৮ম শ্রেণি একাডেমিক প্রোগ্রাম ২০২০

গণিত

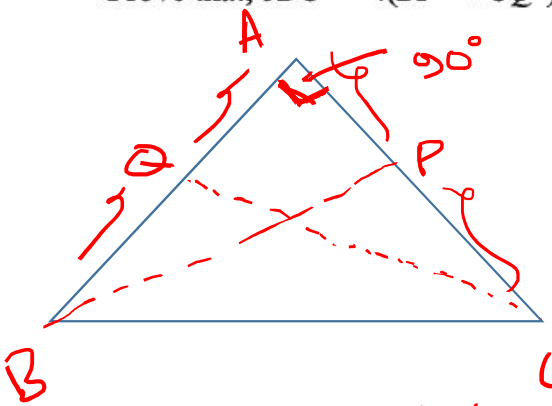
লেকচার : M-01

অধ্যায় ৪ : বীজগণিতীয় সূত্রাবলি ও প্রয়োগ

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

White Board

4) In ΔABC, $\angle A$ is a right angle. BP and CQ is are two medians.
Prove that, $5BC^2 = 4(BP^2 + CQ^2)$



$5BC^2 = 4(BP^2+CQ^2)$

ΔABC ডি,
hypo → BC

$BC^2 = AB^2+AC^2$

ΔABP —
hypo → BP

$BP^2 = AB^2 + AP^2$

ΔAQC →
hypo → CQ

$CQ^2 = AQ^2 + AC^2$

$AQ = BQ$

$AB = AQ+BQ$

$AB = AQ+AQ$

$AB = 2AQ$ —i

$AB^2 = 4AQ^2$

Similarly,

$AP = CP$

$AC = 2AP$ —ii

$AC^2 = 4AP^2$

$4(BP^2+CQ^2)$

$= 4BP^2 + 4CQ^2$

$= 4(AB^2+AP^2) + 4(AQ^2+AC^2)$

$= 4AB^2+4AP^2+4AQ^2+4AC^2$

$= 4(AB^2+AC^2) + 4AP^2+4AQ^2$

$= 4BC^2 + AC^2 + AB^2$

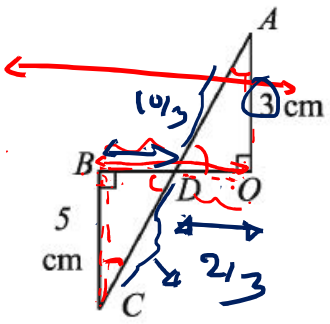
$= 4BC^2 + BC^2$

$= 5BC^2$

proved

White Board

6.



In figure, if $OB = 4$ cm find the length of BD and AC.

$OB = BD + DO$

$4 = BD + 2/3$

$BD = 4 - 2/3$

$= \frac{12-2}{3}$

$= 10/3$ Ans

$\triangle ADO$ & $\triangle BDC$,

$\frac{5}{3} = \frac{BD}{DO}$

$\frac{5}{3} + 1 = \frac{BD}{DO} + 1$

$\frac{5+3}{3} = \frac{BD + DO}{DO}$ (BO)

$\frac{8}{3} = \frac{BO}{DO}$

$\frac{8}{3} = \frac{4}{DO}$

$DO = \frac{4}{8} \times 3$

$= 2/3$

$AD^2 = AO^2 + DO^2$

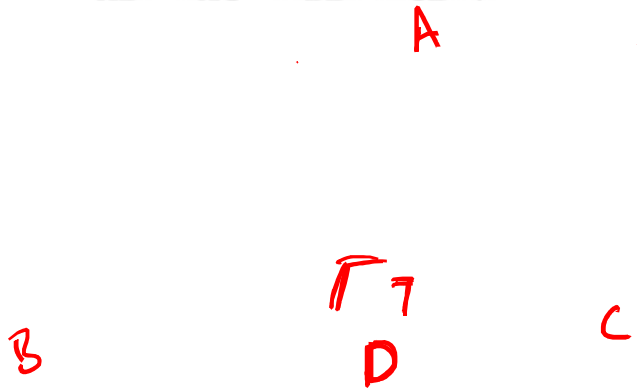
$DC^2 = BD^2 + BC^2$

$AD = \square$, $DC = \square$

$AC = AD + DC$

White Board

10. In ΔABC, AD is the perpendicular to BC and $AB > AC$. Prove that
$AB^2 - AC^2 = BD^2 - CD^2$.



Proof :-

$\Delta ADB \rightarrow$

hypo $\rightarrow$ AB

$AB^2 = AD^2 + BD^2$ — (i)

