CLASS 12 ACADEMIC PROGRAM-2020

Higher math 2nd paper

Lecture : HM-08

Chapter 5 : Binomial Theorem

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Poll Question 01

Find the sum of all the coefficients in expansion of $(x + 4y^3)^{100}$

$x \to 1$, $y \to 1$

(i) $^{100}C_2 * 4^{100}$ (ii) 4^{100} (iii) 5^{100} (iv) $^{100}C_2 * 5^{100}$

$(a+b)^2 = a^2 + 2ab + b^2$

$a = 1$

$b = 1$

$(1+1)^2 = 1 + 2 + 1 = 4$

$= 2^2 = 4$

$x = 1$

$y = 1$

$(x+4y^3)^{100} = (1+4\cdot 1^3)^{100}$

$= 5^{100}$

NEW $W = \{0, 1, 2, 3, 4, \dots\}$

Infinite series of binomial expansion:

$$W \quad (a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n a^{n-n} b^n \qquad n \in W$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots\dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots\dots$$

$0!$ $1!$ 4th

$n \in$ -ve, fraction

$${}^nC_r = \frac{n!}{r!(n-r)!} \qquad (r+1)^{th} \text{ term}$$

This series is an infinite binomial series where,

(i) n is a negative integer or a fraction

(ii) $|x| < 1$ or, $-1 < x < 1$

$$(a+b)^n = a^n + n a^{n-1} b^1 + \frac{n(n-1)}{2!} a^{n-2} b^2 + \frac{n(n-1)(n-2)}{3!} a^{n-3} b^3 + \dots$$

$$(a+b)^{1/2} = a^{1/2} + \frac{1}{2} a^{\frac{1}{2}-1} b^1 + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!} a^{\frac{1}{2}-2} b^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} a^{\frac{1}{2}-3} b^3 + \dots$$

Expansion of $(a+x)^n$ where $n \in \boldsymbol{Q}$ but is n not a natural number:

$\frac{p}{q}$

$4 = \frac{8}{2}$

$\frac{1}{2} > \frac{1}{8}$

(i) If $|a| > |x|$ then, $\left|\frac{x}{a}\right| < 1$:

$|a| > |x|$

$\Rightarrow 1 > \left|\frac{x}{a}\right|$

$\Rightarrow \left|\frac{x}{a}\right| < 1$

$$(a+x)^n = a^n\left(1+\frac{x}{a}\right)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \dots\dots$$

$\left|\frac{x}{a}\right| < 1$

$1+2+4+8+\dots\ r>1$

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots\ r<1$

$1+x+x^2+x^3+x^4+\dots\infty = \frac{1}{1-x} = (1-x)^{-1}$; $|x|<1$

G.R

$\frac{1}{2} = 0.5$

$S_\infty = \frac{a}{1-r}$

G. Ratio $= \frac{x}{1} = \frac{x^2}{x} = x$

$2^3 = 8$

$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$S_n = ar^0 + ar^1 + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$

$n = \infty$

$= \frac{a(r^n-1)}{r-1}$; $|r|>1$ S_∞

$r-1 \neq 0$

$= \frac{a(1-r^n)}{1-r}$; $|r|<1$ S_∞

Expansion of $(a+x)^n$ where $n \in \boldsymbol{Q}$ but is n not a natural number:

(i) If $|a| < |x|$ then , $\left|\frac{a}{x}\right| < 1$:

$$(a+x)^n = x^n\left(1+\frac{a}{x}\right)^n = \overset{x^n}{xn} + n\,a\,xn^{-1} + \frac{n(n-1)}{2!}a^2 x^{n-2} + \ldots\ldots$$

Convergence of infinite binomial expansion:

$$\frac{n(n-1)(n-2)}{3!}x^3$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots\dots + \frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r + \dots\dots \to ??$$

$|\pm x| = |x| < 1$

$(r+1)^{th}$ term

0
1
2
3
4

This series is finite if n is positive integer. But if n is a negative integer or a fraction, then $\frac{n(n-1)(n-2)\dots(n-r+1)}{r!}x^r$ can't be 0. In that case, this series infinite.

Condition for convergence of this series: $|x| < 1$ or, $-1 < x < 1$

$$(1+x)^n \longrightarrow (r+1)^{th} \text{ term} = \frac{n(n-1)(n-2)\dots\underbrace{(n-(r-1))}_{(n-r+1)}}{r!} x^r$$

0, 1, 2, ..., $(r-1)$

Co-eff.

