بِسْمِ اللهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

Class-8: Math (Chapter-11)

Lecture M-14

Previous Homework

Let, O be the center of a circle where AB is a diameter and CD is a chord .

A. find the area of a fish where his diameter is 6.4 meter . 2

B. Prove that, $AB > CD$. → Chapter 10.2, Example-4 4

C. If E is the midpoint of CD, Prove that, $OE \perp CD$. 4

↳ Chapter 10.1, Theorem-1

(A)
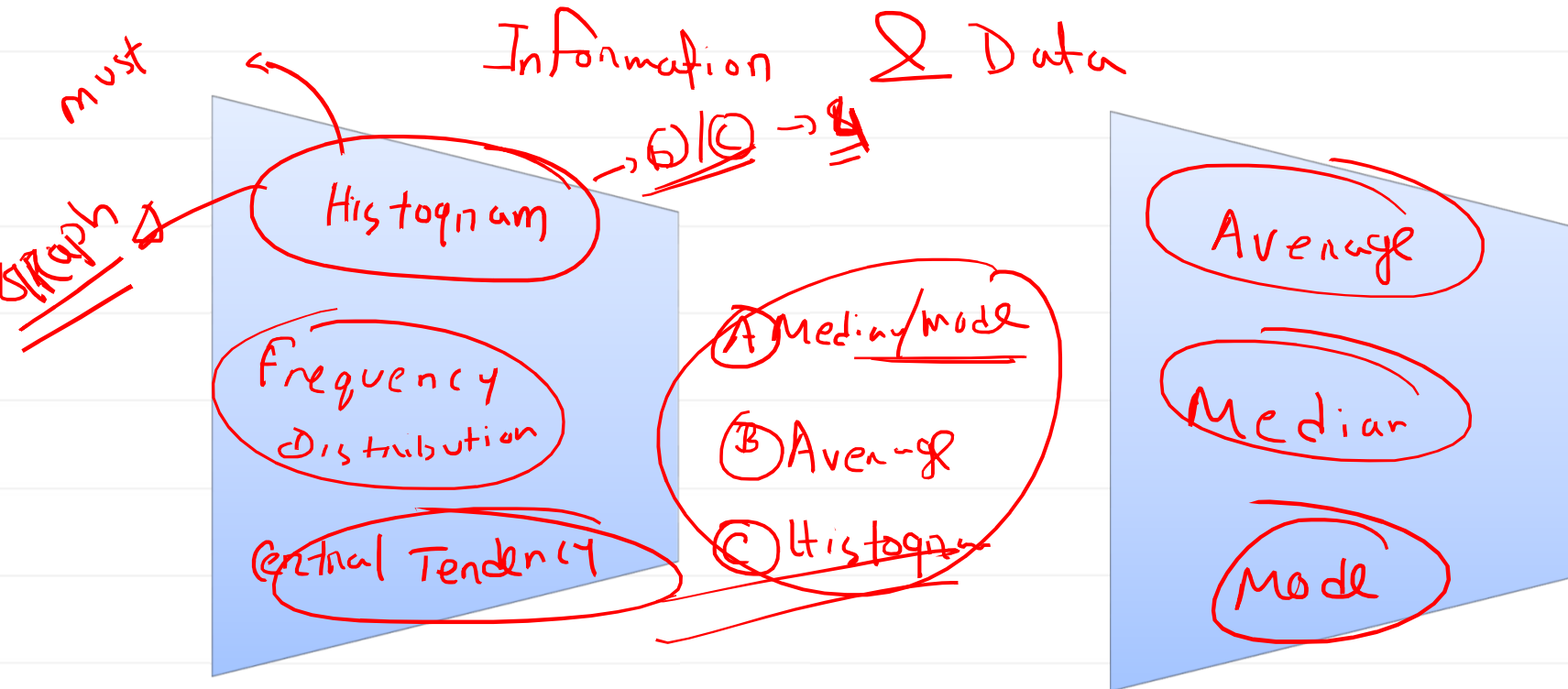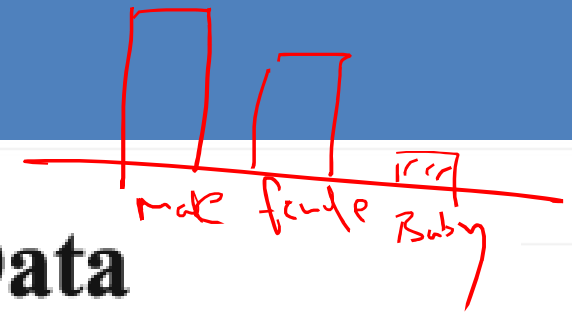
Area of a circle $= \pi r^2$

