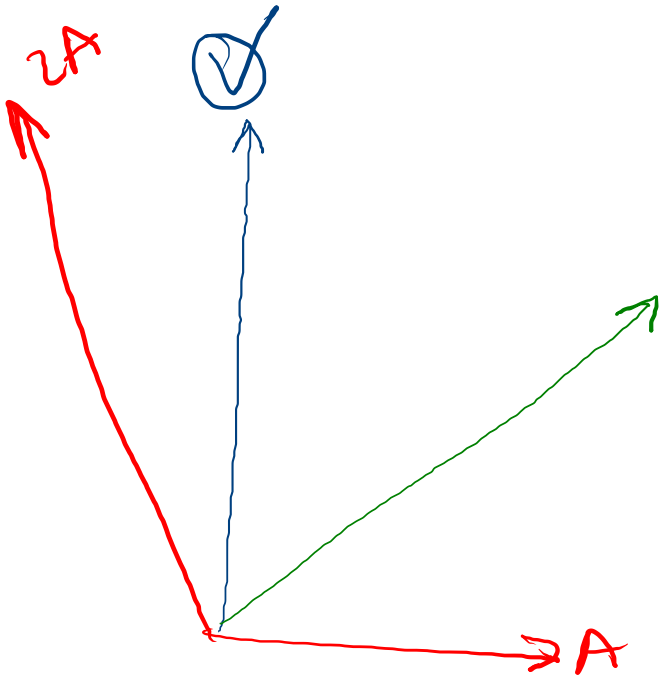
$$\Rightarrow A + 2A\cos\alpha = 0$$

$$\cos\alpha = -\frac{A}{2A} = -1/2 \rightarrow \alpha = 120^\circ$$



$\theta_1 < \theta_2$





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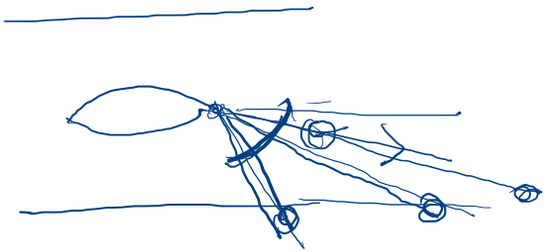
Poll Question 02



Which one is true for Pulling the rope of a boat ?

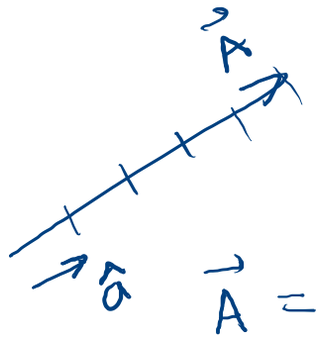
(a) Longer the rope, faster the boat moves

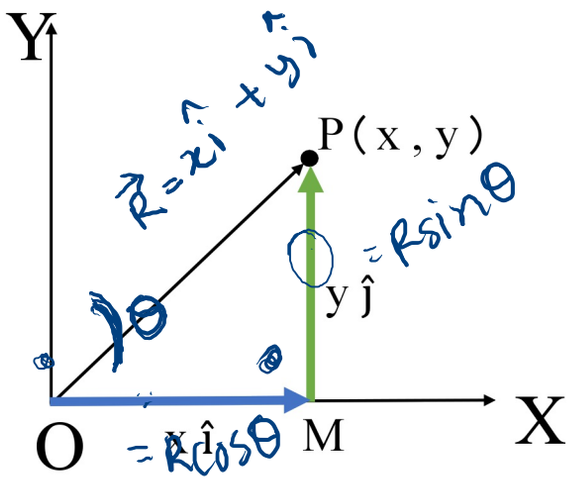
(b) Shorter the rope, faster the boat moves

(c) Shorter the rope, less the work for boatman

(d) Both (b) and (c)

Resolution of Vector in 2-D Coordinate





Applying Triangle Law in ΔOPM,

Vector = magnitude × unit vector

$\overline{OP} = \overline{OM} + \overline{MP}$

$\overline{OM} = x\hat{i} \quad \overline{MP} = y\hat{j}$

$\overline{OP} = x\hat{i} + y\hat{j}$

Known: R, θ

$$x = R\cos\theta$$

$$y = R\sin\theta$$

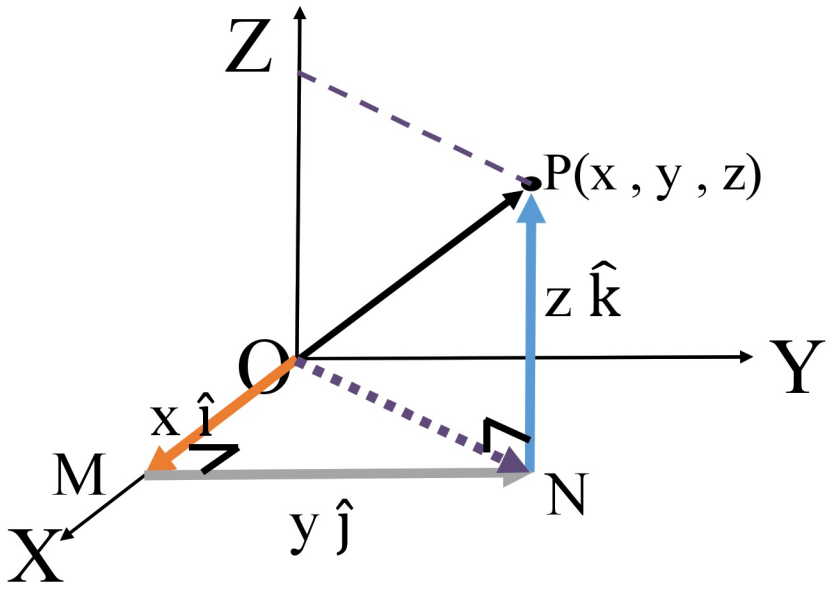
Known: x, y

$$R = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

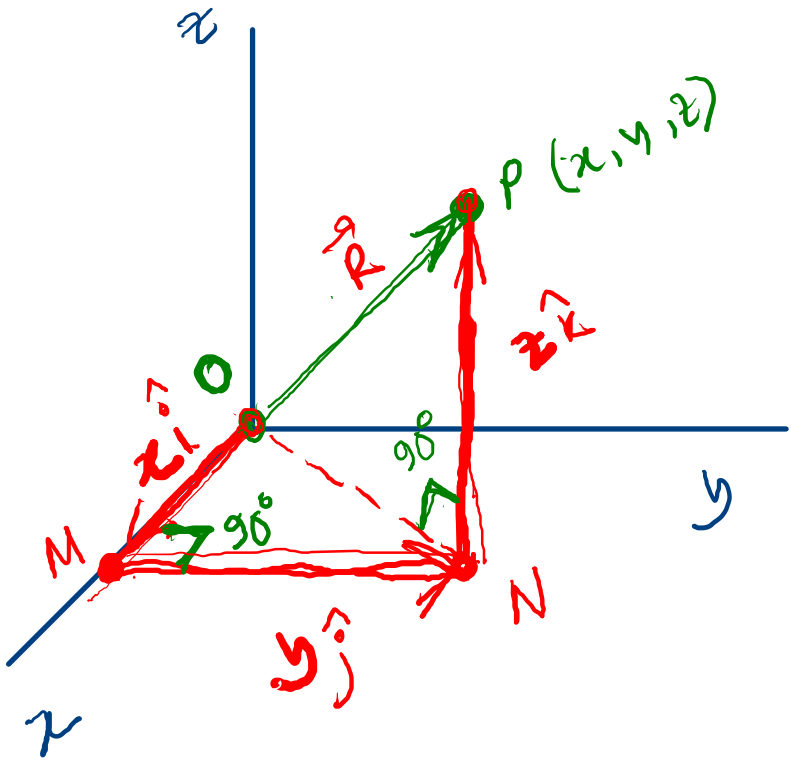
$$\vec{R} = x\hat{i} + y\hat{j}$$

Resolution of Vector in 3-D Coordinate



Applying Triangle Law in ΔOPM,

Applying Triangle Law in ΔONP,



$$\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$$

$\Delta OPN \rightarrow$

$$OP^2 = ON^2 + z^2$$

ΔOMN

$$ON^2 = x^2 + y^2$$

$$OP^2 = x^2 + y^2 + z^2$$

$$OP = \sqrt{x^2 + y^2 + z^2}$$

2D case

$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$R = \sqrt{R_x^2 + R_y^2}$$

3D

$$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

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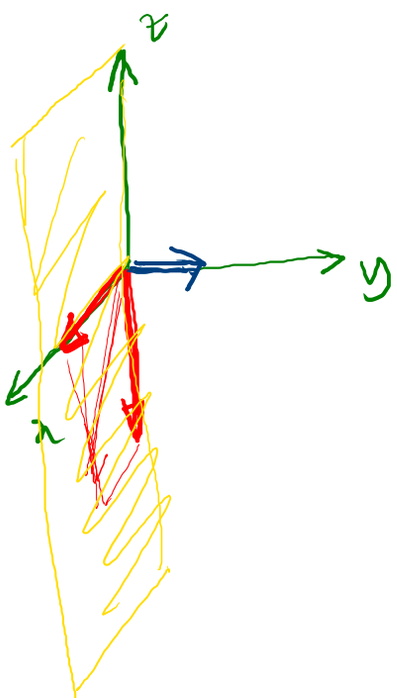
Poll Question 03



$\vec{P} = 5\hat{i} - 12\hat{k}$ Which one is true for $\vec{P}$?