Poll Question 02

Which one is the necessary condition for $(1 - \frac{x}{8})^{\frac{1}{2}}$ being convergent?

(a) $|x| > 8$

(b) $|x| < 8$

(c) None

$\left|\frac{x}{8}\right| < 1$

$\Rightarrow |x| < |8|$

$\Rightarrow |x| < 8$

Type-1: 1 , 2 , 3(a) , 10-15

1. For what value of x, expansion of $\frac{1}{(8-3x)^{1/2}}$ is valid.

Ex: $= \frac{1}{(8-3x)^{1/2}}$

$= (8-3x)^{-1/2}$

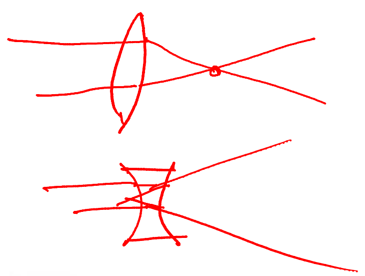
$= 8^{-1/2}\left(1-\frac{3x}{8}\right)^{-1/2}$

$\left|\frac{3x}{8}\right| < 1$

$\Rightarrow -1 < \frac{3x}{8} < 1$

$\Rightarrow -8 < 3x < 8$

$\Rightarrow \boxed{-\frac{8}{3} < x < \frac{8}{3}}$ Ans.



$1+2+4+8+16+\cdots$ G.R $=2$

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots \rightarrow 0$

2. Expand up to 4th term: $\frac{x}{\sqrt{a^2-x^2}}$

$$\frac{x}{\sqrt{a^2-x^2}} = x\left(a^2-x^2\right)^{-1/2}$$

$$= x\,(a^2)^{-1/2}\left(1-\frac{x^2}{a^2}\right)^{-1/2}$$

$$= \frac{x}{a}\left[1-\frac{x^2}{a^2}\right]^{-1/2}$$

$$= \frac{x}{a}\left[1+\left(-\frac{1}{2}\right)\left(-\frac{x^2}{a^2}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(-\frac{x^2}{a^2}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(-\frac{x^2}{a^2}\right)^3+\cdots\right]$$

= H.W.

3. Show that, in expansion of $(1-2x)^{-\frac{1}{2}}$ coefficient of $(r+1)$-th term is $\frac{(2r)!}{(r!)^2 2^r}$

$$(r+1)^{th} \text{ term} = \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\cdots\left(-\frac{1}{2}-r+1\right)}{r!} (-2x)^r$$

$$\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}$$

$$= \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\cdots\left(-\frac{(2r-1)}{2}\right)}{r!} (-2)^r x^r$$

$$a.a.a = a^3$$

$$= \frac{\left(-\frac{1}{2}\right)^r 1.3.5.\cdots(2r-1)}{r!} (-2)^r x^r$$

$$\left(-\frac{1}{2}\right)^r (-2)^r = (-2)^{-r}(-2)^r = (-2)^{-r+r} = (-2)^0 = 1$$

$$= \frac{1.2.3.4.5.6.\cdots(2r-1).2r}{r!\,2.4.6.\cdots 2r} x^r = \frac{(2r)!}{r!\,2^r(1.2.3\cdots r)} x^r = \frac{(2r)!}{r!\,2^r\,r!} x^r = \boxed{\frac{(2r)!}{(r!)^2 2^r} x^r}$$

showed

Some series:

(i) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots\dots + x^r + \dots\dots$

$(1-ax)^{-1} = 1 + ax + a^2x^2 + a^3x^3 + a^4x^4 + \dots\dots + a^r x^r + \dots\dots$

$(1-(-x))^{-1}$

Some series:

$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots + x^r + \cdots \infty$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

1 3 3 1

(ii) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots + (-1)^r x^r + \ldots\ldots$

$+(-1)^1 x^1 + (-1)^2 (x)^2$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \cdots + x^r + x^{r+1} + \cdots \infty$

$(-1)(1-x)^{-2}(-1) = \cancel{0} + 1 + 2x + 3x^2 + 4x^3 + \cdots + r \cdot x^{r-1} + (r+1)x^r + \cdots \infty$

(iii) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots + (r+1)\, x^r + \ldots\ldots$

$(1-x)^{-3} = ?$

$(1-x)^{-1} = 1 + 1\cdot x + 1\cdot x^2 + 1\cdot x^3 + 1\cdot x^4 + \cdots$

$(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \cdots x^4 + \cdots$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

Some series:

(iv) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots + (r+1)(-1)^r x^r + \ldots\ldots$

$(1-x)^{-2} =$ - - - - . differentiate করো

(iii) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \ldots\ldots + \frac{1}{2}(r+1)(r+2)\, x^r + \ldots\ldots$

H.W.

Type-2: 6

4. Show that, $(1+x+x^2+x^3+\cdots)(1+2x+3x^2+\cdots)=\frac{1}{2}(1.2+2.3x+3.4x^2+4.5x^3+\cdots)$

$$\text{L.S.} = (1-x)^{-1}(1-x)^{-2}$$

$$= (1-x)^{-1-2}$$

$$= (1-x)^{-3}$$

$$= 1+3x+6x^2+10x^3+\cdots$$

$$\cancel{= \frac{1}{2}(1.2+\cdots}$$

$$= \frac{1}{2}(2+6x+12x^2+20x^3+\cdots)$$

$$= \frac{1}{2}(1.2+2.3x+3.4x^2+4.5x^3+\cdots)$$

$$= \text{R.S.}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

5. If $y = x + x^2 + x^3 + \cdots$ then show that, $x = y - y^2 + y^3 - y^4 + \cdots$

Sol^n: Given,

$$y = x + x^2 + x^3 + \cdots \infty$$

$$\Rightarrow 1 + y = 1 + x + x^2 + x^3 + \cdots \infty$$

$$\Rightarrow 1 + y = (1-x)^{-1}$$

$$\Rightarrow (1+y)^{-1} = 1 - x$$

$$\Rightarrow 1 - x = (1+y)^{-1}$$

$$\Rightarrow \cancel{1} - x = \cancel{1} - y + y^2 - y^3 + y^4 - \cdots \infty$$

$$\Rightarrow x = y - y^2 + y^3 - y^4 + \cdots \infty$$

Showed

6. If $y = 2x + 3x^2 + 4x^3 + \cdots$ then show that, $x = \frac{1}{2}y - \frac{3}{8}y^2 + \frac{5}{16}y^3 - \cdots$

$\Rightarrow 1+y = 1+2x+3x^2+4x^3+\cdots$

$\Rightarrow 1+y = (1-x)^{-2}$

$\Rightarrow (1+y)^{-1/2} = 1-x$

$\Rightarrow 1-x = (1+y)^{-1/2}$

$=$

$=$ H.W.

7. If $y = 3x + 6x^2 + 10x^3 + \cdots$ then show that, $x = \frac{1}{3}y - \frac{1.4}{3^2.2!}y^2 + \frac{1.4.7}{3^3.3!}y^3 - \cdots$

H.W.

$1+y = (1-x)^{-3}$

$\Rightarrow 1-x = (1+y)^{-1/3}$

Type-3: 3(b,c) , 4 ,5 ,7,17-22,26,27

8. In the expansion of $\frac{2x+1}{1+x^2}$, find the coefficient of x^r

$$+(-1)^r x^r$$

$$(1+x)^{-1} = 1-x+x^2-x^3+x^4-\cdots$$

$$(1+x^2)^{-1} = 1-x^2+x^4-x^6+x^8-\cdots$$

$$(-1)^{r/2} x^r$$

$$\frac{(2x+1)}{1+x^2} = (2x+1)\,\underbrace{(1+x^2)^{-1}}$$

$$=(2x+1)\left(1-x^2+x^4-x^6+x^8-\cdots+x^{r-2}+(-1)^{r/2}x^r+\cdots\right)$$

$$\boxed{(-1)^{r/2}}$$

9. Show that, in expansion of $\frac{(1+x)^n}{1-x}$ coefficient of x^n is 2^n ; $n \in \mathbb{N}$

$$\frac{(1+x)^n}{1-x} = (1+x)^n (1-x)^{-1}$$

$$= \left({}^nC_0 \cancel{1^n x^0} + {}^nC_1 1^{n-1} x^1 + {}^nC_2 1^{n-2} x^2 + \cdots + {}^nC_n 1^{n-n} x^n \right)\left(1 + x + x^2 + x^3 + x^4 + \cdots + x^{n-2} + x^{n-1} + x^n + x^{n+1} \cdots \right)$$

$$\text{Co-eff.} = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$$

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 \cancel{a^{n-1} b^1} + {}^nC_2 a^{n-2} b^2 + \cdots + {}^nC_n b^n$$

$$2^n = (1+1)^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n$$

Partial Fraction:

$$\frac{ax^2+bx+c}{(x-\alpha)(x-\beta)^2(px^2+\gamma)}=\frac{A}{x-\alpha}+\frac{B}{(x-\beta)^2}+\frac{C}{(x-\beta)}+\frac{Dx+E}{\beta x^2+\gamma}$$

$$\boxed{\frac{1}{(1-ax)(1-bx)}}=\frac{1}{(1-ax)\left(1-b\cdot\frac{1}{a}\right)}+\frac{1}{\left(1-a\cdot\frac{1}{b}\right)(1-bx)}$$

$1-\frac{b}{a}$

$1-ax=0$

$\Rightarrow x=\frac{1}{a}$

$$=\frac{a}{(1-ax)(a-b)}+\frac{1}{\left(\frac{b-a}{b}\right)(1-bx)}$$

$$\boxed{=\frac{a}{(1-ax)(a-b)}+\frac{b}{(b-a)(1-bx)}}$$

Poll Question 03

$$\frac{1}{(1-x)(1+x)} = ?$$

(a) $\frac{1}{1-x} + \frac{1}{1+x}$

(b) $\frac{1}{1-x} - \frac{1}{1+x}$

(c) $\frac{1}{2}\left(\frac{1}{1-x} + \frac{1}{1+x}\right)$

(d) $\frac{1}{2}\left(\frac{1}{1-x} - \frac{1}{1+x}\right)$

$$\frac{1}{(1-x)(1+1)} + \frac{1}{(1-(-1))(1+x)}$$

$$= \frac{1}{(1-x)2} + \frac{1}{2(1+x)}$$

$$= \frac{1}{2}\left[\frac{1}{1-x} + \frac{1}{1+x}\right]$$

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10. Show that, in expansion of $(1 - 5x + 6x^2)^{-1}$ coefficient of x^r is $3^{r+1} - 2^{r+1}$

$$\frac{1}{(1-ax)(1-bx)}$$

$$= \frac{a}{(1-ax)(a-b)} + \frac{b}{(b-a)(1-bx)}$$

$$= \frac{1}{a-b}\left[\frac{a}{1-ax} - \frac{b}{1-bx}\right]$$

$$= \frac{1}{a-b}\left[a(1-ax)^{-1} - b(1-bx)^{-1}\right]$$

$a^r x^r$ $\quad b^r x^r$

$x^r \to$ coeff. $= \frac{1}{a-b}\left[a \cdot a^r - b \cdot b^r\right] = \frac{a^{r+1} - b^{r+1}}{a-b}$

$$(1 - 5x + 6x^2)^{-1}$$

$$= \frac{1}{1 - 5x + 6x^2}$$

$$= \frac{1}{6x^2 - 5x + 1} = \frac{1}{(1-3x)(1-2x)}$$

$a = 3$, $b = 2$

$$\frac{3^{r+1} - 2^{r+1}}{3-2}$$

$$= 3^{r+1} - 2^{r+1}$$

11. Find the coefficient of x^4 in expansion of $(1 - x + x^2 - x^3)^{-1}$

H.W.

Type-4:

12. Show that, in expansion of $\frac{1}{(1-x)(3-x)}$ coefficient of x^n is $\frac{1}{2}(1-\frac{1}{3^{n+1}})$

$$\frac{1}{(1-x)\,3(1-\frac{x}{3})}$$

$$=\frac{1}{3}\cdot\frac{1}{(1-x)(1-\frac{1}{3}\cdot x)}$$

$a=1$ $\quad b=1/3$

Do as before.

Type-5: 9

13. $1 + \frac{1}{3} + \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \cdots = ?$ $= (1+x)^n$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots$

$nx = \frac{1}{3}$ —— (i)

$\frac{n(n-1)}{2!}x^2 = \frac{1.\cancel{3}}{\cancel{3}.6}$

$\Rightarrow \frac{n^2 - n}{\cancel{2}} x^2 = \frac{1}{\cancel{6}\,3}$

$\Rightarrow \underbrace{n^2x^2}_{(nx)^2} - \underbrace{nx^2}_{nx.x,\ 1/3} = \frac{1}{3}$

$\Rightarrow \frac{1}{9} - \frac{1}{3}x = \frac{1}{3}$

$\Rightarrow x = \boxed{\ }$

$x \rightarrow (i)$

$n = 3$

H.W.

14. $1 + 2.\frac{1}{3^2} + \frac{2.5.1}{1.2.34} + \frac{2.5.8.1}{1.2.3.36} + \cdots = ?$

H.W.

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