$AD^2 = AB^2 - BD^2$

$\Delta ADC \rightarrow$

hypo $\rightarrow$ AC

$AC^2 = AD^2 + CD^2$ — (ii)

$AD^2 = AC^2 - CD^2$

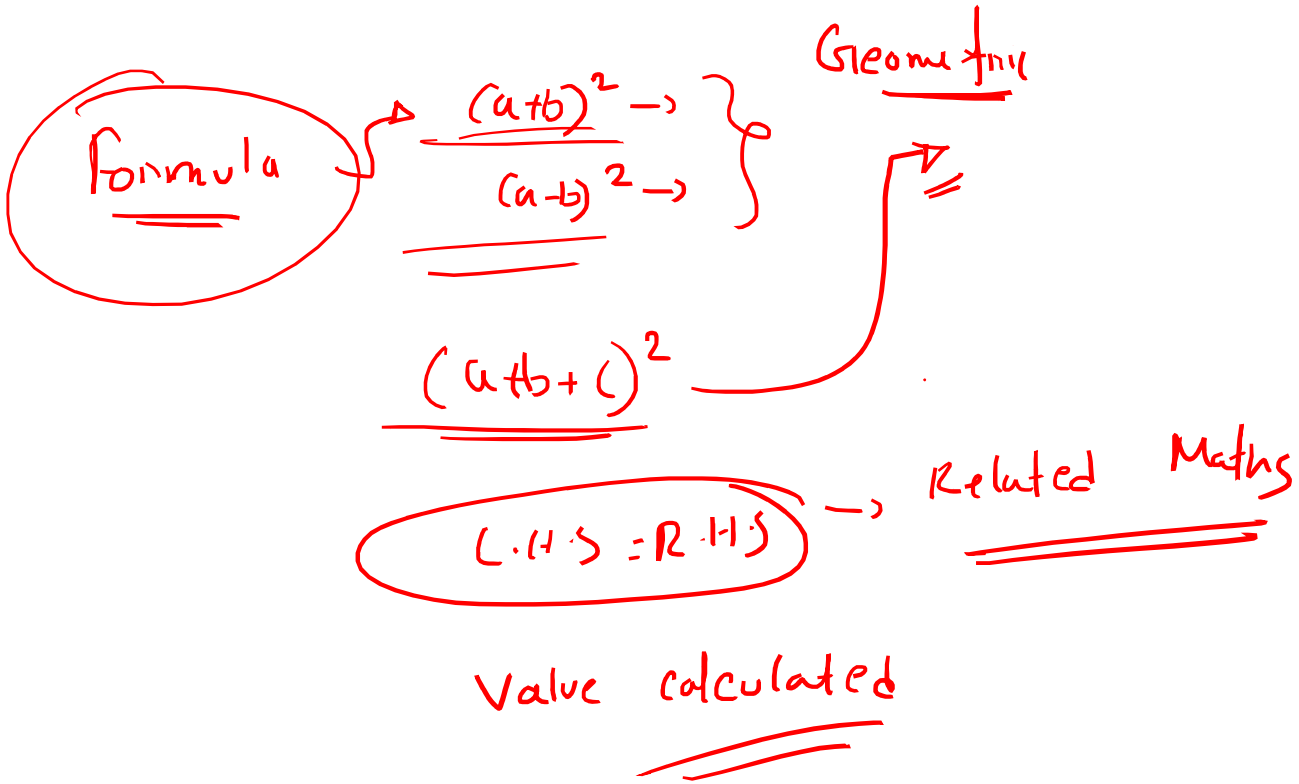
$AD^2 = AD^2$

$AB^2 - BD^2 = AC^2 - CD^2$

$AB^2 - AC^2 = BD^2 - CD^2$

[proved]

What will we learn from 4.1 ?



Algebraic formulae

The first four formulae and the corollaries related to them have been elaborately discussed in class VII. Here, those are repeated.

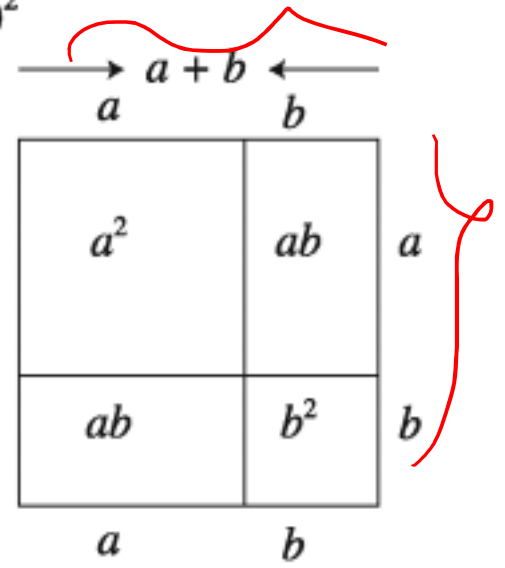
The geometric explanation of $(a+b)^2$ is as follows:

The area of the whole square $= (a+b) \times (a+b) = (a+b)^2$

$\therefore \ (a+b)^2 = a \times (a+b) + b \times (a+b)$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

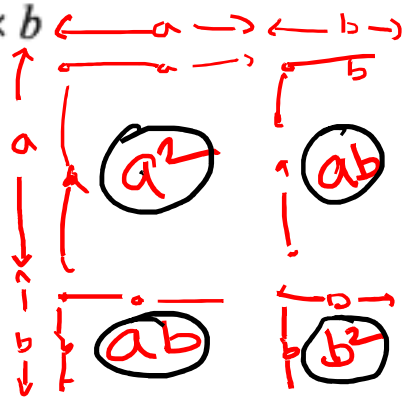


Again, the sum of the areas of the parts of the square

$a \times a + a \times b + b \times a + b \times b$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$



$(a+b)^2 = a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

Algebraic formulae

In class VII, Formulae and corollary, which we have known, are as follows:

Formula 1. $(a+b)^2 = a^2 + 2ab + b^2$

In words, the square of the sum of two quantities = the square of first quantity + 2 × first quantity × second quantity + the square of second quantity.

Formula 2. $(a-b)^2 = a^2 - 2ab + b^2$

In words, the square of the difference of two quantities = the square of first quantity -2 × first quantity × second quantity + the square of second quantity.

Formula 3. $a^2 - b^2 = (a+b)(a-b)$

In words, the difference of squares of two quantities = the sum of two quantities × the difference of two quantities.

Formula 4. $(x+a)(x+b) = x^2 + (a+b)x + ab$

$(x+a)(x+b) = x^2 + ax + bx + ab$

$= x^2 + x(a+b) + ab$

In words, if the first terms of two binomial expressions are the same, their product will be equal to the sum of square of the first term, product of the first term with the sum of their second terms with their usual signs and product of the second two terms with their usual signs. That is, $(x + a)(x + b) = x^2 +$ (algebraic sum of a and b) $x +$ (product of a and b).

Algebraic formulae

Corollary 1. $a^2 + b^2 = (a+b)^2 - 2ab$

Corollary 2. $a^2 + b^2 = (a-b)^2 + 2ab$

Corollary 3. $(a+b)^2 = (a-b)^2 + 4ab$

Corollary 4. $(a-b)^2 = (a+b)^2 - 4ab$

Corollary 5. $2(a^2 + b^2) = (a+b)^2 + (a-b)^2$

Corollary 6. $4ab = (a+b)^2 - (a-b)^2$

or, $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

a^2+b^2

$= a^2 + 2ab + b^2 - 2ab$

$= a^2 + b^2$

Mathematical problems

- *Find the square of* $3x + 5y$.

a b

$$(3x+5y)^2 = (3x)^2 + 2 \cdot 3x \cdot 5y + (5y)^2$$

$$= 9x^2 + 30xy + 25y^2$$

Ans.

Mathematical problems

❑ Find the square of $4x - 7y$.

$(a-b)^2 \rightarrow$

$$(4x-7y)^2 = (4x)^2 - 2 \cdot 4x \cdot 7y + (7y)^2$$

$$= 16x^2 - 56xy + 49y^2$$

Ans

POLL

Find the square of x+2y ?

[Dhaka Board -16]

a) $x^2 + 2xy + y^2$

b) $x^2 + 4xy + 4y^2$

c) $x^2 + 2xy + 4y^2$

d) $x^2 + xy + y^2$

$(x+2y)^2$

$= x^2 + 2 \cdot x \cdot 2y + (2y)^2$

$= x^2 + 4xy + 4y^2$

Mathematical problems

- If $a - b = 7$ and $ab = 60$, find the value of $a^2 + b^2$.

$a^2+b^2 = (a+b)^2 - 2ab$

$= a^2+2ab+b^2 - 2ab$

$= a^2+b^2$

$a^2+b^2 = (a-b)^2 + 2ab$

$a^2-2ab+b^2 + 2ab$

$= a^2+b^2$

a^2+b^2

$= (a-b)^2 + 2ab$

$= 7^2 + 2 \cdot 60$

$= 49 + 120$

$= 169$ Ans.

Mathematical problems

❑ If $x - \frac{1}{x} = 5$,find the value of $\left(x + \frac{1}{x}\right)^2$.

$(a+b)^2 = (a-b)^2 + 4ab$

$= a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$\left(x + \frac{1}{x}\right)^2$

$= \left(x - \frac{1}{x}\right)^2 + 4 \times \frac{1}{x}$

$= 5^2 + 4$

$= 25 + 4 = 29$ Ans

POLL

$a^2+b^2 = (a-b)^2+2ab$

If $a^2 - 1 = 5a$, *what is the value of* $a^2 + \frac{1}{a^2}$? [Ctg. B.-16]

A)21 B)23

C)25 D)27

$a^2 + \frac{1}{a^2}$

$= \left(a-\frac{1}{a}\right)^2 + 2 \cdot a \cdot \frac{1}{a}$

$= \left(a-\frac{1}{a}\right)^2 + 2$

$= 5^2 + 2 = 25 + 2 = 27$

$a^2 - 1 = 5a$

$\Rightarrow \frac{a^2-1}{a} = 5$

$\Rightarrow \frac{a^2}{a} - \frac{1}{a} = 5$

$a - \frac{1}{a} = 5$

Mathematical problems

- Multiply $5m+8$ by $5m+9$ *by an appropriate formula*.

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$(5m+8)(5m+9) = (5m)^2 + (8+9)5m + 8 \cdot 9$

$= 25m^2 + (17 \cdot 5)m + 72$

$= \boxed{25m^2 + 85m + 72}$ Ans.

Mathematical problems

- Express $(x+6)(x+4)$ as the difference of two squares .

Difference of two squares

$(x+6)(x+4) = (x+5)^2 - 1^2$ Ans.

$\Rightarrow ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$(x+6)(x+4) = \left(\frac{x+6+x+4}{2}\right)^2 - \left(\frac{x+6-x-4}{2}\right)^2$

$= \left(\frac{2x+10}{2}\right)^2 - \left(\frac{2}{2}\right)^2 = \{2(x+5)\}^2 - 1^2$

POLL

If x+y=6 and x-y=4 , 4xy=?
[D.B.-15]
a)5 b)13 c)20 d)52

$4xy = (x+y)^2 - (x-y)^2$

$= 6^2 - 4^2$

$= 36 - 16$

$= 20$

$x + y = 6$

$x - y = 4$

$2x = 10$

$\therefore x = 5$

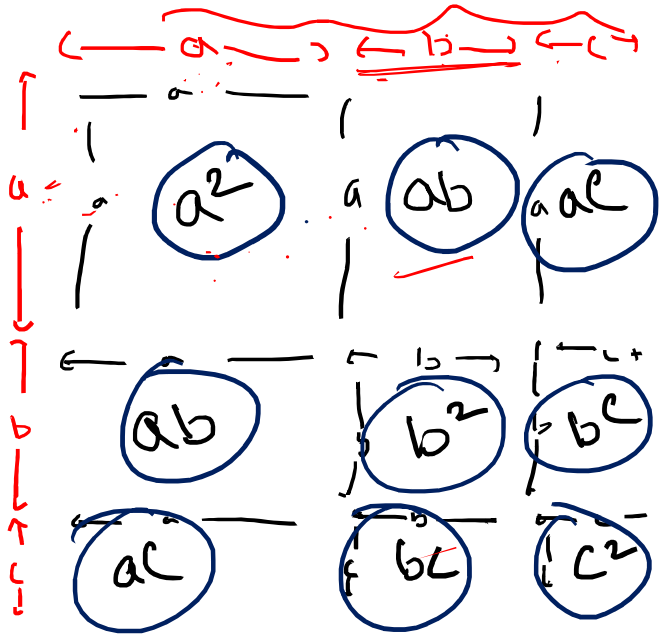
$y = 6 - 5 = 1$

$4 \times y$

$= 4 \cdot 5 \cdot 1 = 20$

Mathematical problems

- Geometric explanation of $(a+b+c)^2$:



Each side = a+b+c

Area = $(a+b+c)^2$

$(a+b+c)^2 = a^2+ab+ac+ab+b^2+bc+ac+bc+c^2$

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Mathematical problems

- Find the square of $2x + 3y + 5z$.

(a) (b) (c)

$$(2x+3y+5z)^2$$

$$= (2x)^2 + (3y)^2 + (5z)^2 + 2(2x)(3y) + 2(3y)(5z) + 2(2x)(5z)$$

$$a^2 \quad b^2 \quad c^2 \quad 2ab \quad 2bc \quad 2ac$$

POLL

What is the square of a+b-c?
[ctg. B.-16]

a) $a^2 + b^2 + c^2$
b) $a^2 + b^2 - c^2 + 2ab - 2bc - 2ca$
c) $a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$
d) $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

$a + b - c$

$a + b + (-c)$

$a^2 + b^2 + (-c)^2 + 2ab + 2b(-c) + 2(-c)a$

$= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

Mathematical problems

*Simplify:- $(x+y)^2 + 2(x+y)(x-y) + (x-y)^2$

$(a+b)(a-b)$
a^2-b^2

↳

$= x^2+2xy+y^2 + 2\{x^2-y^2\} + x^2-2xy+y^2$

$= x^2+2xy+y^2+2x^2-2y^2+x^2-2xy+y^2$

$= 4x^2$ Ans

Mathematical problems

6. If $a+\frac{1}{a}=4$, what is the value of $a^4+\frac{1}{a^4}$?

$a^2+b^2=(a+b)^2-2ab$

$a^4+\frac{1}{a^4}$

$=\left(a^2\right)^2+\left(\frac{1}{a^2}\right)^2$

$=\left(a^2+\frac{1}{a^2}\right)^2-2\cdot a^2\cdot\frac{1}{a^2}$

$=\left(a^2+\frac{1}{a^2}\right)^2-2$

$=\left\{\left(a+\frac{1}{a}\right)^2-2\cdot a\cdot\frac{1}{a}\right\}^2-2$

$=\{(4)^2-2\}^2-2$

$=\{16-2\}^2-2$

$=14^2-2$

$=196-2$

$=194$ Ans.

Mathematical problems

8. If $a-\frac{1}{a}=m$, show that $a^4+\frac{1}{a^4}=m^4+4m^2+2$

$a^2+b^2 = (a-b)^2+2ab$

$a^2+b^2 = (a+b)^2-2ab$

L.H.S

$a^4+\frac{1}{a^4}$

$=(a^2)^2+\left(\frac{1}{a^2}\right)^2$

$=\left(a^2+\frac{1}{a^2}\right)^2-2\cdot a^2\cdot\frac{1}{a^2}$

$=\left(a^2+\frac{1}{a^2}\right)^2-2$

$=\left\{\left(a-\frac{1}{a}\right)^2+2\cdot a\cdot\frac{1}{a}\right\}^2-2$

$=\left(m^2+2\right)^2-2$

$=(m^2)^2+2\cdot m^2\cdot 2+4-2$

$=m^4+4m^2+2$

$=$ R.H.S

L.H.S = R.H.S

POLL

$(a+b)^2 = (a-b)^2 + 4ab$

$(x+\frac{1}{x})^2 = (x-\frac{1}{x})^2 + 4 \cdot x \cdot \frac{1}{x}$

$= 4^2 + 4$

$= 16 + 4$

$= 20$ Ans.

If $x - \frac{1}{x} = 4$, what is the value of $(x + \frac{1}{x})^2$?

a)10 b)18

c)16 d)20

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে