একাদশ শ্রেণি একাডেমিক প্রোগ্রাম ২০২০

পদার্থবিজ্ঞান ১ম পত্র

লেকচার : P-05

অধ্যায় ০২ : ভেক্টর

আলোচ্য বিষয়াবলি

- ক্যালকুলাস পরিচিতি
- অন্তরীকরণ
- আংশিক অন্তরীকরণ
- গ্রেডিয়েন্ট
- ডাইভারজেন্স
- কার্ল

ক্যালকুলাস পরিচিতি

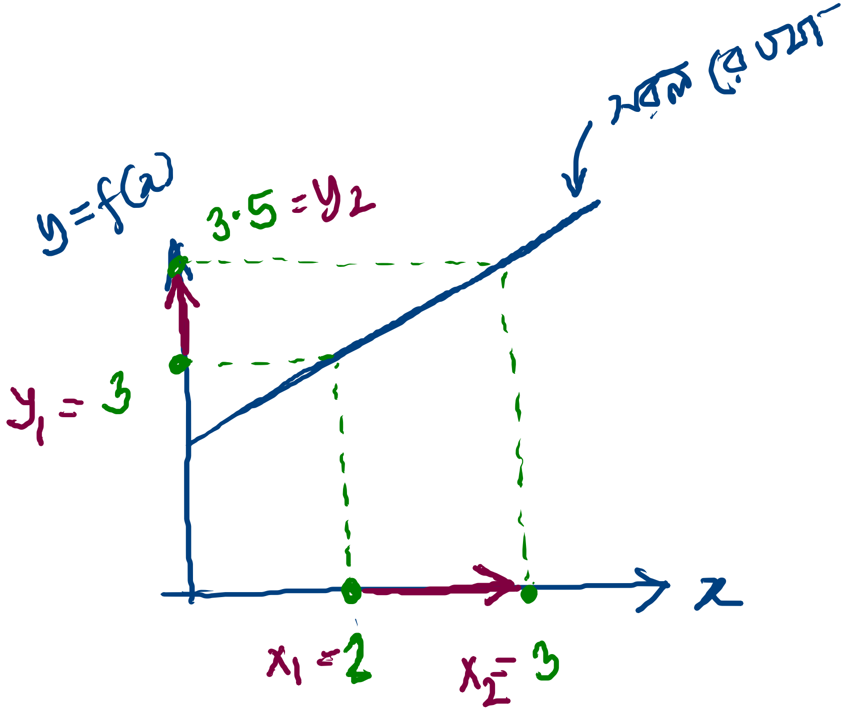
Newton

LEIBNITZ

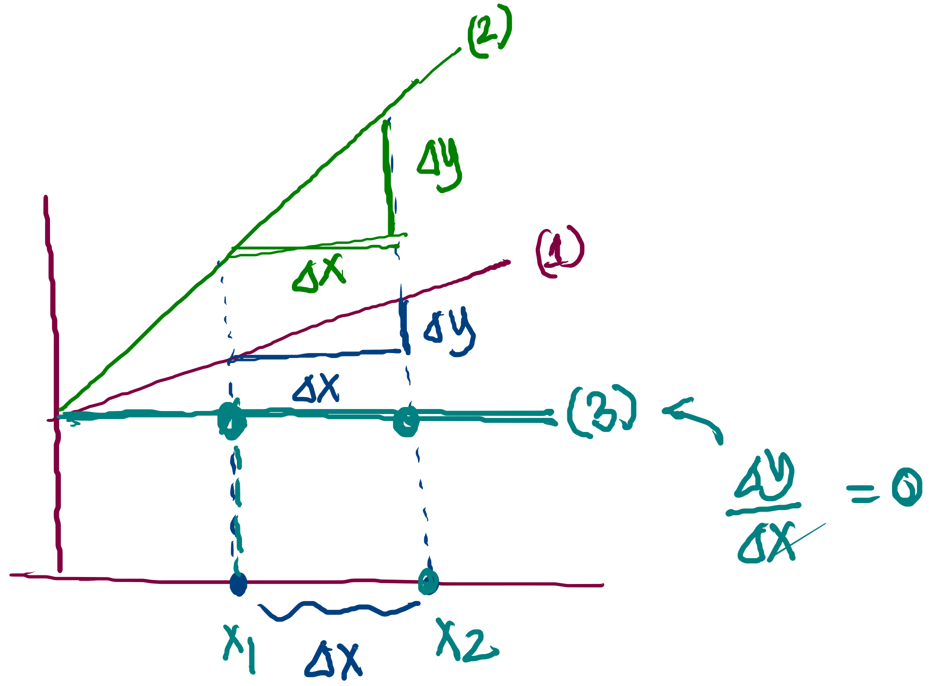
$$(x_2 - x_1) \longrightarrow (y_2 - y_1)$$

$$1 \longrightarrow \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\Delta y}{\Delta x}$$

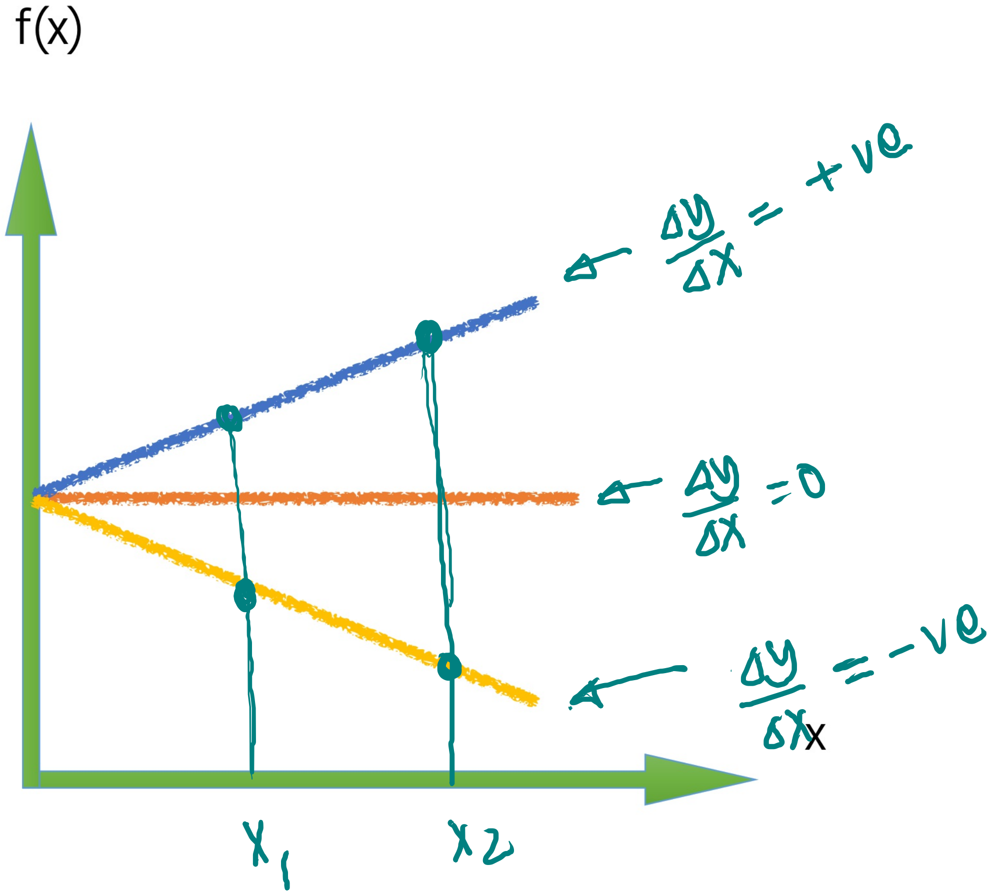


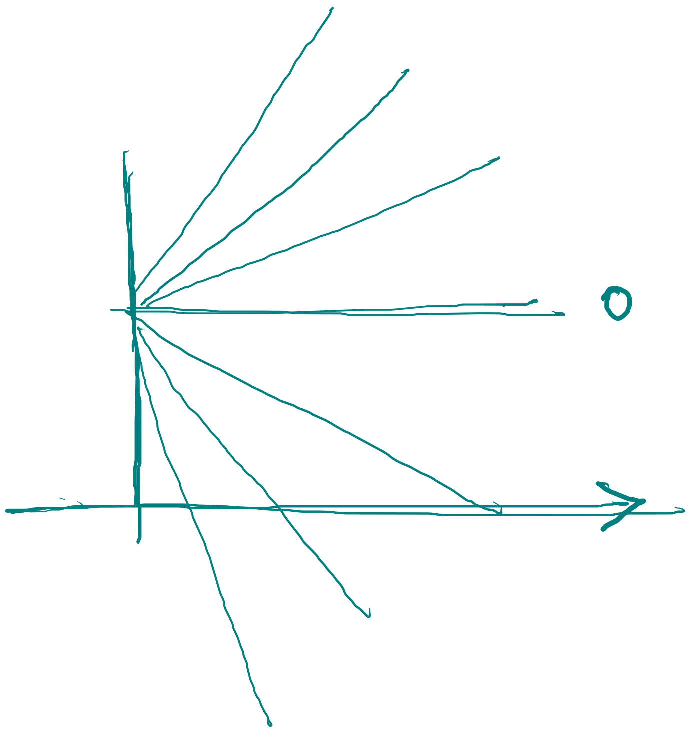
$$\frac{\Delta y}{\Delta x}$$



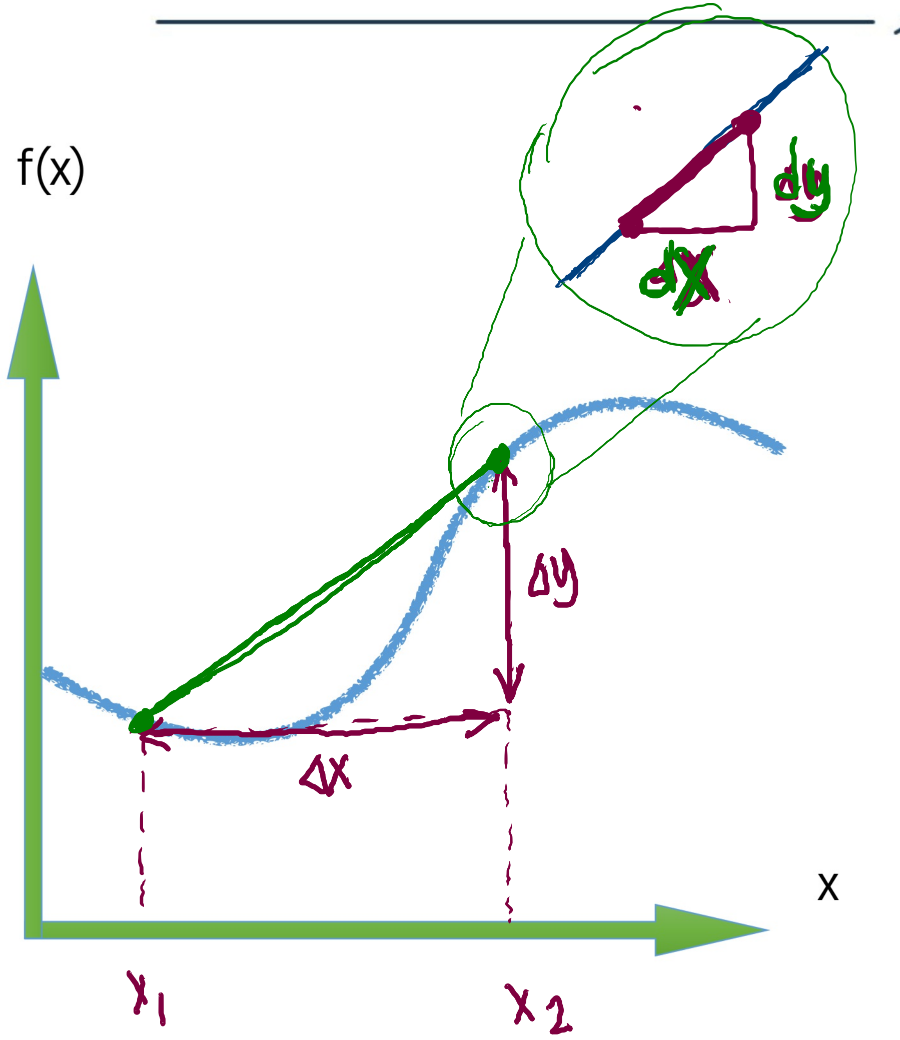
- যেই রেখার ঢাল বেশি থাকবে তার পরিবর্তনের হার তত বেশি

অন্তরীকরণ : পরিবর্তনের হার





অন্তরীকরণ : পরিবর্তনের হার



$\frac{\Delta y}{\Delta x} \neq$ পরিবর্তনের হার

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} =$ পরিবর্তনের হার

$\frac{dy}{dx}$

$dx =$ x-এর অতিক্ষুদ্র পরিবর্তন

$\frac{dy}{dx}$

$\boxed{y = f(x)}$

$\frac{df(x)}{dx}$

অন্তরীকরণ করা
Derivative নেওয়া
Differentiate করা

$\rightarrow$ x-এর সাপেক্ষে পরিবর্তনের হার

$* \quad \frac{d}{dx}(x^n) = nx^{n-1}$

$* \quad \frac{d}{dx}(c) = 0$

c = constant

$* \quad \frac{d}{dx}(c\underbrace{x^n}_{f(x)}) = c\,n\,x^{n-1}$

$$\frac{d}{dx}\left(x^5\right) = 5x^4$$

$$\frac{d}{dx}(3) = 0$$

$$\frac{d}{dx}\left(3\underline{\underline{x^5}}\right) = 3 \times 5x^4$$

$$= 15x^4$$

Practice Problem

$f(x) = 0.1x^2 + 3$ কে x এর সাপেক্ষে অন্তরীকরণ করো।

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}(0.1x^2 + 3)$$

$$= \frac{d}{dx}(0.1x^2) + \frac{d}{dx}(3)$$

$$= 0.1 \times 2x^1 + 0$$

$$= 0.2x$$

$\rightarrow x = 0 \rightarrow 0$

Poll Question - 01

$f(x) = x^2 + 3$ এর $x=0$ বিন্দুতে পরিবর্তনের হার = ?