$r = \dfrac{d}{2}$

$r = \dfrac{6.4}{2}$

$\left[3.14 \times \left(\dfrac{6.4}{2} \right)^2 \right] m^2$

sq m

Information and Data

Information and data have an important role in and contribution to the wide expansion and rapid development of knowledge and science. Based on information and data, research is carried out and continuous research results in the unthinkable development of knowledge and science. The use of numbers has expanded largely in the presentation of information and data. The number based information is statistics. So, the fundamental concepts and related contents of statistics are essential to learn. The basic contents of statistics have been presented in the previous class gradually. In continuation of the presentation, the central tendency and its measure namely mean, median and mode have been discussed in detail in this chapter.
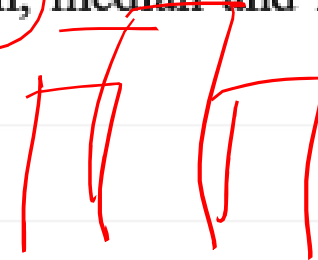
11·2 Frequency Distribution Table

The following steps are used to make a frequency distribution table :

Determination of (1) range (2) number of classes (3) class interval (4) frequency using tally. Range of data to be investigated = (highest number – lowest number) + 1.

Class Interval : After the determination of the range of data under investigation it is required to find the class interval. The data are divided into some class taking convenient intervals. Generally, the classification is made depending upon the number of data. There is no hard and fast rule of classification. But usually, the limit of class interval is maintained between minimum 5 and maximum 15. Hence, there is a highest and a lowest value of each interval. The lowest value of any class is its lower limit and the highest value is its higher limit. The difference between the higher and lower limits of any class is its class interval. For example, let, 10-20 be a class; its minimum value is 10 and maximum value is 20 and (20-10) = 10 and its class interval = 10 + 1 = 11. It is always better to keep the class intervals equal.

Number of class : The range divided by the class interval is number the of classes.

Hence, number of classes = $\dfrac{\text{range}}{\text{class interval}}$ (converted into integer)

Tally Marks : The numerical information of the data must belong to some class. For a numerical value, tally mark is put against the class. If the number of tally in a class is 5, the 5th one is put crosswise.

Frequency : The numerical values of information in the classes are expressed by tally marks and frequency is determined by the numbers of tally marks. The number of frequency of a class will be the number of the tally marks, which is written in frequency column against the tally marks.

Range, class interval and number of classes of the above data under consideration are as follows :

Range = (highest numerical value of the data − lowest numerical value) + 1

$= (50 - 20) + 1 = 31.$

If the class interval is taken to be 5, the number of classes will be $\dfrac{31}{5} = 6\cdot2$

which will be 7 after converting into integer. Hence the number of classes is 7. In respect of above discussion, the frequency distribution table of the stated data is:

| Class interval	Tally marks	Frequency
20-24	||	2
25-29	||	2
30-34	||||	4
35-39	||	2
40-44	||||	4
45-49	ᚋ	5
50-54	|	1
total	20	20

11·3 Diagram

The presentation of information and data by diagram is a widely used method. If the data used in any statistics are presented through diagram, they become easy to comprehend and convenient to draw conclusion. Moreover, the data presented through diagram also become attractive. That is why frequency distribution of data is presented in diagram for easy comprehension and for drawing conclusion. Though there are different types of diagrams in presenting the frequency diagrams, here only Histogram and Pie-chart will be discussed.

Histogram : One of the diagrams of frequency distribution is histogram. For drawing histogram x-axis and y-axis are drawn in a graph paper. The class interval and the frequency are placed along x-axis and y-axis respectively and the histogram is drawn. The base of rectangle is the class interval and height is the frequency.
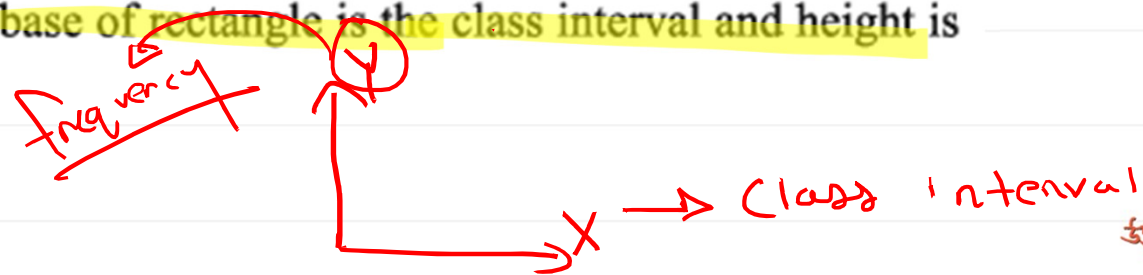
Poll Question: 01

When we put the data in a class interval ,what will be the indicator ?