$P = \sqrt{(5)^2 + (-12)^2}$

$= 13$

(a) Stands on X-Z plane

(b) Magnitude is 13 unit

(c) $2\hat{j}$ is perpendicular to $\vec{P}$

(d) All three

Addition and Subtraction of Resolved Normal Vectors

Let,

$$\vec{A} = Ax\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$$
$$\vec{B} = B_x\,\hat{\imath} + B_y\,\hat{\jmath} + B_z\,\hat{k}$$

And if, $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$

$$\vec{R} = \vec{A} \pm \vec{B} = \mathrm{Rx}\,\hat{\imath} + \mathrm{R_y}\,\hat{\jmath} + \mathrm{R_z}\,\hat{k}$$

$$\vec{A} \pm \vec{B} = (A_x \pm Bx)\,\hat{\imath} + (A_y \pm By)\,\hat{\jmath} + (A_z \pm Bz)\,\hat{k}$$

$$R_x = Ax \pm Bx$$
$$R_y = Ay \pm By$$
$$R_z = Az \pm Bz$$

$$\bar{A} = 3\hat{i} + 2\hat{j} + 0\hat{k}$$

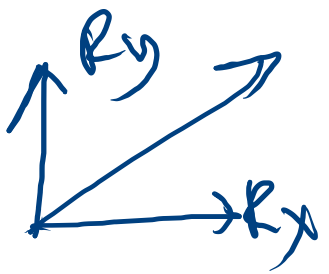
$$\bar{B} = 6\hat{i} + 5\hat{j} - 10\hat{k}$$

$$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

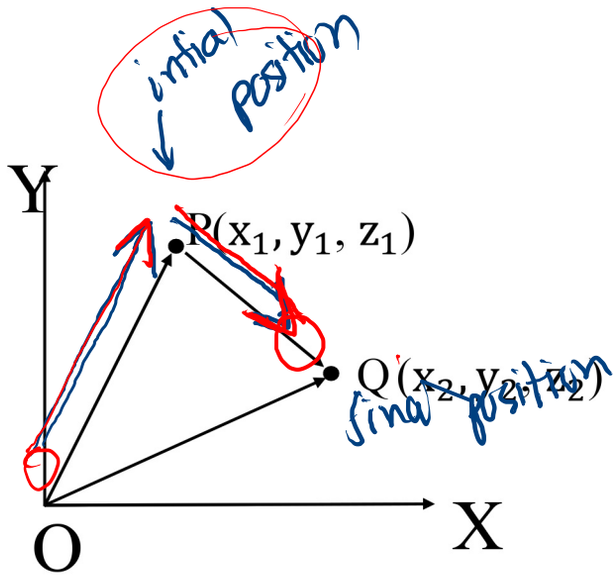
$$\bar{A} + \bar{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

$\vec{R}_x$ $\vec{R}_y$



$A_x \longrightarrow B_x$

Expression of a Vector From the Coordinates of Initial and Final Point



$$\overrightarrow{OP} = x_1\,\hat{i} + y_1\,\hat{j} + z_1\hat{k}$$

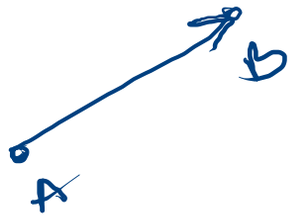
$$\overrightarrow{OQ} = x_2\,\hat{i} + y_2\,\hat{j} + z_2\hat{k}$$

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$(x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\hat{k}$$

$\overrightarrow{OP} + \overrightarrow{PQ} = \overrightarrow{OQ}$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$



Practice Problem

If A(2,1,3) and B(4,3,2)

Find a unit vector along $\vec{AB}$.

$$\vec{P} = P\hat{p}$$

$$\hat{p} = \frac{\vec{P}}{P}$$

$$\frac{\vec{AB}}{|\vec{AB}|} = \frac{(2\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{2^2+2^2+1^2}}$$

$$= \frac{1}{3}(2\hat{i} + 2\hat{j} - \hat{k})$$

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে।