



CLASS XII ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : M-09

Chapter 6 : Conics



$$x = \sqrt{\frac{c^2}{c} + c} - \frac{b}{2}$$



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Conic

If the ratio of distance of some points from a fixed point a fixed straight line is constant, then the locus created by those points are called a conic.

This fixed point is **Focus**, the fixed line is **Directrix** and the ratio is called **Eccentricity(e).**

Poll Question-01

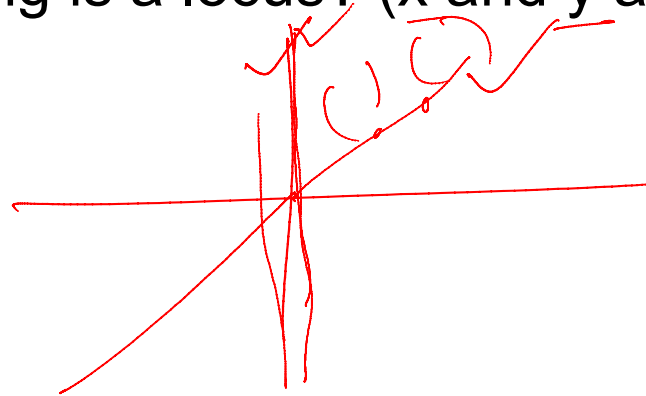
Which one of the following is a locus? (x and y are variable)

(a) $x = y$

(b) $ax = 0$

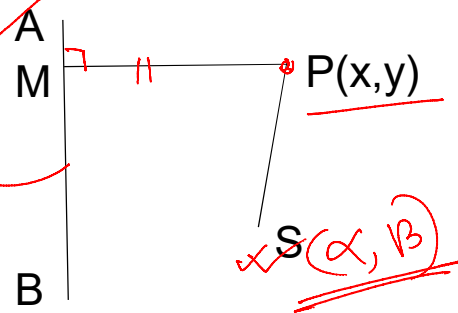
(c) $x^2 + a^2 = 0$

(d) All



General equation of conic

Let, the focus point is $S(\alpha, \beta)$ and the directrix is AB: $lx + my + n = 0$. If $P(x, y)$ is a point on the conic and PM is perpendicular to AB, then from definition of conic we know, $\frac{SP}{PM} = e$.



$$\frac{SP}{PM} = e$$

$$\Rightarrow SP = e \cdot PM$$

$$\Rightarrow \sqrt{(x-\alpha)^2 + (y-\beta)^2} = e \frac{|lx+my+n|}{\sqrt{l^2+m^2}}$$

$$\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = e^2 \frac{(lx+my+n)^2}{l^2+m^2}$$

or, $SP = e \cdot PM$

or, $\sqrt{(x-\alpha)^2 + (y-\beta)^2} = e \frac{lx+my+n}{\sqrt{l^2+m^2}}$

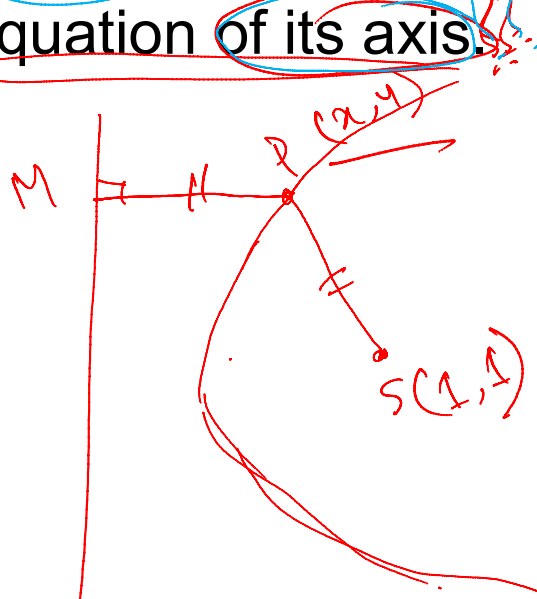
or, $(x-\alpha)^2 + (y-\beta)^2 = e^2 \frac{(lx+my+n)^2}{l^2+m^2}$

$e > 1$ → Hyper.

by the defⁿ

Mathematical Problem

Find the parabola with focus at $(1,1)$ and the directrix $3x + 4y = 1$. Also find the equation of its axis.



$$SP = e \cdot PM$$

$$\Rightarrow 3x + 4y - 1 = 0$$

$$e = 1$$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = \frac{|3x+4y-1|}{\sqrt{3^2+4^2}} \quad \begin{matrix} SP \\ PM \end{matrix} = 1 \Rightarrow \boxed{SP = PM}$$

$$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = \frac{9x^2 + 16y^2 + 1 + 24xy - 3x - 4y}{25}$$

$$\Rightarrow \underline{\underline{H.W.}}$$



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Higher Math 1st Paper
Chapter 6 : Conics

General equation of conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

General equation of a conic can be written as $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

(i) If $a = b$ and $h = 0$ then the conic is a circle.

(ii) If $ab = h^2$ then the conic is a parabola.

(ii) If $ab - h^2 > 0$ then the conic is an ellipse.

(iv) If $ab - h^2 < 0$ then the conic is a hyperbola.

$$\Rightarrow ab - h^2 = 0$$

for which value of m it will be a circle?

$$5x^2 + 5y^2 + 10mxy + 25x + 25y - 25 = 0$$

$2h = 10m$
 $h = 5m$
 $a = 5$
 $b = 5$
 $h = 5m \Rightarrow m = 0/5 = 0$

Poll Question-02

For which value of K, $2x^2 + 2y^2 + 2Kxy + 3x + 4y + 5 = 0$ is parabola?

(a) 2

(b) -2

(c) a&b

(d) None

$$a = 2$$

$$b = 2$$

$$2h = 2K$$

$$h = K$$

$$ab = h^2$$

$$\Rightarrow 2 \times 2 = K^2$$

$$K^2 = 4$$

$$\Rightarrow K = \pm 2$$



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Parabola

A locus created by some equidistant points from a fixed point and a fixed straightline is a parabola.

$$e = 1$$

$$SP = PM$$

$$\Rightarrow \sqrt{(x-a)^2 + (y-0)^2} = x+a$$

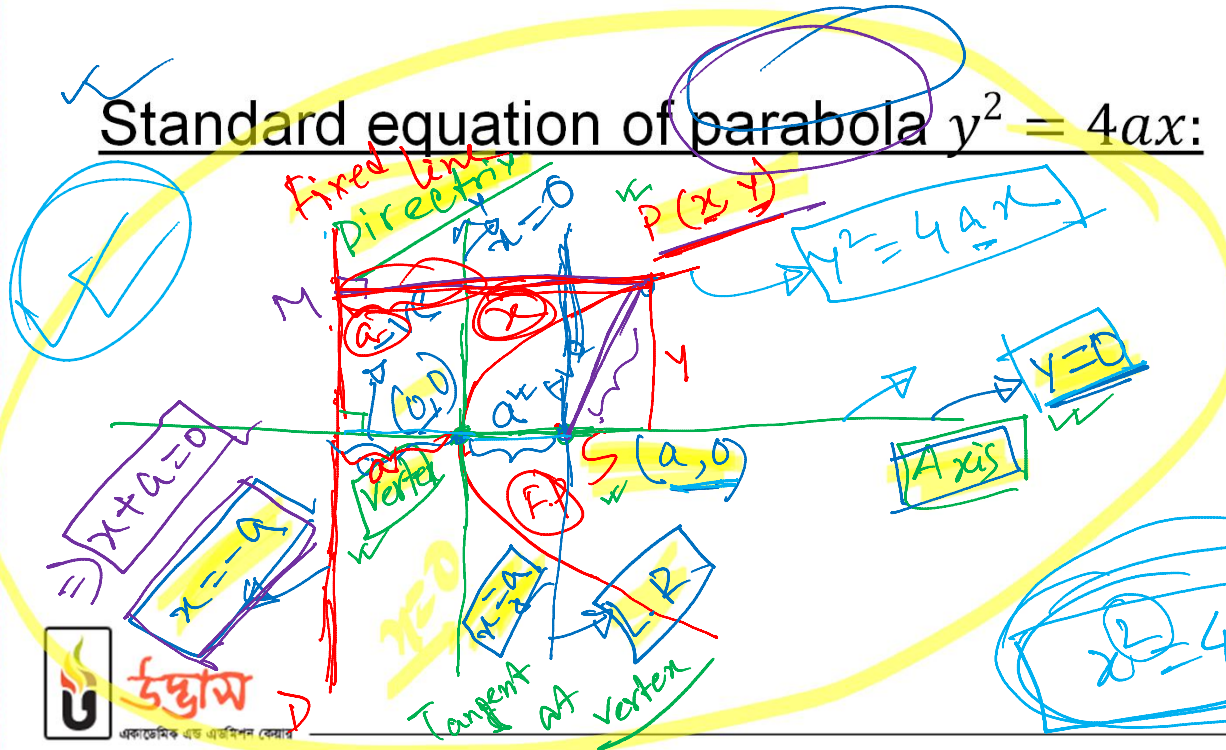
$$\Rightarrow x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$

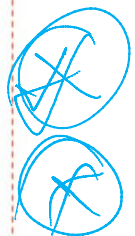
$$\Rightarrow y^2 = 4ax$$

St. eqⁿ of a parabola

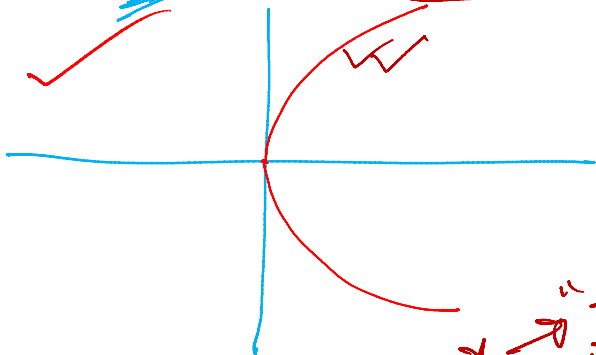
Axis \rightarrow

$$y^2 = 4ax$$



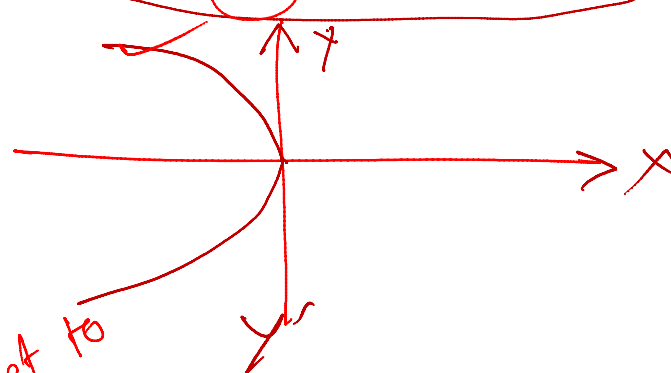


$$y^2 = 4ax ; \boxed{a > 0}$$

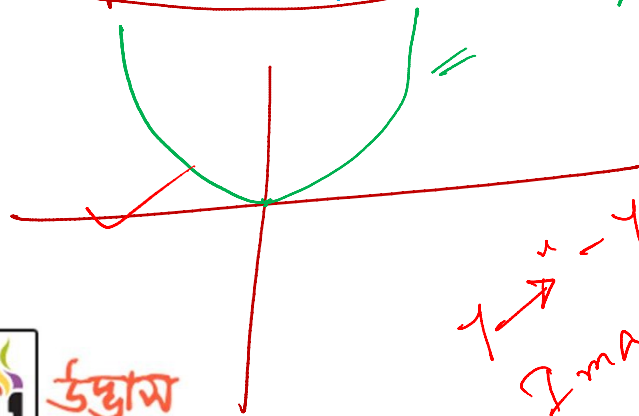


$x \rightarrow "x"$ with respect to
Image y -axis

$$y^2 = -4ax = 4a(-x)$$

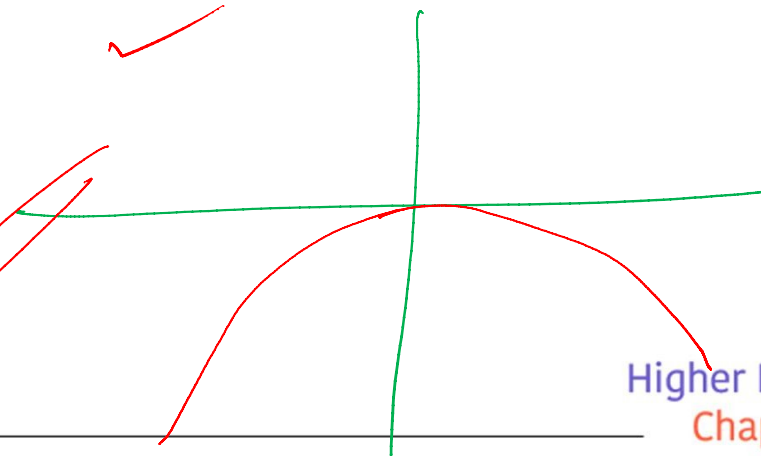


$$\boxed{x^2 = 4ay} \Rightarrow y = \frac{x^2}{4a}$$



$y \rightarrow "y"$ with respect to
Image x -axis

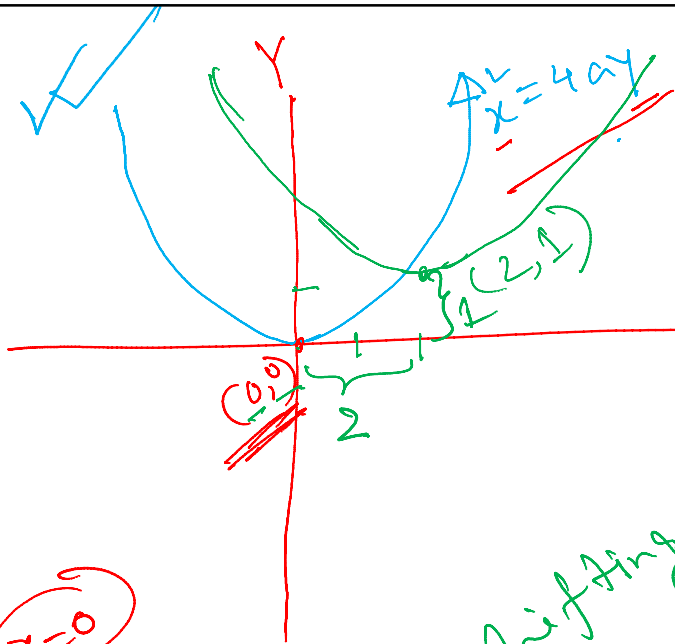
$$x^2 = -4ay = 4a(-y)$$



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Higher Math 1st Paper
Chapter 6 : Conics



$$x^2 = 4ay$$

$$(x-2)^2 = 4a(y-1)$$

$$y-1=0$$

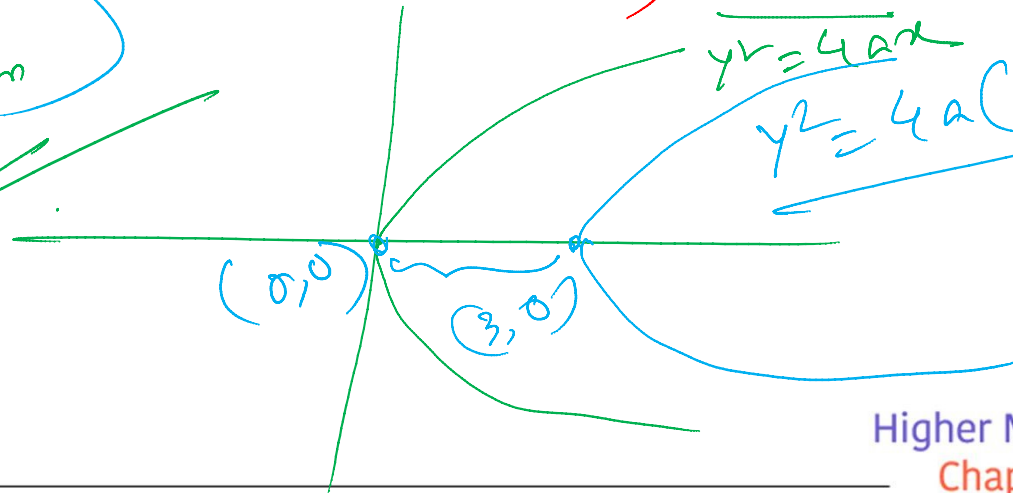
$$y=1$$

$$x-2=0$$

$$\Rightarrow x=2$$

$$y^2 = 4ax$$

$$y^2 = 4a(x-3)$$



$$x=0$$

$$y=0$$

Right shifting

Left

upward

downward

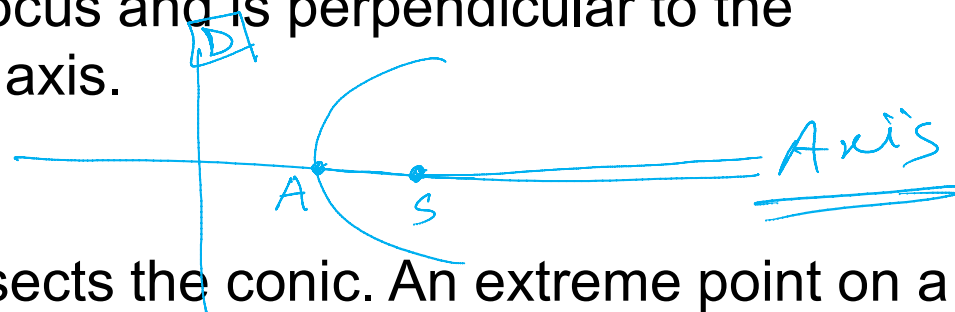


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Some definitions related to conic

Axis: If a straight line goes through focus and is perpendicular to the directrix, when the line is called axis.

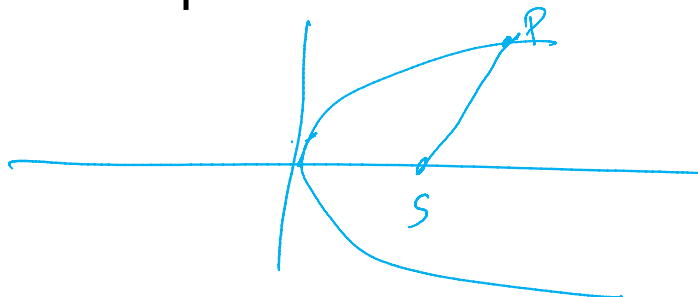


Vertex: In which points the axis intersects the conic. An extreme point on a conic section.

Foot of directrix: Intersection points of conics and directrix.

Some definitions related to conic

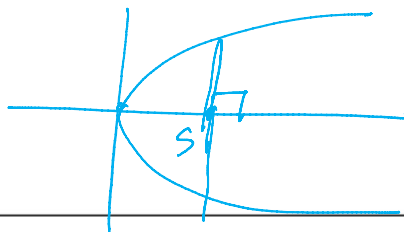
Focal distance: Distance between and point on conic and the focus is called focal distance.



Focal chord: If a chord of a conic passes through its focus point, then the chord is called a focal chord.



Latus rectum: If a focal chord is perpendicular to the axis, then it is called latus rectum.



Poll Question-03

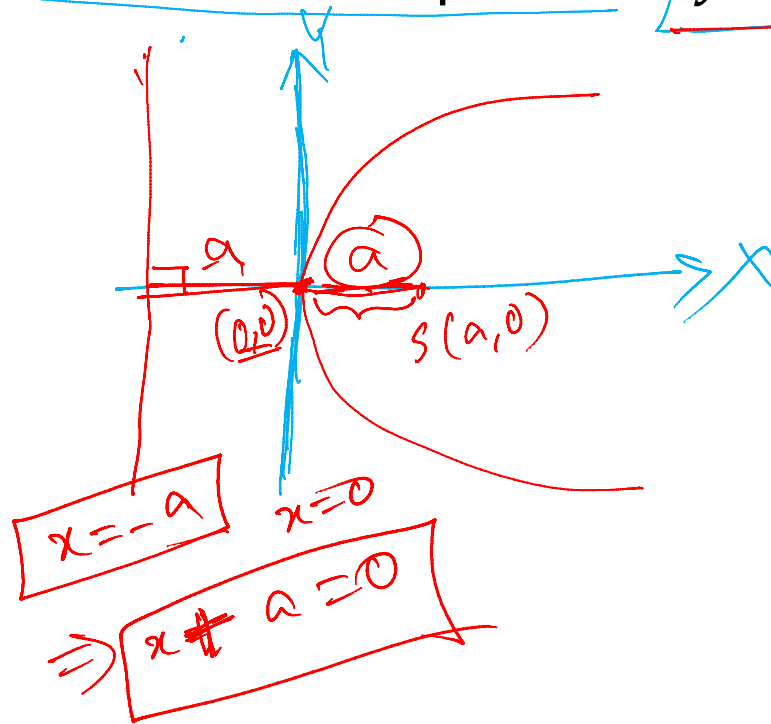
Which one is the equation of directrix of the parabola $y^2 = 4ax$? ($a > 0$)

(a) $x = 0$

(b) $ax = 0$

☒ (c) $x + a = 0$

(d) $x - a = 0$



Other shapes of parabola

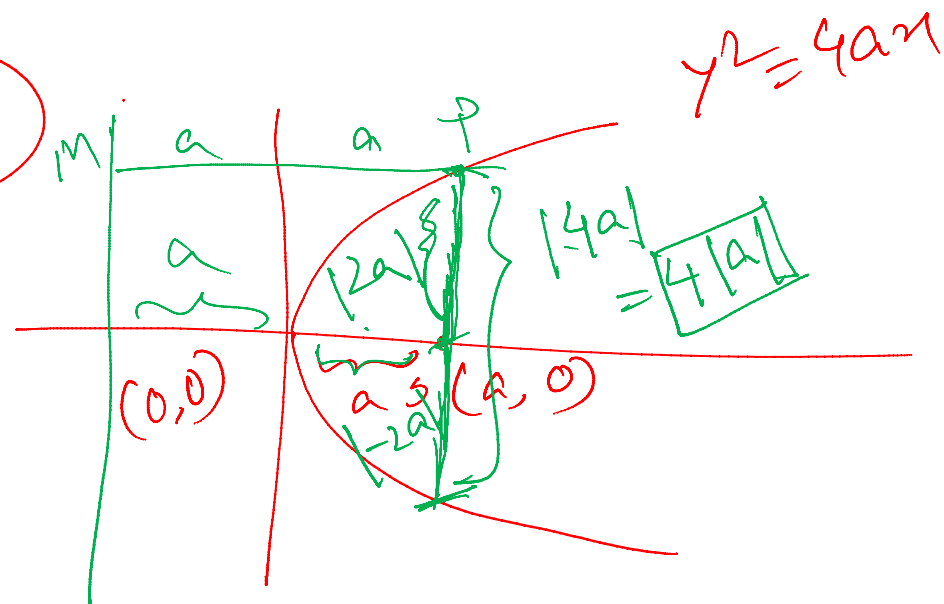
If $a > 0$, $y^2 = -4ax$ lies on left side of y axis and symmetrical to x axis.

If $a > 0$, $x^2 = 4ay$ lies on upper side of x axis and symmetrical to y axis.

If $a > 0$, $x^2 = -4ay$ lies on lower side of x axis and symmetrical to y axis.

Length of lastus rectum of $y^2 = 4ax$:

Length of latus rectum, $LL' = 4|a|$ units



$$PM = 2a$$
$$SP = 2a$$

Poll Question-04

Find the vertex of parabola $(y - 3)^2 = 4a(x - 6)$?

(a) (0,0)

(b) (3,6)

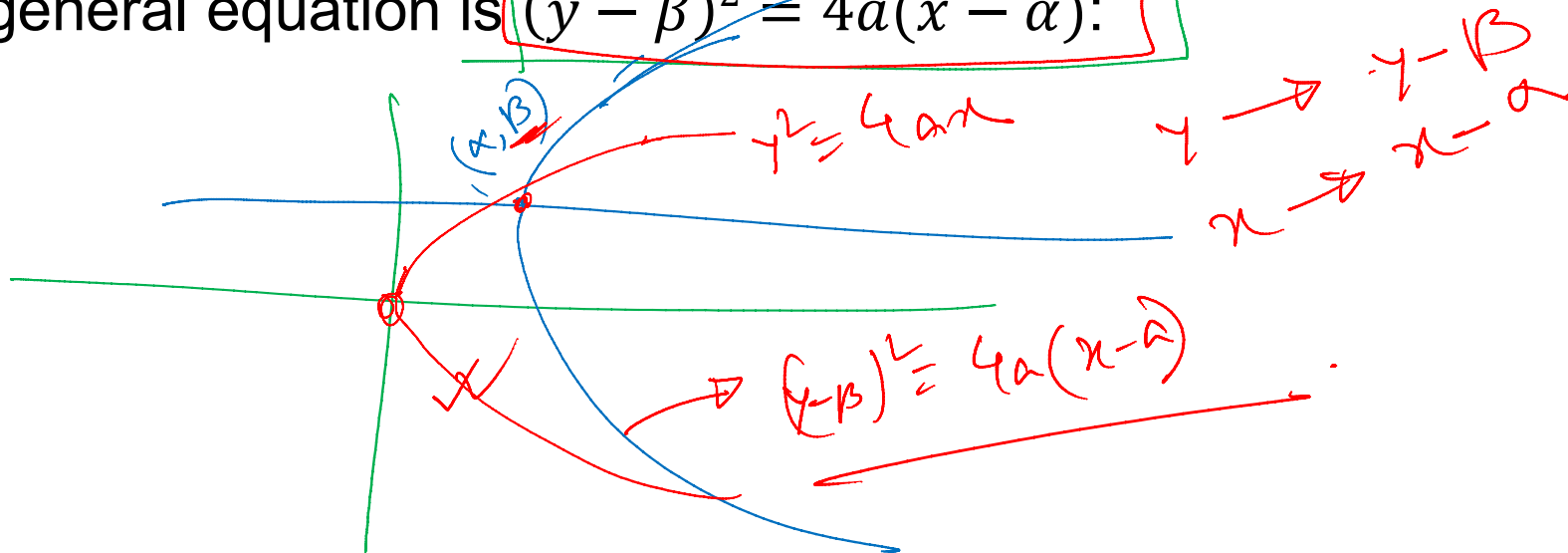
(c) (-3,-6)

(d) (3,0)

$$x=6, \quad y=3$$

(6,3)

If vertex of a parabola is (α, β) and axis is parallel to x axis, then its general equation is $(y - \beta)^2 = 4a(x - \alpha)$:



	$y^2 = 4ax$	$y^2 = -4ax$		
✓ Vertex				
✓ Focus				
✓ Foot of the directrix				
✓ Axes				
✓ Directrix				
✓ Latus rectum				
✓ Length of latus rectum				
✓ Co-ordinate of the end point of latus rectum				
✓ Tangent at the vertex				

Mathematical Problem

* For the parabola $5x^2 + 30x + 2y + 59 = 0$, find vertex, focus, length of latus rectum and equations of axis and directrix.

$$5x^2 + 30x + 2y + 59 = 0$$

$$\Rightarrow x^2 + 6x + \frac{2}{5}y + \frac{59}{5} = 0$$

$$\Rightarrow x^2 + 2 \cdot x \cdot 3 + 9 + \frac{2}{5}y + \frac{59}{5} - 9 = 0$$

$$\Rightarrow (x+3)^2 = -\frac{2}{5}y - \frac{59}{5} + 9$$

$$\Rightarrow (x+3)^2 = 4\left(-\frac{1}{10}\right)y - \frac{59-45}{5}$$

$$(x+3)^2 = 4\left(-\frac{1}{10}\right)y - \frac{14}{5}$$

Mathematical Problem

For the parabola $y^2 = 4(x - 2)$, find vertex, focus, length of latus rectum and equations of axis and directrix.

$$y^2 = 4(x - 2) \quad \boxed{a=1}$$

$$y^2 = 4ax$$

Vertex

$$y=0 \quad \& \quad x=0$$

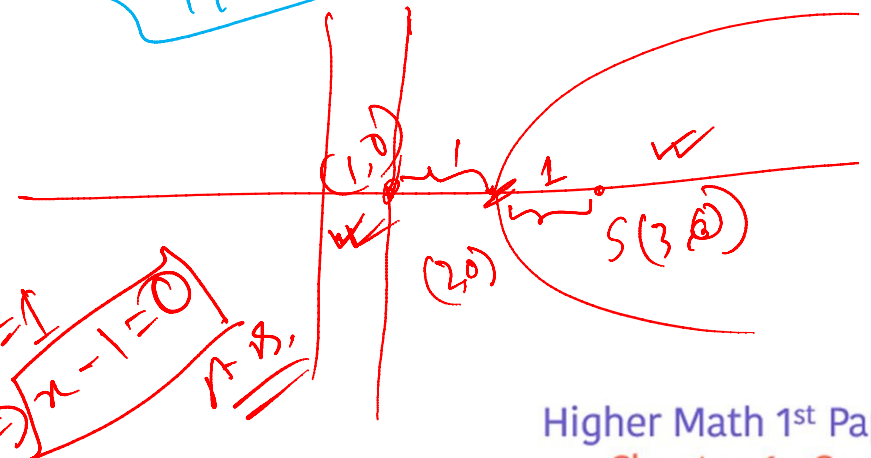
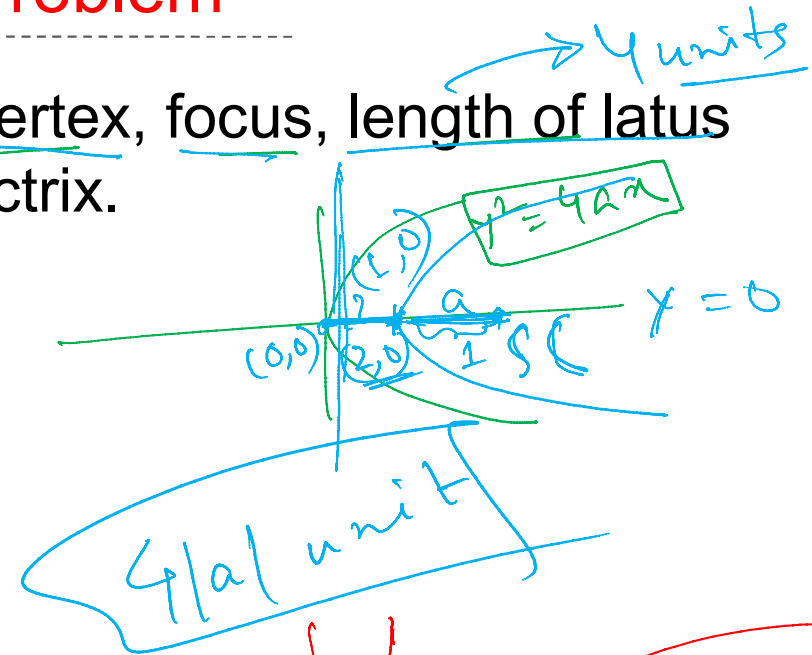
$$\Rightarrow y=0 \quad x-2=0 \Rightarrow x=2$$

Focus

$$S(a+x, 0)$$

$$S(1+2, 0) = (3, 0)$$

Dir.



Mathematical Problem

* Vertex of the parabola $y = ax^2 + bx + c$ lies at $(-2, 3)$ and this parabola goes through the point $(0, 5)$. Find the values of a, b, c .

Imp.

$$y = ax^2 + bx + c$$

⇒ $(-2, 3)$
⇒ $(0, 5)$

$$(x+2)^2 = 4 \cdot \frac{1}{2} (y-3)$$

$$\Rightarrow x^2 + 4x + 4 = 2y - 6$$

$$\Rightarrow 2y = x^2 + 4x + 4 + 6$$

$$\Rightarrow 2y = x^2 + 4x + 10$$

$$\Rightarrow y = \frac{1}{2}x^2 + 2x + 5$$

$$(x+2)^2 = 4A(y-3)$$

$$(0+2)^2 = 4 \cdot A(5-3)$$

$$\Rightarrow 4 = 4A \cdot 2$$

$$\Rightarrow A = \frac{1}{2}$$

$$a = \frac{1}{2} \quad b = 2 \quad c = 5$$



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Higher Math 1st Paper
Chapter 6 : Conics

If $y = mx + c$ is a tangent to parabola $y^2 = 4ax$

$$c = \frac{a}{m}$$

$$(mx + c)^2 = 4ax$$

$$\Rightarrow m^2x^2 + 2mcx + c^2 - 4ax = 0$$

$$\Rightarrow m^2x^2 + 2(mc - 2a)x + c^2 = 0$$

$$D = \{2(mc - 2a)\}^2 - 4 \cdot m^2c^2 = 0$$

$$\Rightarrow 4(mc - 2a)^2 = 4m^2c^2$$

$$\Rightarrow \cancel{m^2c^2} - 2 \cdot mc \cdot 2a + 4a^2 = \cancel{m^2c^2}$$

$$\Rightarrow -4mca + 4a^2 = 0$$

$$\Rightarrow 4a^2 = 4mca \Rightarrow a = mc \Rightarrow c = \frac{a}{m}$$

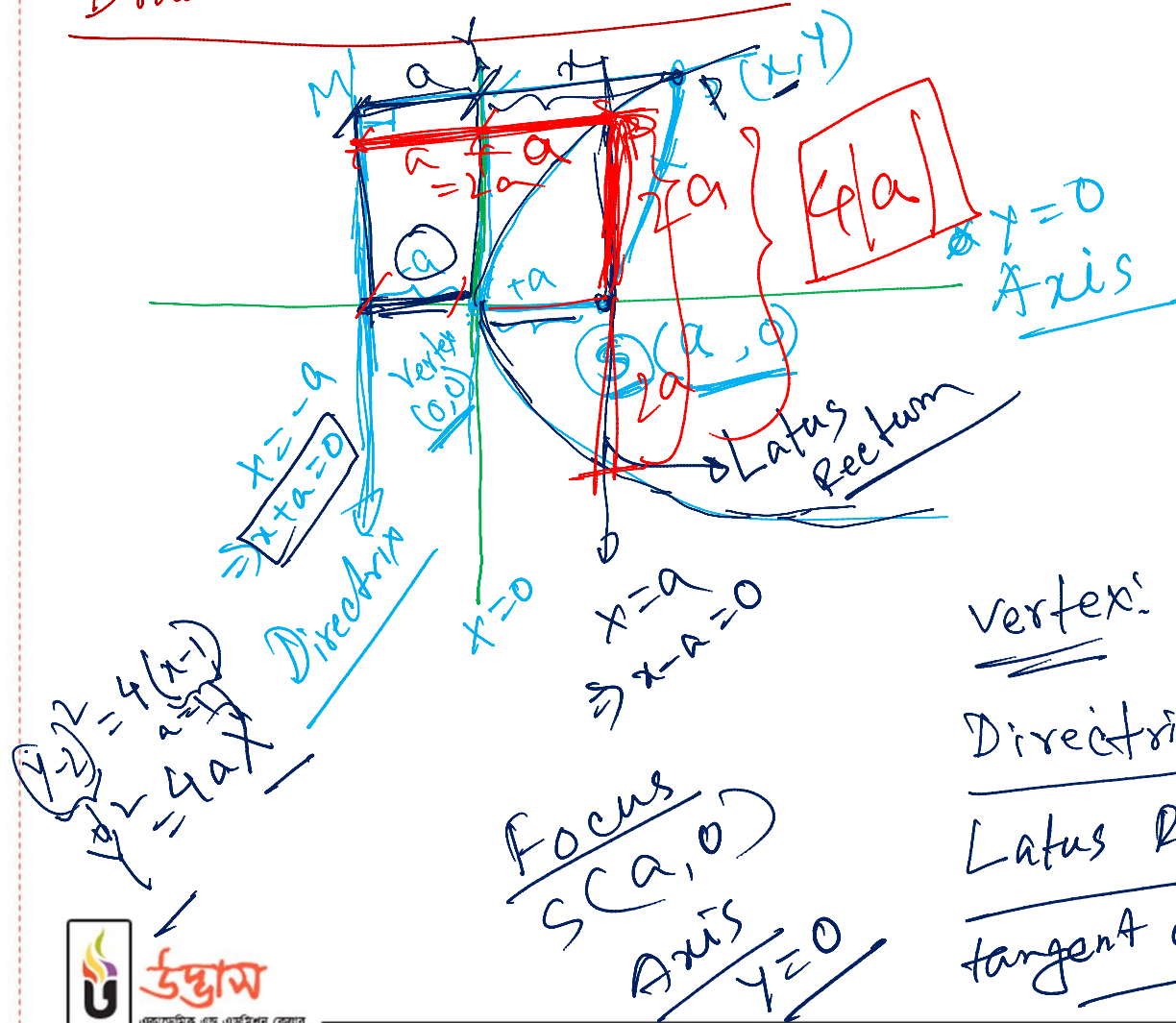
$$y = mx + c$$

$$y^2 = 4ax$$

$$ax^2 + bx + c = 0$$

$$D = b^2 - 4ac$$

Draw & Find Properties:



$$SP = PM$$

$$\Rightarrow \sqrt{(x-a)^2 + (y-0)^2} = x+a$$

$$\Rightarrow x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$

$$\Rightarrow y^2 = 4ax$$

Vertex: $x=0$ $y=0 \rightarrow (0,0)$

Directrix: $x = -a$

Latus Rectum: $x = a$

tangent at Vertex: $x = 0$

Mathematical Problem

$$y = mx + c \rightarrow y^2 = 4am \quad \frac{c}{4a} = \frac{1}{m}$$

Show that, the straightline $lx + my + n = 0$ is a tangent to the parabola $y^2 = 4ax$ if $ln = am^2$.

$$\begin{aligned} lx + my + n &= 0 \\ \Rightarrow my &= -lx - n \\ \Rightarrow y &= -\frac{l}{m}x - \frac{n}{m} \end{aligned}$$

$$\begin{aligned} m &= -\frac{l}{\frac{1}{m}} \\ c &= -\frac{n}{m} \end{aligned}$$

$$\Rightarrow +\frac{n}{m} = \frac{a}{+\frac{l}{m}}$$

$$\begin{aligned} \Rightarrow \frac{n}{m} &= \frac{am}{l} \\ \Rightarrow ln &= am^2 \end{aligned}$$

showed

Poll Question-05

For which value of K , $x=k-y$ is a tangent to the parabola $x^2=x-y$?

- (a) $k = 1$
- (b) $k = 0$
- (c) $k = -1$
- (d) $k = 3$

Practice

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ধ্বংস করে

$$X = \frac{c \rho V^2}{2S}$$

$$X = \frac{c \rho V^2}{2S}$$

$$E = mc^2$$

$$x = \sqrt{\frac{c^2}{c}} + c - \frac{b}{2}$$



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