(a) 0

(b) 3

(c) বলা সম্ভব নয়

$$\frac{d}{dx}(x^2+3)$$

$$= \frac{d}{dx}(x^2) + \cancel{\frac{d}{dx}(3)} \rightarrow 0$$

$$= 2x$$

$x=0$

$$= 0$$

চলক

$f(x) = x^2 + 3$ ⟶ function - একটি মাত্র variable x -এর উপর নির্ভর করে

↓

এক চলক বিশিষ্ট function

$f(x,y,z) = x^2 + y^2 + 2xyz$ ⟶ 3টি variable (x,y,z) -এর উপর নির্ভর করে।

↓

বহুচলক বিশিষ্ট function

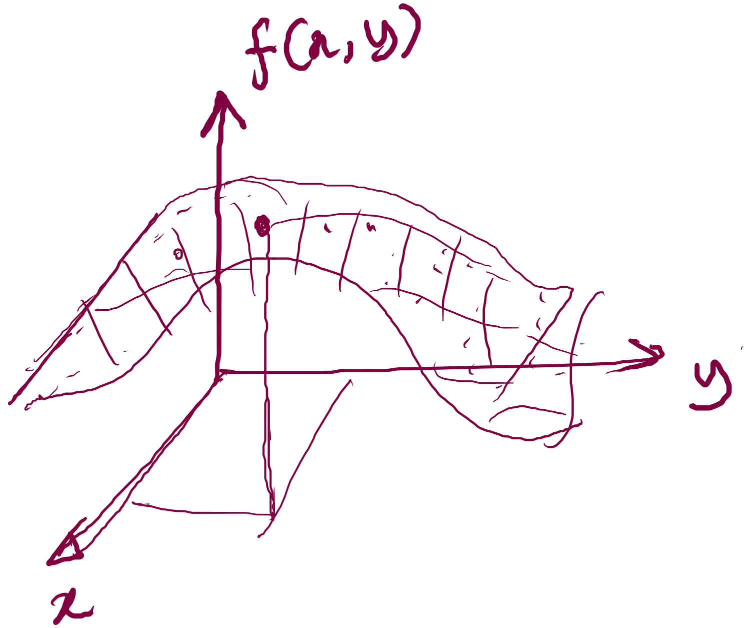
(multi variable function)

single variable function

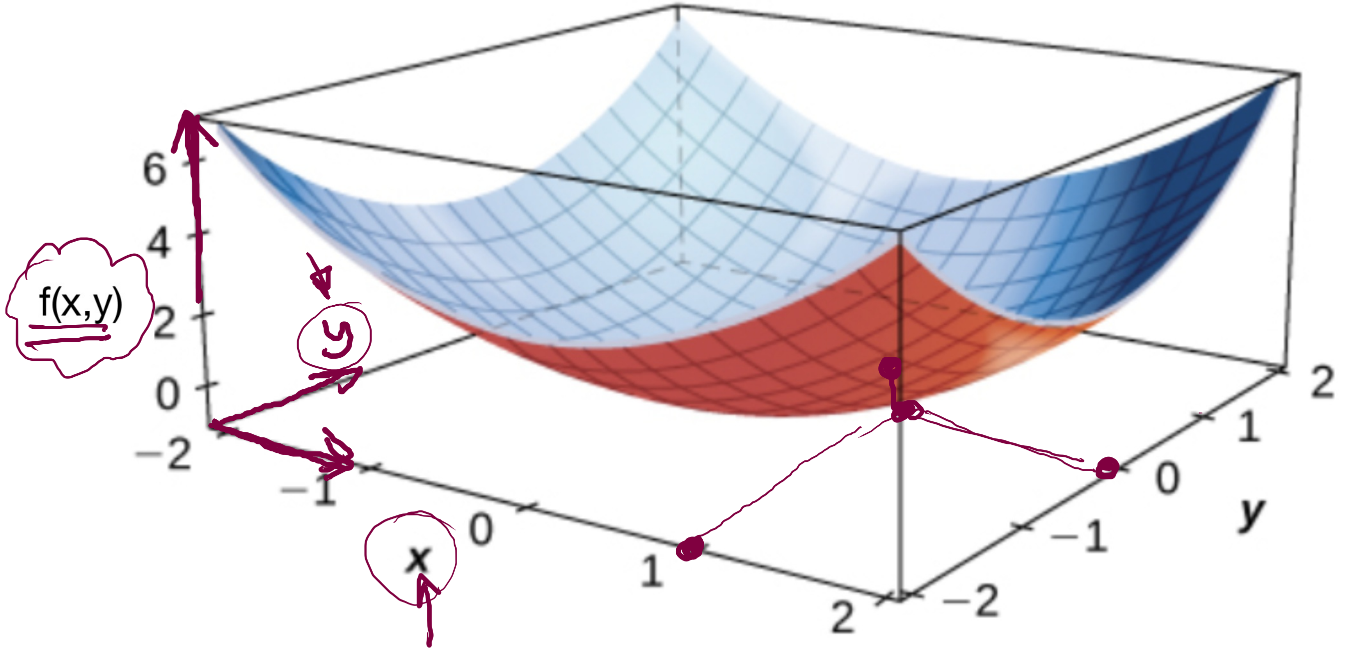
$f(x)$

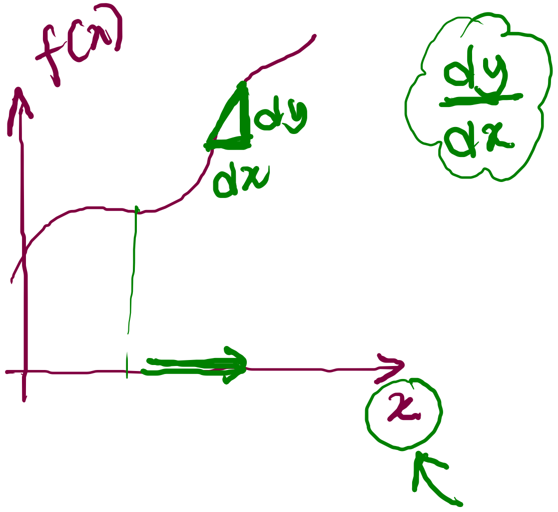
Multivariable function

$f(x, y)$



বহুচলক বিশিষ্ট ফাংশন




f(x)
dy
dx
x
dy/dx

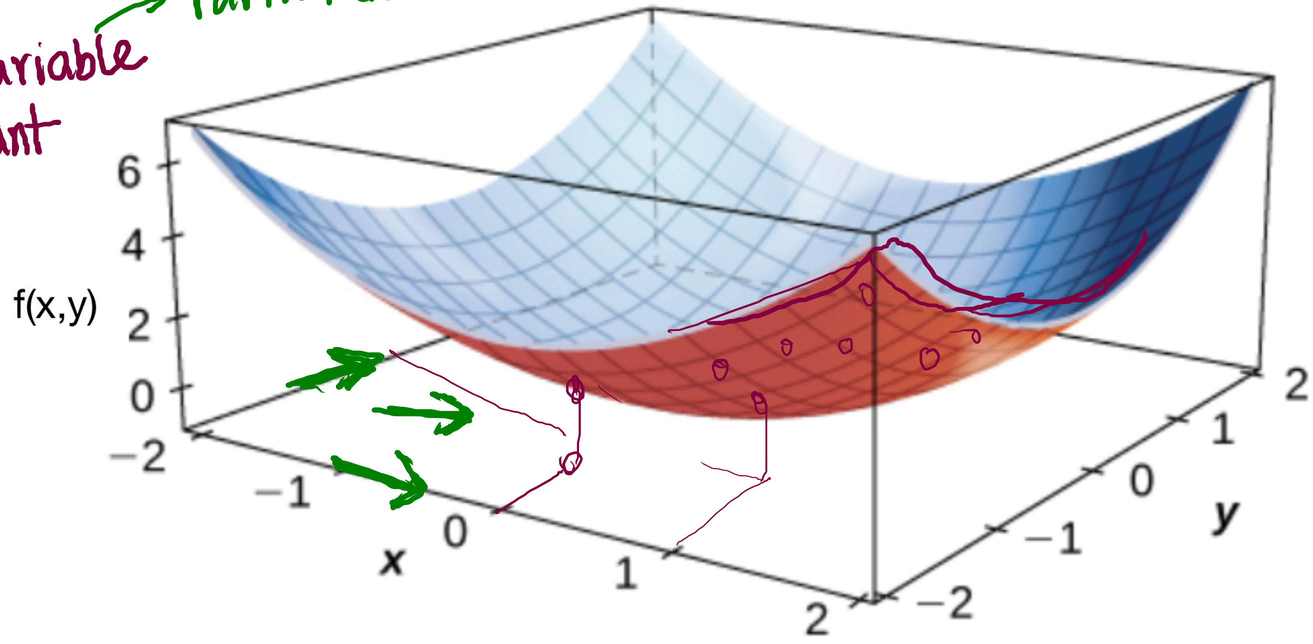
আংশিক অন্তরীকরণ

* পরিবর্তনের হার = ? (কোন দিকে ?)

$\partial \rightarrow$ delta

$\frac{\partial}{\partial x} f(x,y) \rightarrow$ x - এর দিকে পরিবর্তনের হার | অন্য variable constant $\rightarrow$ Partial derivative

$\frac{\partial}{\partial y} f(x,y) \rightarrow$ y এর দিকে



$$f(x,y,z) = x^2 + 3xy + z^2$$

$$\frac{\partial f(x,y,z)}{\partial x} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial x}(3x^{1}y) + \frac{\partial}{\partial x}(z^2)$$

$$= 2x + 3y + 0$$

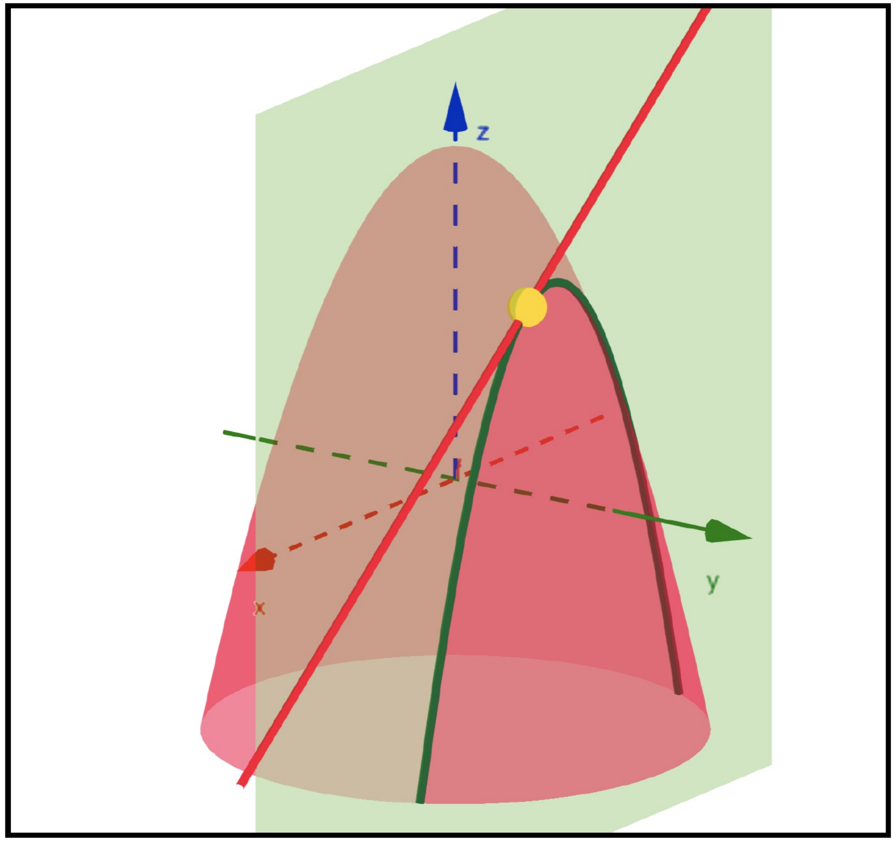
$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2) + \frac{\partial}{\partial y}(3xy^{1}) + \frac{\partial}{\partial y}(z^2)$$

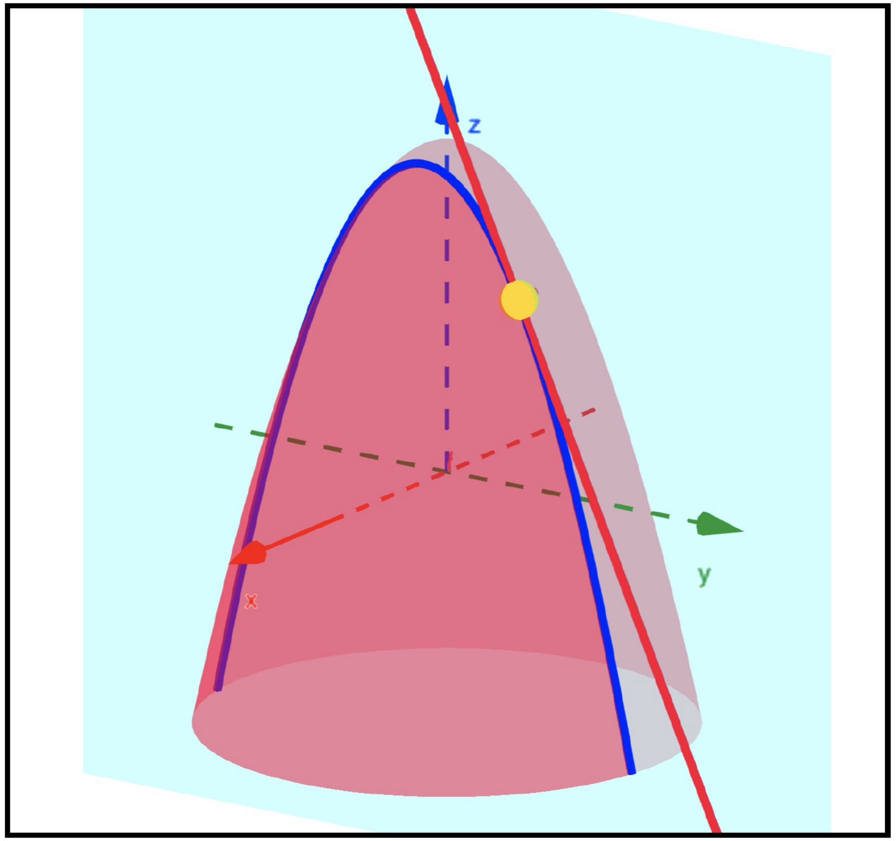
$$= 0 + 3x\underbrace{y^{0}}_{1} + 0$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(x^2) + \frac{\partial}{\partial z}(3xy) + \frac{\partial}{\partial z}(z^2)$$

$$= 0 + 0 + \boxed{2z}$$

আংশিক অন্তরীকরণ





Practice Problem

$f(x,y) = -0.1x^2 - 0.1y^2 + 32.4$ কে x ও y এর সাপেক্ষে আংশিক অন্তরীকরণ করো।

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(-0.1x^2) - \frac{\partial}{\partial x}(0.1y^2) + \frac{\partial}{\partial x}(32.4)$$

$$= -0.1 \times 2x + 0 + 0$$

$$= -0.2x$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(-0.1x^2) - \frac{\partial}{\partial y}(0.1y^2) + \frac{\partial}{\partial y}(32.4)$$

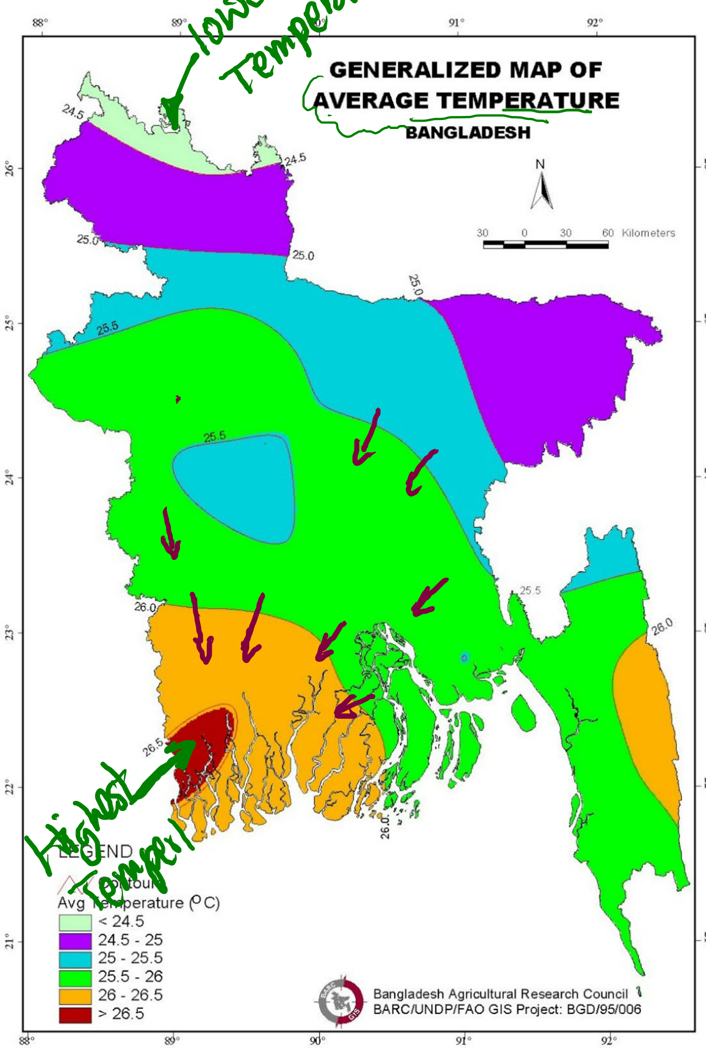
$$= 0 - 0.1 \times 2y + 0 = -0.2y$$

<u>Field</u>: প্রত্যেক point -এ একটা Quantity (রাশি) আরোপিত হলো।

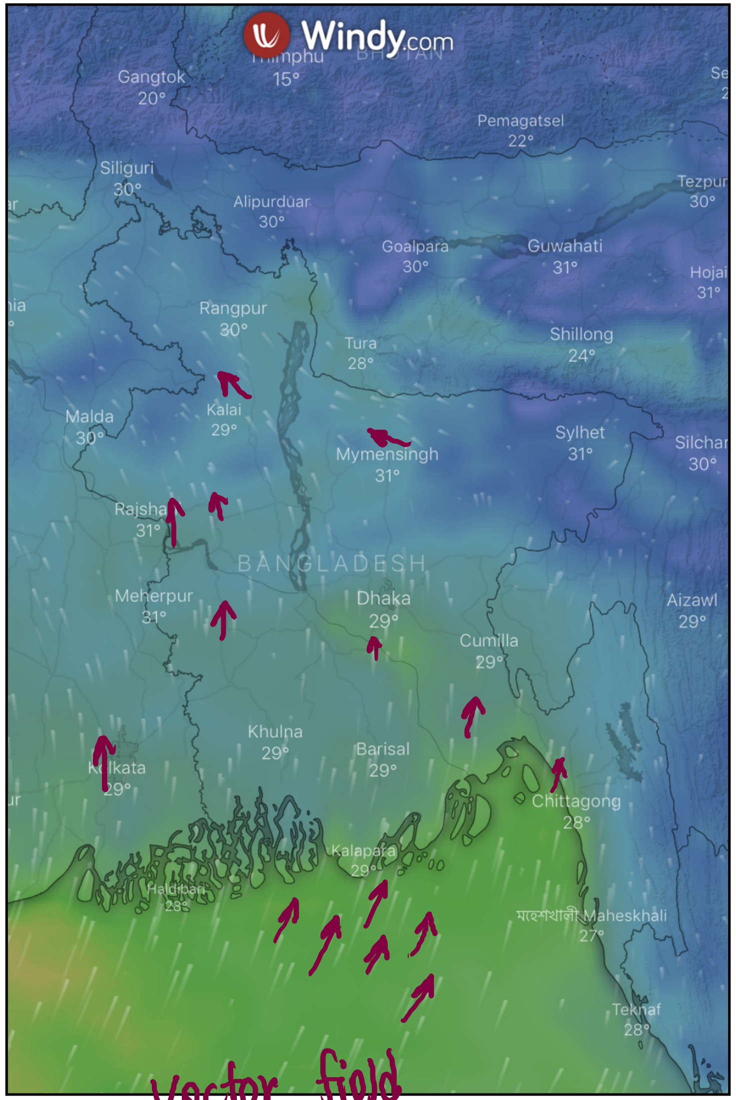
Scalar

Vector

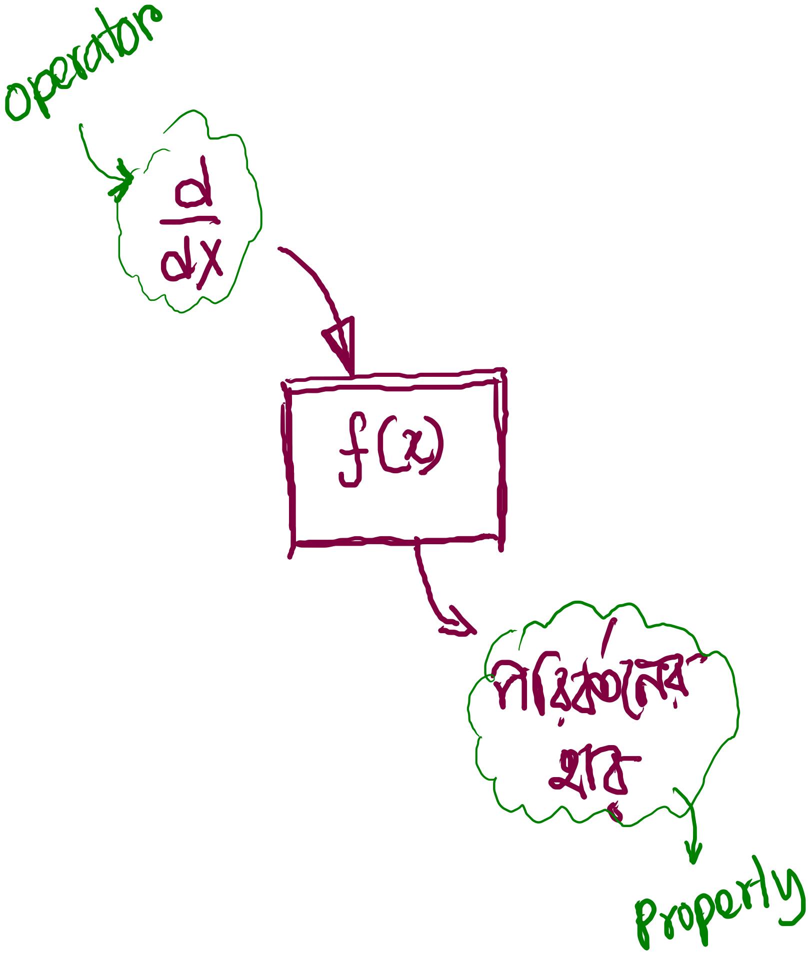
ভেক্টর ও স্কেলার ফিল্ড



Scalar field



Vector field



$$\frac{d}{dx}\{f(x)\} = \text{Property}$$

operator

$$\frac{\partial}{\partial x}$$

Nabla Operator

Scalar / Vector field

Property

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} \quad \frac{\partial}{\partial z}\hat{k}$$

"Vector Operator"

গ্রেডিয়েন্ট

স্কেলার ফিল্ড : $f(x,y,z)$

$\vec{\nabla} f(x,y,z)$ = Gradient of $f(x,y,z)$ = vector field

Vector operator

Scalar field

$f(x,y,z)$ বা Scalar field -এর সর্বোচ্চ পরিবর্তনের মান ও দিক

$$\vec{E} = -\vec{\nabla} V$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$* \; f(x,y,z) = x^2 + y^2 + z^2$$

$$\frac{\partial f}{\partial x} = 2x$$

$$\frac{\partial f}{\partial y} = 2y$$

$$\frac{\partial f}{\partial z} = 2z$$

$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) f(x,y,z)$$

$$= \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$\vec{\nabla} f = 2x\,\hat{i} + 2y\,\hat{j} + 2z\,\hat{k}$$

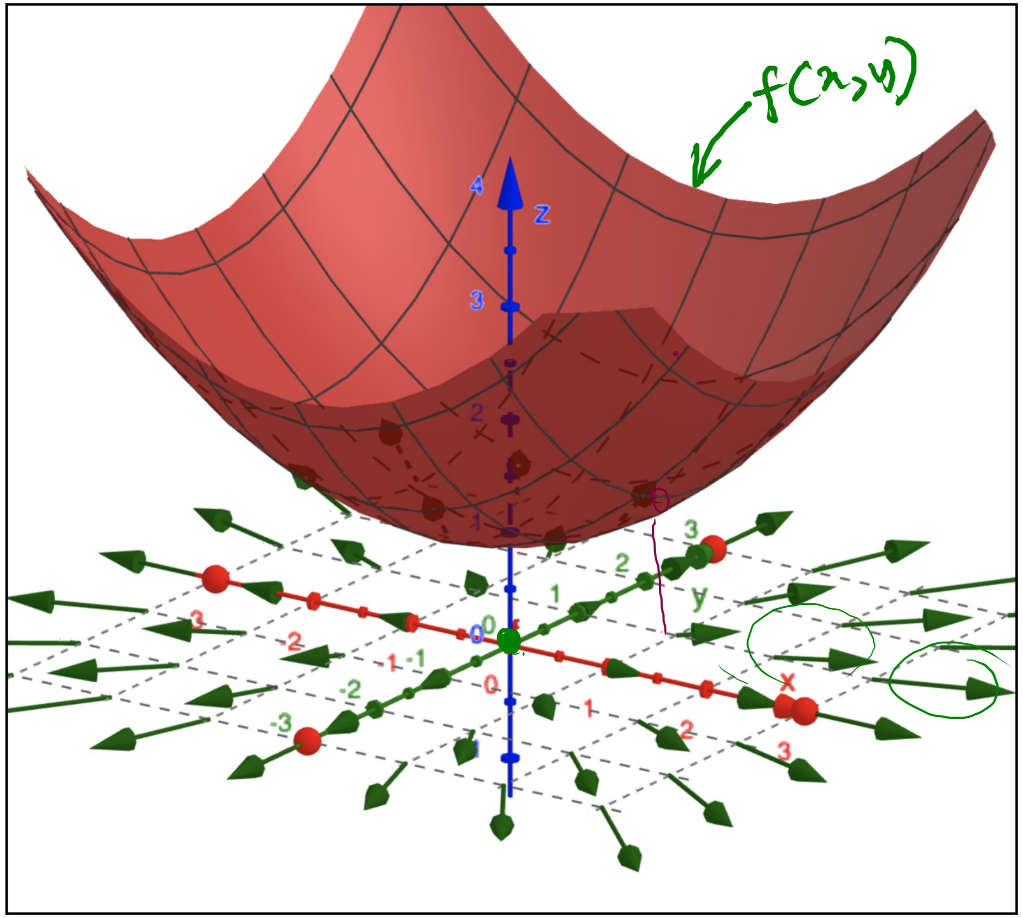
Practice Problem

x^2

$f(x)$

$= 0.2x^2 + 0.2y^2 + 1$

$f(x,y) = 0.2(x^2 + y^2) + 1$ এর গ্রেডিয়েন্ট নির্ণয় করো ।



$$\vec{\nabla} f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) f(x,y,\ldots)$$

$$= \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

H.W

$$= 0.4x\hat{i} + 0.4y\hat{j} + 0\hat{k}$$

Nabla

∇

$$\boxed{\begin{array}{c}\text{Vector field}\\ \vec{F}(x,y,z)\end{array}}$$

Property

$\vec{\nabla} \circ \vec{F}$

vector field

Vector operator

dot $\rightarrow$ Divergence

cross $\rightarrow$ Curl

ডাইভারজেন্স : Source / sink

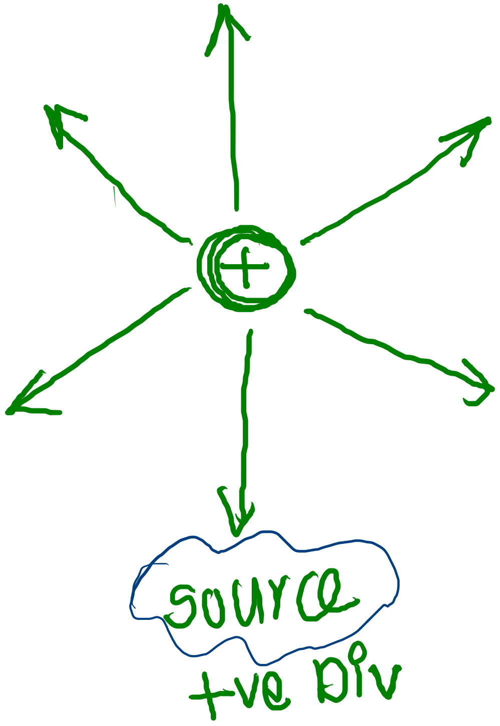
$\vec{F}(x,y,z)$ = Vector field

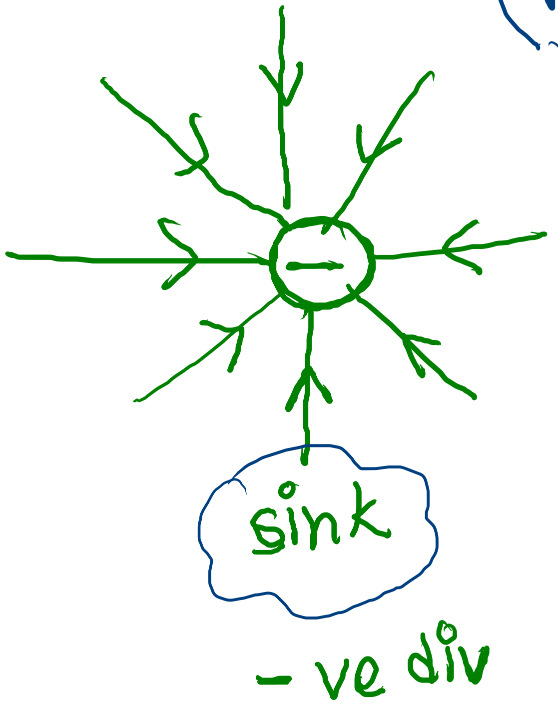
$\vec{\nabla}\cdot\vec{F}$ = Scalar field → মান → +ve, -ve

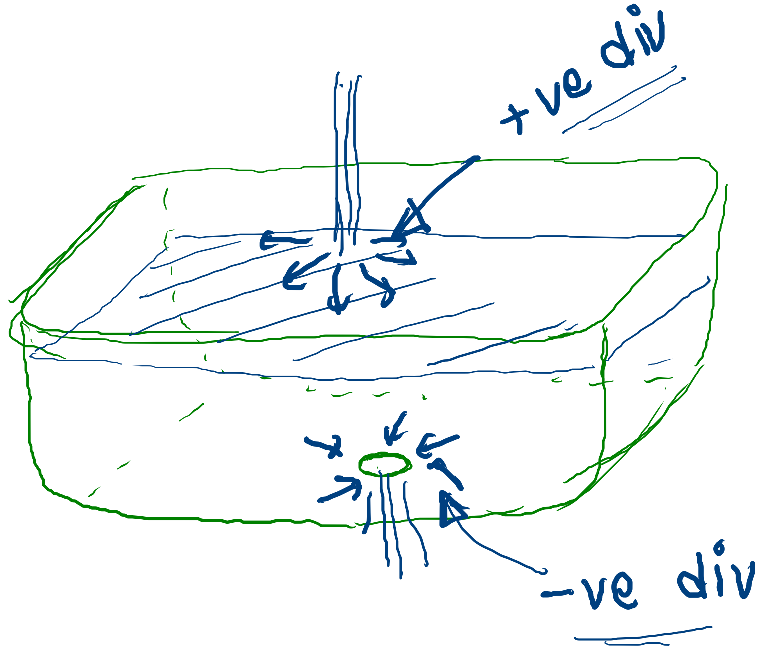
$\vec{\nabla}\cdot\vec{F} = 0$ at all points

↓

($\vec{F}$ সলিনয়ডাল)



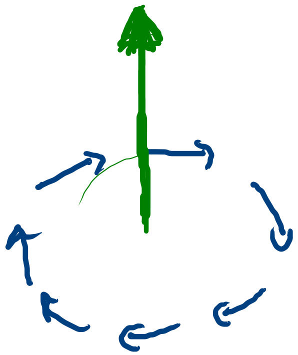


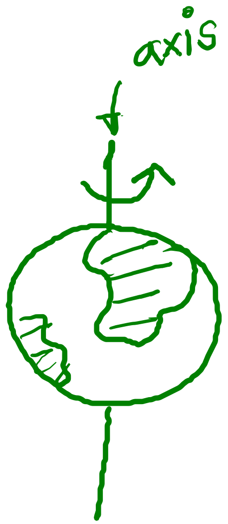

+ve div
-ve div

কার্ল (Curl)

$\vec{\nabla} \times \vec{F}$ = vector field

- মান → rotation
- দিক → rotation axis





$\vec{\nabla} \times \vec{F} = 0$ →
- irrotational (অঘূর্ণনশীল)
- conservative (সংরক্ষণশীল)

Poll Question - 02

কোনটির ডাইভারজেন্স নির্ণয় সম্ভব নয় ?

$\vec{\nabla} \cdot \vec{E}$

(a) তাড়িত-চুম্বকীয় বল → vector

(b) বাতাসের বেগ → vector

(c) তাপমাত্রা → scalar

Practice Problem

$\vec{F} = i + 2j$ এর ডাইভারজেন্স ও কার্ল নির্ণয় করো।

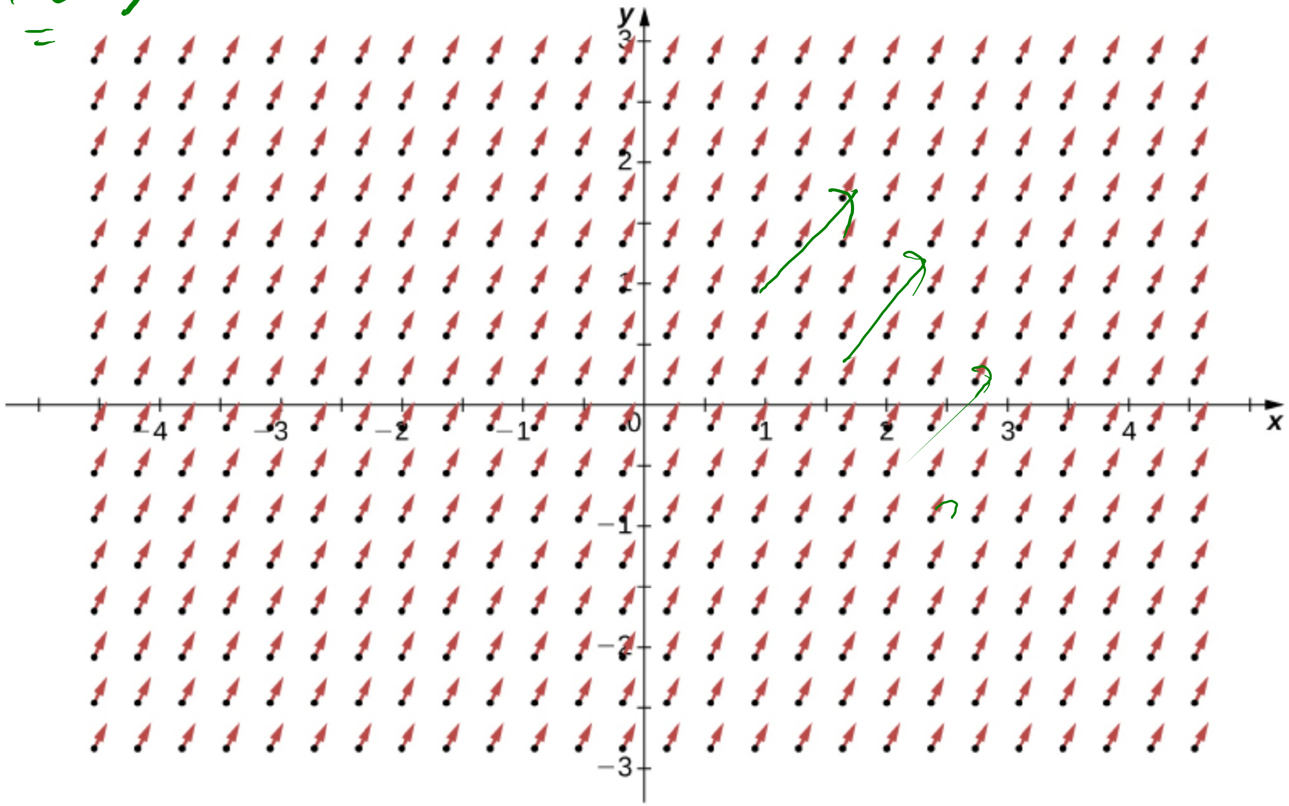
(i) $\vec{\nabla}\cdot\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(\hat{i} + 2\hat{j} + 0\hat{k}\right)$

$= \cancel{\frac{\partial}{\partial x}(1)}^{0} + \frac{\partial}{\partial x}(2) + \frac{\partial}{\partial z}(0)$

$= 0$

(ii) $\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & 2 & 0 \end{vmatrix}$

$= 0$ (গুণ করে দেখব)



Practice Problem

$\boldsymbol{F} = -x\boldsymbol{i} - y\boldsymbol{j}$ এর ডাইভারজেন্স ও কার্ল নির্ণয় করো।

+ve

-ve

$$\vec{\nabla}\cdot\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(-x\hat{i} - y\hat{j} + 0\hat{k}\right)$$

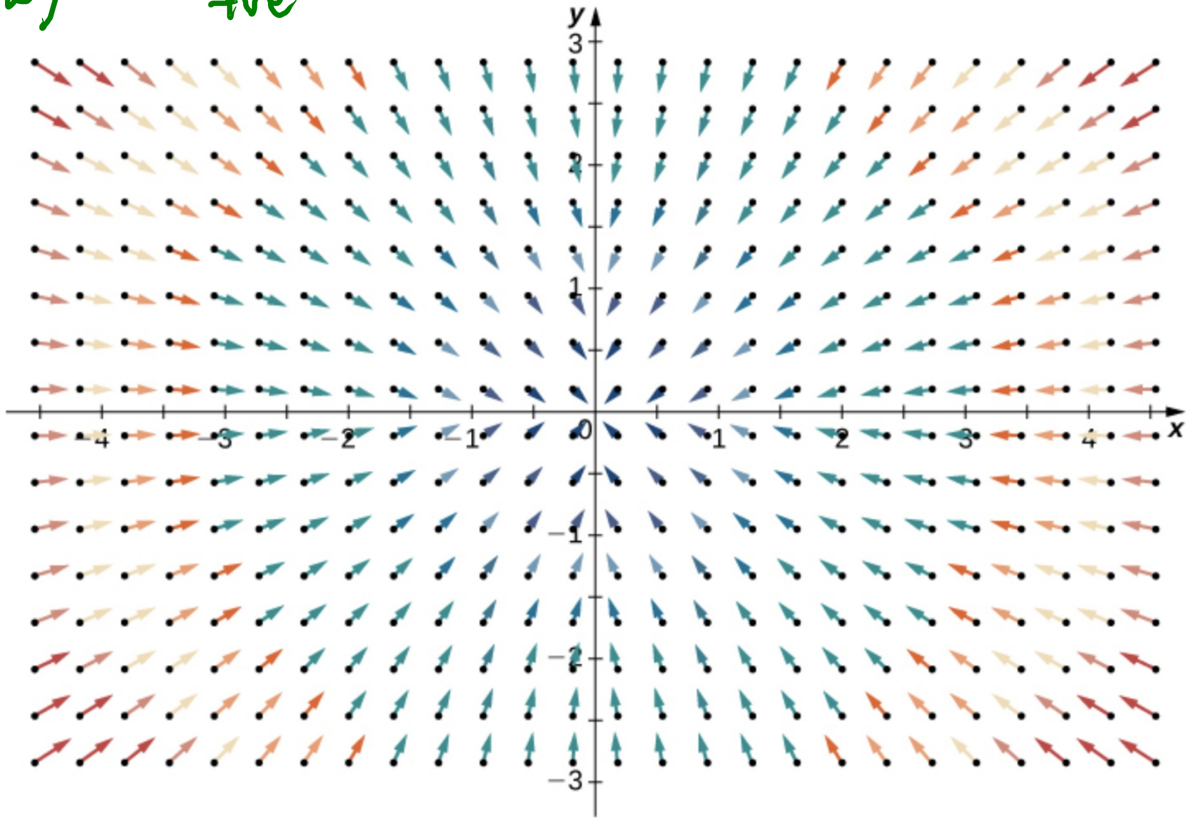
$$= \frac{\partial}{\partial x}(-x) + \frac{\partial}{\partial y}(-y) + \frac{\partial}{\partial z}(0)$$

$$= -1 - 1$$

$$= -2$$

-ve

sink



$$\vec{\nabla}\times\vec{F} =$$

$\vdots$

$= 0$

$\boxed{\text{HoW}}$

Practice Problem

Guess
Div $= 0$
Curl $= \hat{k}$

$\vec{F} = -y\boldsymbol{i} + x\boldsymbol{j}$ এর ডাইভারজেন্স ও কার্ল নির্ণয় করো।

$\vec{\nabla}\cdot\vec{F} = \ldots$

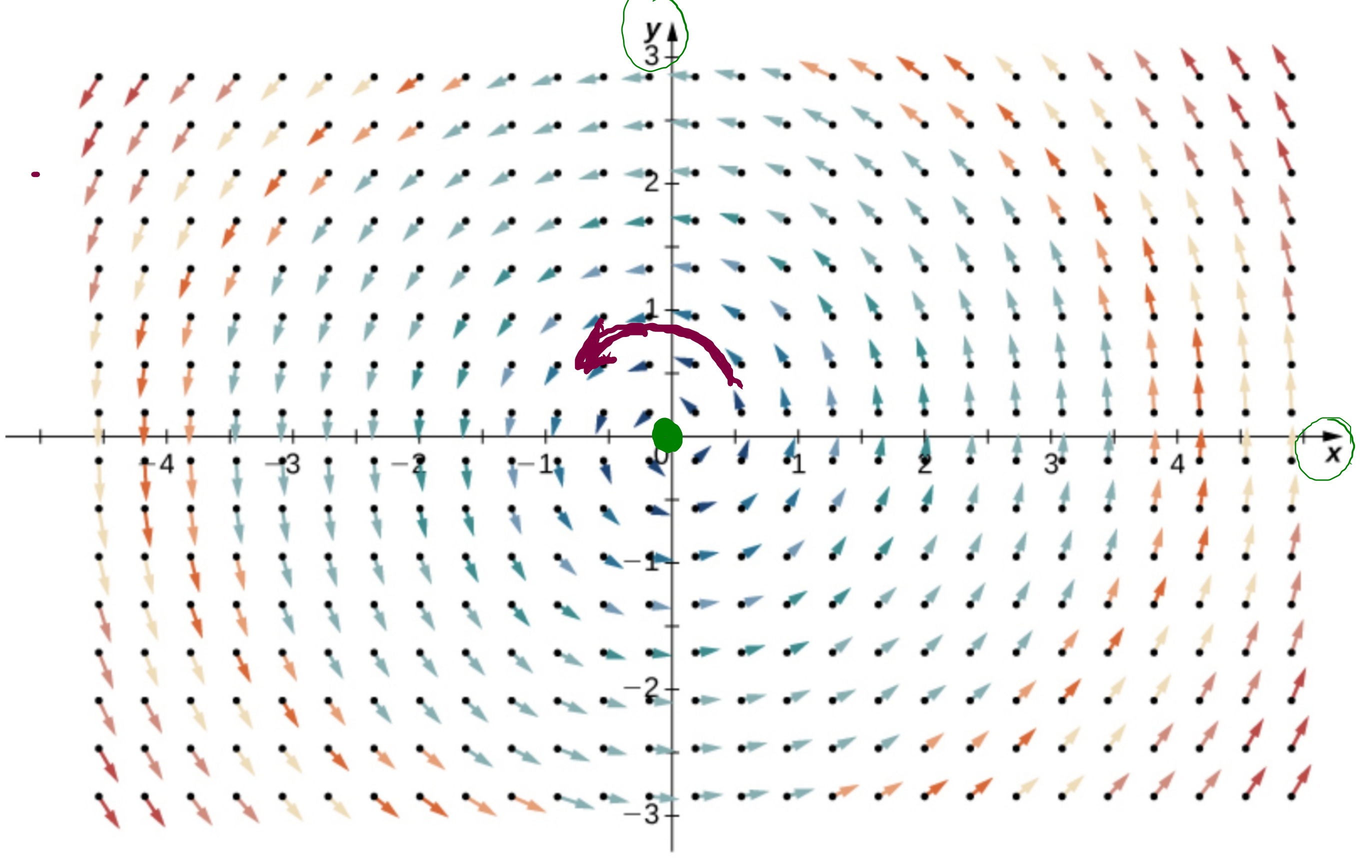
$\ldots = 0$ (H.W)

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & +x & 0 \end{vmatrix}$$

$$= \hat{i}\left(\cancel{\frac{\partial}{\partial y}(0)}^{\,0} - \cancel{\frac{\partial}{\partial z}(x)}^{\,0}\right) - \hat{j}\left(\cancel{\frac{\partial}{\partial x}(0)}^{\,0} - \cancel{\frac{\partial}{\partial z}(-y)}^{\,0}\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x}(x) - \frac{\partial}{\partial y}(-y)\right)$$

$$= +2\hat{k}$$



Resources

Derivative: https://www.geogebra.org/m/WbcrsCm7

https://www.geogebra.org/m/DSEBMEyM

Partial Derivative: https://www.geogebra.org/m/EWMQ8qnr

Gradient: https://www.geogebra.org/m/QhfcuhqA

Divergence & Curl: https://www.geogebra.org/m/GmJqrGsC#material/xacMPzSj

https://openstax.org/books/calculus-volume-3/pages/6-5-divergence-and-curl