A) Frequency   B) Class mid value

C) Range   D) Cumulative frequency

Example 1. The frequency distribution table of the heights of 50 students is as follows. Draw a histogram.

Class interval of heights (in cm)	114-124	124-134	134-144	144-154	154-164	164-174
Frequency (number of students)	3	5	10	20	8	4

Considering one unit of graph paper to represent 2 of the class interval along the x-axis and one unit of graph paper to denote 1 of the frequency along the y-axis, the histogram of frequency distribution has been drawn. The broken segments from the origin of x-axis to 114 indicate that the previous intervals are omitted.
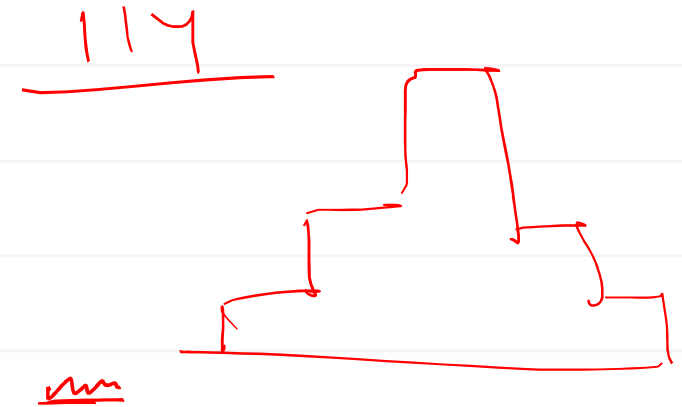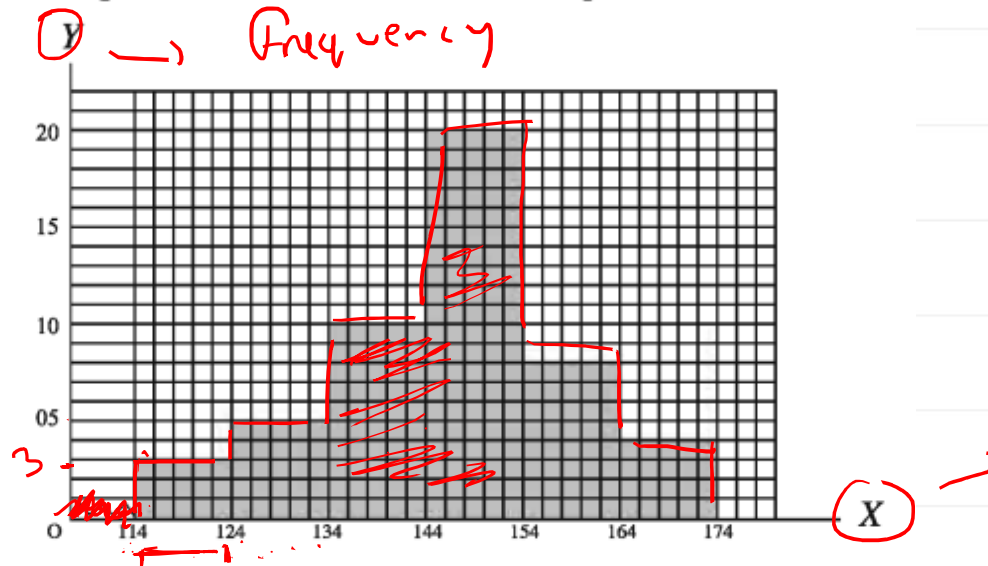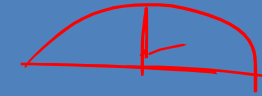
Pie-chart : A pie-chart is also a diagram. Sometimes the collected statistics consists of the sum of the elements or it is divided into some classes. If these classes are expressed by different slices of a circle, the diagram thus obtained is a pie-chart. A pie-chart is also known as a circular diagram. We know that the angle subtended at the centre of a circle is 360°. If statistics is presented as a part of 360°, it will be a pie-chart.

We know that the runs are scored by 1, 2, 3, 4 and 6 in a cricket game. Extra runs are also scored by no-ball and wide ball. The runs scored by Bangladesh cricket team in a game is placed in the following table.

Run scored	1	2	3	4	6	Extra	Total
Scored Run in different ways	66	50	36	48	30	10	240

Pie-chart formula

$$\frac{x}{\text{total result}} \times 360°$$

If the data of cricket game is shown by a pie-chart, it becomes attractive as well as so easy to understand. When a data is presented through a circle, the diagram is called a pie-chart. Hence a pie-chart is a circular diagram. We know that the angle subtended at the centre is 360°. If the above stated data is presented as a parts of 360°, we get the pie-chart of the data.

For 240 runs, the angle is $360°$

$$\therefore \quad " \quad 1 \quad " \quad " \quad " \quad \frac{360°}{240}$$

$$\therefore \quad " \quad 66 \quad " \quad " \quad \frac{66 \times 360°}{240} = 99°$$
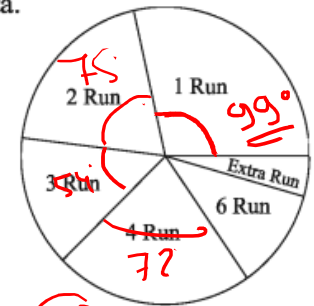
Similarly for 50 runs, the angle will be $\frac{50}{240} \times 360° = 75°$

$$" \quad " \quad 36 \quad " \quad " \quad " \quad " \quad \frac{36}{240} \times 360° = 54°$$

$$" \quad " \quad 48 \quad " \quad " \quad " \quad " \quad \frac{48}{240} \times 360° = 72°$$

$$" \quad " \quad 30 \quad " \quad " \quad " \quad " \quad \frac{30}{240} \times 360° = 45°$$

$$" \quad " \quad 10 \quad " \quad " \quad " \quad " \quad \frac{10}{240} \times 360° = 15°$$

Here, the angles obtained are drawn as parts of 360°, which is the pie-chart of the data.

Example 2. The table of death due to accidents in a year is given below. Draw a pie-chart :

Accident	bus	truck	car	vessel	total
Number of deaths	450	350	250	150	1200

Solution :

The angle for death of 450 due to bus accident $= \dfrac{450}{1200} \times 360 = 135°$

The angle for death of 350 due to truck accident $= \dfrac{350}{1200} \times 360° = 105°$

The angle for death of 250 due to car accident $= \dfrac{250}{1200} \times 360° = 75°$

The angle for death of 150 due to vessel accident $= \dfrac{150}{1200} \times 360° = 45°$
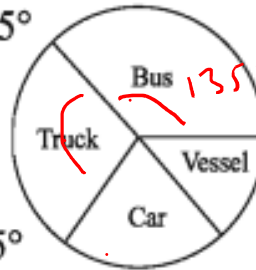
Here, the angles are drawn as parts of 360° to form the required pie-chart.

11.4 Central Tendency

Let the time (in second) taken by 25 girl students to solve a problem be as follows:

22,16,20,30,25,36,35,37,40,43, 40,43,44,43,44,46,45,48,50,64,50,60,55,62,60.

The numbers arranged in ascending order are :

16, 20, 22, 25,30,35, 36, 37,40,40,43, 43, 43,44,44,45,46,48, 50, 50, 55,60,60, 62, 64. The stated data are centred round the middle value of 43 or 44. This tendency is also seen in frequency distribution table. The frequency distribution table of the data is

Interval	16-25	26-35	36-45	46-55	56-65
Frequency	4	2	10	5	4

From this frequency distribution table, it is to be noted that the maximum of the frequency occurs in the class 36−45. Hence, it is clear from the above discussion that the data cluster round the value at centre or middle. The tendency of clustering of the data to the value at middle or centre is called central tendency. The central value of the data is a representative number which measures the central tendency. Generally, measurement of central tendency are (1) Arithmetic Average, (2) Median, (3) Mode.

11·5 Arithmetic Mean

We know that if the sum of the numerical values of data is divided by the number of data, we get Arithmetic mean. Let the number of data be n and their numerical values are $x_1, x_2, x_3 \ldots\ldots x_n$. If the arithmetic mean of the data is $\bar{x}$,

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots\ldots x_n}{n} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Example 4. Out of 50 in an examination, the marks obtained by 20 students are 40,41,45,18,41,20,45,41,45,25,20,40,18,20,45,47,48,48,49,19.Find the arithmetic mean of the marks.

Solution : Here n = 20, $x_1 = 40$, $x_2 = 41$, $x_3 = 45$,.......... etc.

If the arithmetic mean is $\bar{x}$, $\bar{x} = \dfrac{\text{sum of numbers}}{\text{number of numbers}}$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \sum_{c=1}^{20} \frac{x_i}{20} = \frac{40 + 41 + 45 + \ldots\ldots + 19}{20} = \frac{715}{20} = 35 \cdot 75$$

$\therefore$ Arithmetic Mean is 35.75

Arithmetic Mean of disorganized Data (short-cut method) :

If the numbers of data are large, to find the arithmetic mean by the previous method is difficult and there is every possibility to make mistakes in finding the sum of such large numbers of the data. In this context, it is convenient to use a short-cut method.

In the short-cut method, the possible arithmetic mean is estimated through proper and careful observation of central tendency. Through careful observation of central tendency of the above example, it is clear that the arithmetic mean is a number between 30 and 46. Let the mean be 30. Here the estimated arithmetic mean 30 has to be subtracted from each of the numbers to determine the subtracted value. If the number is larger than 30, the result will be positive and if the number is less than 30, the result will be negative. Then the algebraic sum of the differences has to be determined. The two successive differences are added to find the cumulative sum and the process contiues. The sum of all differences is equal to the final cumulative sum . Here the arithmetic mean of the data used in the above example can be determined by the short-cut method. Let, the data is x_i (i = 1, 2,....., n) and the estimated mean of the data is a, (a = 30).

Data x_i	$x_i - a$	Cumulative Sum	Data x_i	$x_i - a$	Cumulative Sum
40	40 − 30 = 10	10	20	20 − 30 = − 10	61 − 10 = 51
41	41 − 30 = 11	10 + 11 = 21	40	40 − 30 = 10	51 + 10 = 61
45	45 − 30 = 15	21 + 15 = 36	18	18 − 30 = − 12	61 − 12 = 49
18	18 − 30 = −12	36 − 12 = 24	20	20 − 30 = −10	49 − 10 = 39
41	41 − 30 = 11	24 + 11 = 35	45	45 − 30 = 15	39 + 15 = 54
20	20 − 30 = − 10	35−10 = 25	47	47 − 30 = 17	54 + 17 = 71
45	45 − 30 = 15	25 + 15 = 40	48	48 − 30 = 18	71 + 18 = 89
41	41 − 30 = 11	40 + 11 = 51	48	48 − 30 = 18	89 + 18 = 107
45	45 − 30 = 15	51 + 15 = 66	49	49 − 30 = 19	107 + 19 = 126
25	25 − 30 = −5	66 − 5 = 61	19	19 − 30 = − 11	126−11 = 115

It is evident from the above table that the sum of the differences = 115

∴ The average of the differences $= \dfrac{115}{20} = 5 \cdot 75$

Hence actual mean   = Estimated mean + average of differences

$$= 30 + 5.75$$
$$= 35.75$$

How is the arithmetic mean of 8,12,16,17,20 these numbers ?

A)10.5   B)12.5

C)13.6   D)14.6

$$\frac{8+12+16+17+20}{5} = \underline{14.6}$$

Arithmetic Mean of Organized Data

Of the marks obtained in Mathematics by 20 students in example 4, more than one student have obtained the same marks. The frequency distribution table of the marks obtained is placed below :

Marks obtained $x_i \, i = 1, ..., k$	Frequency $f_i \, i = 1, ..., k$	$f_i x_i$
18	2	36
19	1	19
20	3	60
25	1	25
40	2	80
41	3	123
45	4	180
47	1	47
48	2	96
49	1	49
$k = 10$	$k = 10, n = 20$	Total $= 715$

$$\text{Arithmetic Mean} = \frac{\text{Sum of } f_i x_i}{\text{Total Frequency}} = \frac{715}{20}$$

$$= 35.75$$

Formula 1. Arithmetic Mean (Organized Data) : If frequency of k numbers of $x_1, x_2, x_3 \ldots \ldots, x_k$ of n number of data is $f_1, f_2, \ldots \ldots f_k$, arithmetic mean of the data

$$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n} = \frac{1}{n} \sum_{i=1}^{k} f_i x_i \text{ where } n \text{ is the number of data}$$

Example 5. The frequency distribution table of the marks obtained in Mathematics by 100 students of a class is as follows. Find the arithmetic mean.

Class Interval	25-34	35-44	45-54	55-64	65-74	75-84	85-94
Frequency	5	10	15	20	30	16	4

Solution : It is not possible to know the individual marks of the students as the class interval is given. In this case, it is necessary to find the class mid-value of the class

$$\text{Class mid-value} = \frac{\text{class higher value} + \text{class lower value}}{2}$$

$$\frac{25 + 34}{2} = 29.5$$

If the class mid-value is x_i ($i = 1, 2, \ldots \ldots, k$), the table containing mid-values will be as follows :

Class interval	Class mid-value (x_i)	Frequency (f_i)	$(f_i x_i)$
25 – 34	29.5	5	147.5
35 – 44	39.5	10	395.0
45 – 54	49.5	15	742.5
55 – 64	59.5	20	1190.0
65 – 74	69.5	30	2085.0
75 – 84	79.5	16	1272.0
85 – 94	89.5	4	348.0
	Total	100	6190.00

Required arithmetic mean $= \dfrac{1}{n} \sum_{i=1}^{k} f_i x_i = \dfrac{1}{100} \times 6190$
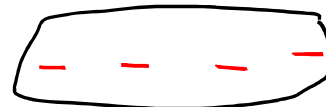
$$= 61.9$$

11·6 Median

→ ▷ Middle value

We have already learnt about median of the data under consideration in statistics in class VII.

Let 5, 3, 4, 8, 6, 7, 9, 11, 10 be a few numbers. If arranged in ascending order, they will be 3, 4, 5, 6, 7, 8, 9, 10, 11. If the ordered arranged numbers are divided into two equal parts, they will be

| 3, | 4, | 5, | 6, | 7 | 8, | 9, | 10, | 11 |

It is evident that the number 7 divides the numbers in two equal parts and its position is in the middle. Hence, here the mid-term is the 5th term. The value of the 5th term or mid-term is 7. Therefore, the median of the numbers is 7. Here, the number given data is odd. If the number of data is even such as 8, 9, 10, 11, 12, 13, 15, 16, 18, 19, 21, 22, what will be the median ? If the numbers are divided into two equal parts, they will be

| 8, | 9, | 10, | 11, | 12, | 13, 15 | 16, | 18, | 19, | 21, | 22 |

It is evident from the above that 13 and 15 divide the numbers into two equal

n = 9

$\dfrac{9+1}{2} = 5th$

parts and their positions are in the middle. Here mid-terms are 6th and 7th terms. Therefore, the median will be average of the numbers of 6th and 7th terms. The

average of the numbers of 6th and 7th terms is $\dfrac{13+15}{2}$ or 14 i.e. the median is 14.

From the above discussion, we can conclude that if there is n number of data and if n is added, the median of the data will be the value of $\dfrac{n+1}{2}$th term. But if n is even number, the median will be average of the numerical values of $\dfrac{n}{2}$th and $\left(\dfrac{n}{2}+1\right)$th terms.

Example 6. Find the median of the following numbers : 23, 11, 25, 15, 21, 12, 17, 18, 22, 27, 29, 30, 16, 19.

Solution : The numbers if arranged in ascending order will be

11, 12, 15, 16, 17, 18, 19, 21, 22, 23, 25, 27, 29, 30

Here the number of data is even, i.e. $n = 14$

$$\therefore \text{Median} = \frac{\text{Value of sum of } \frac{14}{2}\text{th and } \left(\frac{14}{2}+1\right)\text{th terms}}{2}$$

$$= \frac{\text{value of sum of } 7^{\text{th}} \text{ and } 8^{\text{th}} \text{ terms}}{2}$$

$$\therefore \text{Median} = \frac{19+21}{2} = \frac{40}{2} = 20$$

Therefore, median is 20.

How is the median of 10,12,14,18,19,25 these numbers ?

A) 11.5   B) 14.6
C) 16   D) 18.6

$$\frac{14+18}{2}$$

$n = 6$

$$\frac{n}{2}, \quad \frac{n}{2}+1$$

3, 4 th

11·7 Mode

Let 11, 9, 10, 12, 11, 12, 14, 11, 10, 20, 21, 11, 9 and 18 be a data. If the data are arranged in ascending order, it will be

9, 9, 10, 10, 11, 11, 11, 11, 12, 12, 14, 18, 20, 21.

It is to be noted that in arranged data, 11 appears 4 times which is maximum times of repetition. Since 11 appears maximum times, 11 is the mode of the data.

The number which appears maximum times is the mode of the data.

Example 8. The marks obtained in social science by 30 students in annual examination are as follows. Find the mode of the data.
75,35,40,80,65,80,80,90,95,80,65,60,75,80,40,67,70,72,69,78,80,80,65, 75,75,88,93,80, 75,65.

Solution: The data are arranged in ascending order: 35, 40, 40, 60, 65, 65, 65, 65,67,69, 70, 72, 75, 75, 75, 75, 75, 78,80,80,80,80,80,80,80,80,88,90, 93, 95.

In presentation of the data, 40 repeats 2 times, 65 repeats 4 times, 75 repeats 5 times, 80 repeats 8 times and the rest appears once. Hence the mode is 80.
∴ Required mode is 80.

What is the mode of 6,12,7,12,11,12,11,7,11 ?

A) 11 and 7   B) 11 and 12

C) 7 and 12   D) 6 and 7

Solve:

The frequency distribution table of the mean obtained in mathematics by 50 stadenes of a class in as follows.

Class interval	51-56	56-60	61-65	66-70	71-75	76-80
Frequency	6	8	13	10	8	5

(a) Draw a cumulative frequency distribution table.

(b) Find the anithmetic mean from the table

•Draw the histrogram of frequency distribution.

(B) Arithmetic mean table:

Class interval	Class mid-value(x_i)	Frequency (fi)	Fixi
51-55	53	6	318
56-60	58	8	464
61-65	63	13	819
66-70	68	10	680
71-75	10	8	584
76-80	73	5	390

(A) Cumulative frequency table

Class interval	Frequency	Cumulative frequency
51-55	6	6
56-60	8	14
61-65	13	27
66-70	10	37
71-75	8	45
76-80	5	50

$$\therefore \text{average} = \frac{\overset{k}{\underset{i+1}{\Sigma}} f_i x_i}{n} = \frac{3266}{50} = 65.3$$

So, our arithmetic mean is 65.1

51-65 □ 56

56   55.5

C) Table for drawing histogram:

Class Interval	Mid-value	Frequency
51-55	50.5-55.5	6
56-60	55.5-60.5	8
61-65	60.5-65.5	13
66-70	65.5-70.5	10
71-75	70.5-75.5	8
76-80	75.5-80.5	5

In our graph , taking 1 unit of mid values per square unit along the X axis where , the broken segments indicates 0-50.5 and taking each square unit as 1 unit of frequency along Y axis the histogram has been drawn .

→ Mid value Must
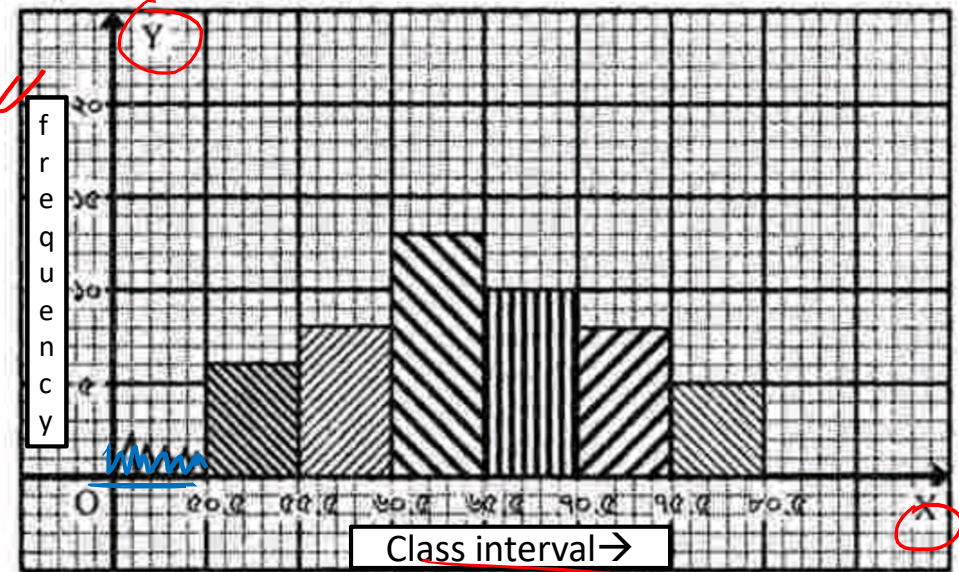
Class interval→

Solution

The frequency table of Daily savings of Class-8 are given below :

Savings (taka)	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	7	9	15	13	11	5

A) Find the median of 3rd interval .   2
B) Find the arithmetic mean from the table.   4

A)Here, 3rd interval is 61-70 ………..$\frac{61+70}{2}$

so, Median = $\frac{}{2}$ = 65.5

So, the median of 3rd interval is 65.5

B) The arithmetic mean table :

Saving(taka) class interval	Mid-value (xi)	Mid-value (xi)	$f_i x_i$
41-50	45.5	7	318.5
51-60	55.5	9	499.5
61-70	65.5	15	982.5
71-80	75.5	13	981.5
81-90	85.5	11	940.5
91-100	95.5	5	477.5
Total		$n = 60$	$\Sigma f_i x_i = 4200$

$$\therefore \text{average} = \frac{\sum_{i=1}^{k} f_i x_i}{n} = \frac{4200}{60} = 70$$

So, Required daily average saving is 70 taka.

C) Table for drawing histogram:

Class Interval	Mid-value	Frequency
41-50	40.5-50.5	7
51-60	50.5-60.5	9
61-70	60.5-70.5	15
71-80	70.5-80.5	13
81-90	80.5-90.5	11

In our graph, taking 2 units of mid values per square unit along the X axis where , the broken segments indicates 0-40.5 and taking each square unit as 1 unit of frequency along Y axis the histogram has been drawn .

Savings(taka
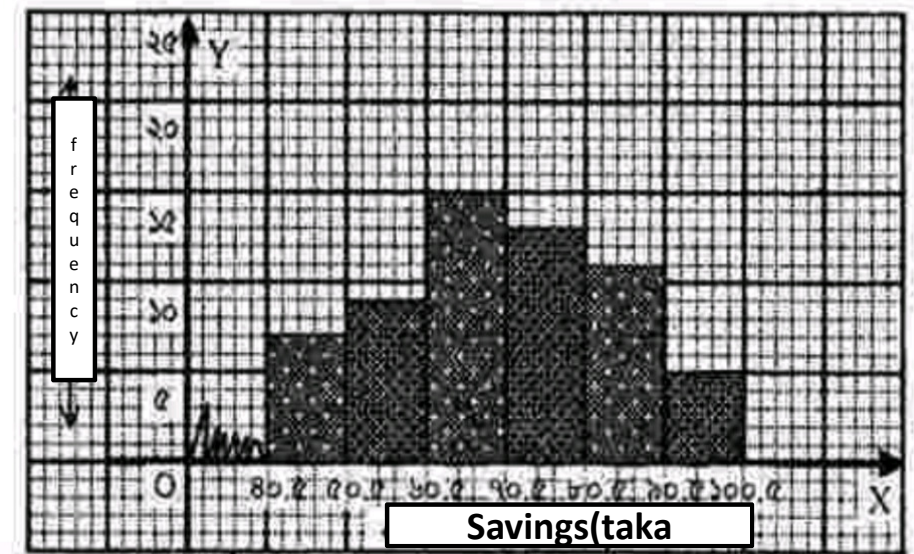
The frequency distribution table of the marks obtained by the students of class 7 in science are given below :

Marks obtained	31-41	41-50	51-60	61-70	71-80	81-90	91-100
Frequency	4	9	12	15	9	8	5

A) Find the value of ~~median~~ of mode interval.   2

B) find the arithmetic mean from the table.   4

C) Draw a histogram from given data.   4

A) Here, Mode interval is 61-70 ………61+70

so, Median =   2   = 65.5

So, the median of mode interval is 65.5

B) The arithmetic mean table :

Marks obtained	Frequency(f_i)	Mid-value(x_i)	$f_i x_i$
31-40	4	35.5	142
41-50	9	45.5	409.5
51-60	10	55.5	555
61-70	15	64.5	982.5
71-80	9	75.5	679.5
81-90	8	85.5	684
91-100	5	95.5	477.5
	$n = 60$		$\Sigma f_i x_i = 3930$

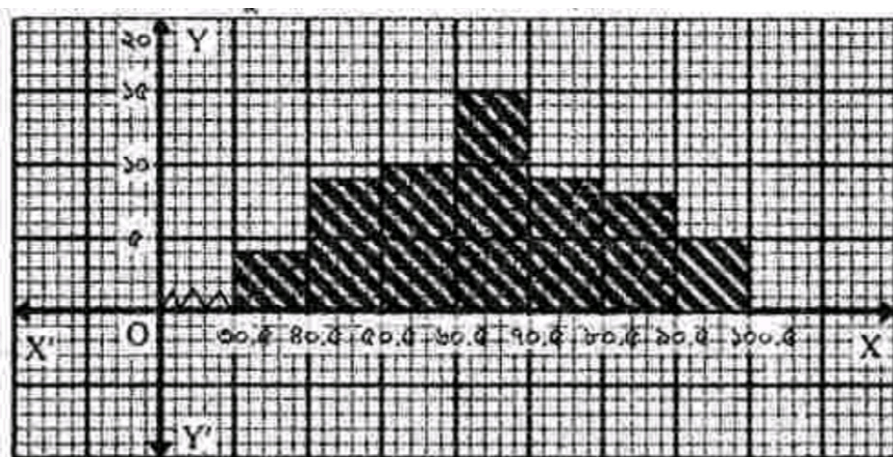$$\therefore \text{average} = \frac{\Sigma f_i x_i}{n} = \frac{3930}{60} = 65.5$$

Required Arithmetic mean is 65.5

(c) Table for drawing histogram:

Class Interval	Discrete class interval	Frequency
31-40	30.5-40.5	4
41-50	40.5-50.5	9
51-60	50.5-60.5	10
61-70	60.5-70.5	15
71-80	70.5-80.5	9
81-90	80.5-90.5	8
91-100	90.5-100.5	5

লেগে থাকো সৎভাবে,
    স্বপ্ন জয় তোমারই হবে

উদ্ভাস-উন্মেষ শিক্ষা পরিবার