CLASS NINE ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : HM-07

Chapter 3.1 : Geometry

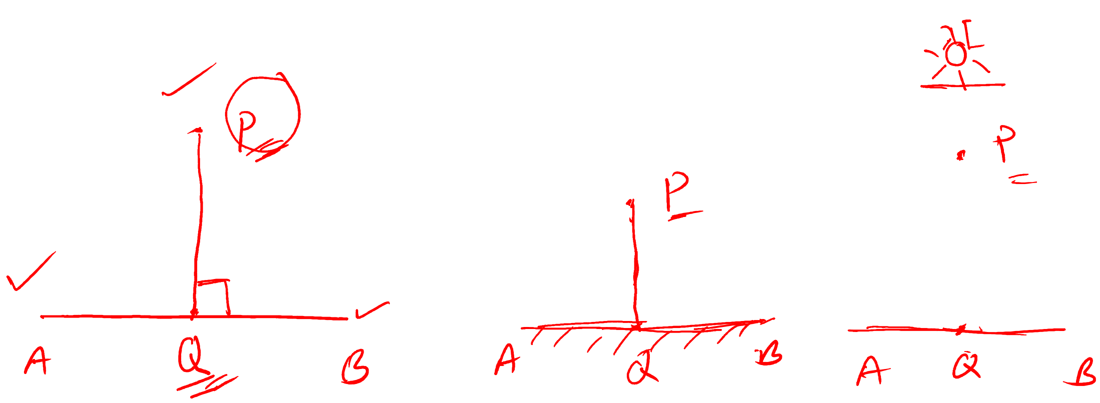
উদ্ভাস

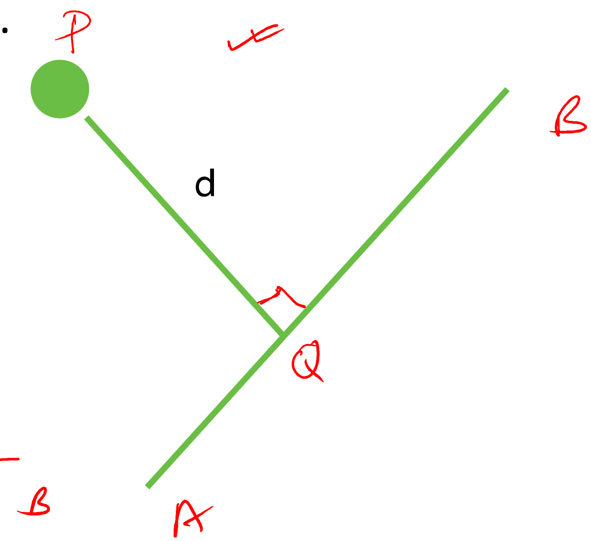
একাডেমিক এন্ড এডমিশন কেয়ার

www.udvash.com

Orthogonal Projection of a point

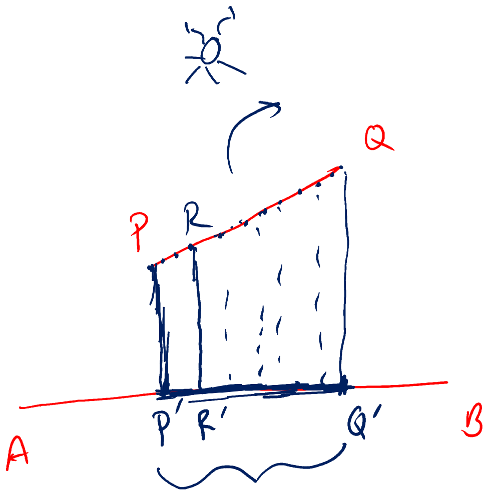
The orthogonal projection of any point on any definite line is the foot of the perpendicular drawn from that point on the line. In easy words, if the line is a mirror, then the reflection of that point on the mirror is the orthogonal projection of that point.

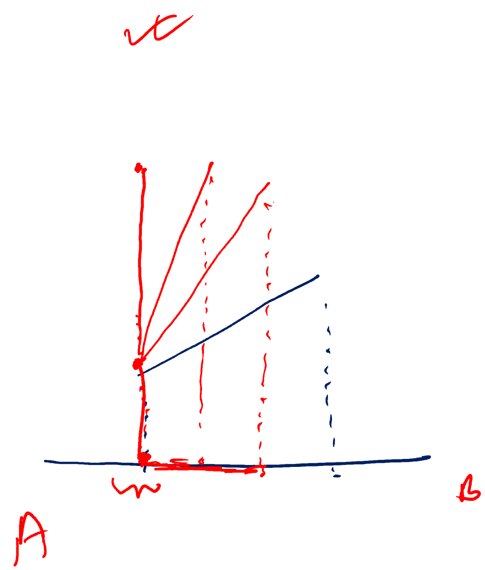


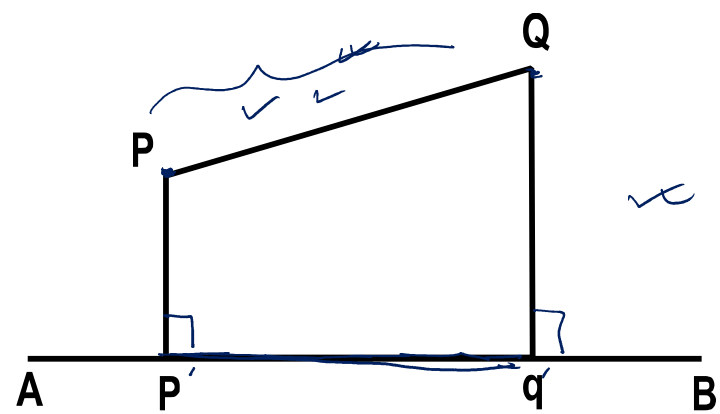


Orthogonal Projection of a line

The orthogonal projection of any line on any definite line is the joining line of the foots of the perpendiculars drawn from the end points of the line on the definite line.







Poll Question 01

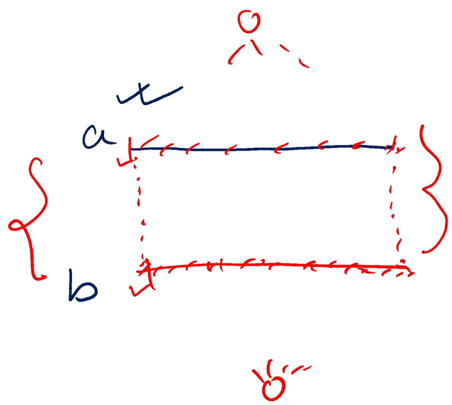
Two equal and parallel lines will have orthogonal project of one on another, which is-

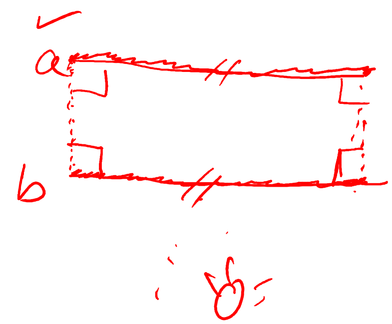
(a) Equal to each other

(b) Greater than second one

(c) Equal to the main lines

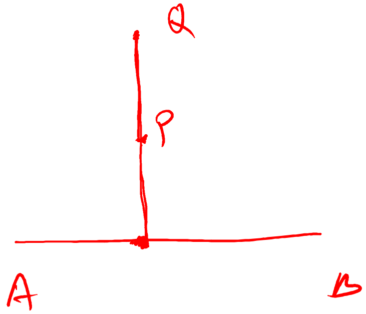
(d) Both a & b

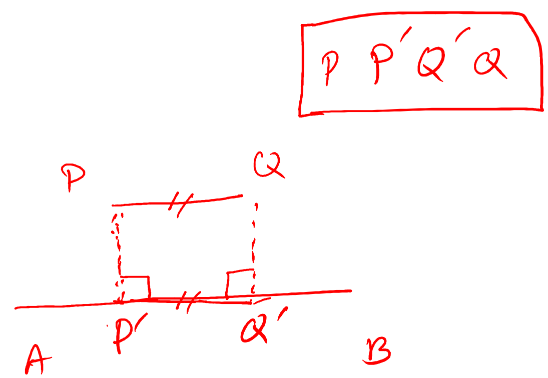




Fun Fact #1

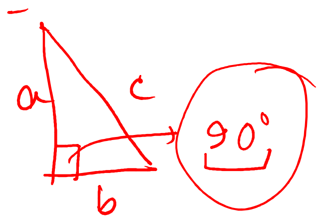
- If the line is perpendicular to the definite line, then the orthogonal projection of first line will be a point
- If the line is parallel to the definite line, then the orthogonal projection of first line will be of the same length as the first line.



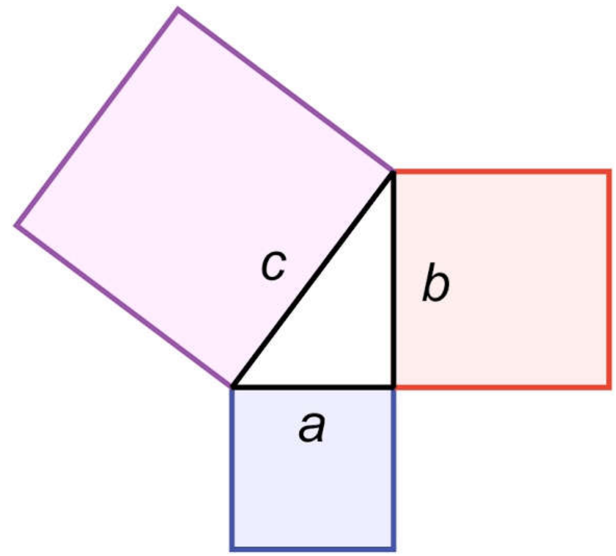


Theorem 1 (Recalling the Theorem of Pythagoras)

Statement: In a right angled triangle, the area of the square drawn on tha hypotenuse is equal to the sum of the areas of the two squares drawn on the other two sides.

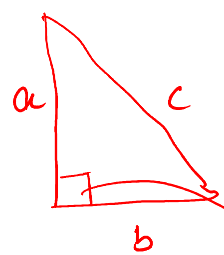


$$\underbrace{a^2+b^2=c^2}_{\checkmark}$$



Theorem 2 (The Converse one)

Statement: If the area of the square on one side of a triangle is equal to the sum of the squares on the other two sides, the angle included in the latter two sides is a right angle.



$$\boxed{a^2 + b^2 = c^2}$$

given

90°

Poll Question 02

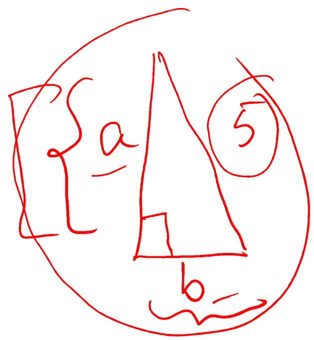
How many right angled triangles can be drawn if the hypotenuse is 5 cm?

$\boxed{\text{(a) 1}}$

(b) 2

$\boxed{\text{(c) Infinite}}$ ✓

(d) None



$a^2+b^2=5^2$

$b^2=5^2-a^2$

1, 2, 2.5, 3, 4, 1.5

$a>0$

$a<5$

$3, \cancel{4}, 5$

$a=1,\quad b=\sqrt{5^2-1^2}=\sqrt{24}$

$a=2,\quad b=\sqrt{5^2-2^2}=\sqrt{21}$

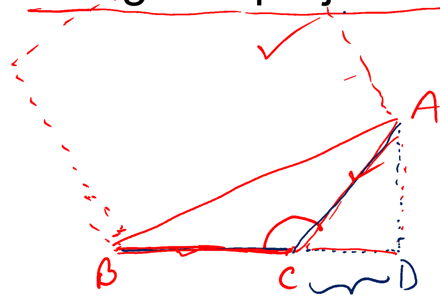
$a=5 \Rightarrow b=\sqrt{5^2-5^2}=0$

Theorem 3 (Theorem related to obtuse angles)

AB

Statement: The area of the square drawn on the opposite side of the obtuse angle of an obtuse angled triangle is equal to the total sum of the two squares drawn on the other two sides and product of twice the area of the rectangle included by anyone of the two other sides and the orthogonal projection of the other side on that side

AC, BC, CD, BC

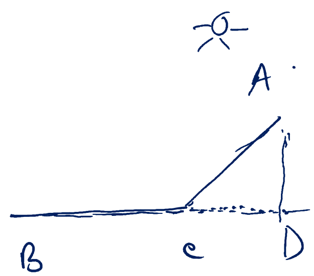


$$AB^2 = AC^2 + BC^2 + 2 \cdot BC \cdot CD$$

x

y

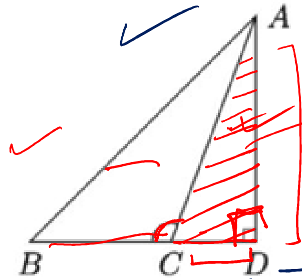
xy



Proof

Special Nomination: Suppose, in the triangle ABC, $\angle BCA$ is an obtuse angle, AB is the opposite side of the obtuse angle and the sides adjacent ot obtuse angle are BC and AC respectively.

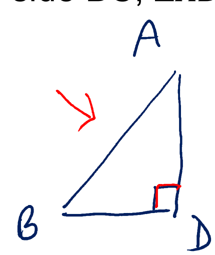
CD is the orthogonal projection of the side AC on the extended side BC (figure below). It is to be proved that,
$AB^2 = AC^2 + BC^2 + 2.BC.CD$



$AC^2 = AD^2 + CD^2$

$BD = BC + CD$

Proof: As CD is the orthogonal projection of the side AC on the extended side BC, ΔABD is a right angled triangle and $\angle ADB$ is a right angle.
So, according to the theorem of Pythagoras
$AB^2 = AD^2 + BD^2$
$= AD^2 + (BC + CD)^2$ [$\therefore BD = BC + CD$]
$= AD^2 + BC^2 + CD^2 + 2.BC.CD$
$\therefore AB^2 = AD^2 + CD^2 + BC^2 + 2.BC.CD$ …………. (1)
Again ΔACD is a right angled triangle and $\angle ADC$ is right angle.
$\therefore AC^2 = AD^2 + CD^2$ ……….. (2)
From the equation (2), putting the value $AD^2 + CD^2 = AC^2$ in equation (1), we get,
$AB^2 = AC^2 + BC^2 + 2.BC.CD$ [Proved]



$AB^2 = AD^2 + BD^2$
$= AD^2 + (CD + BC)^2$
$= AD^2 + CD^2 + BC^2 + 2.BC.CD.$

$\therefore AB^2 = AC^2 + BC^2 + 2.BC.CD$

Theorem 4 (Theorem related to acute angles)

AB

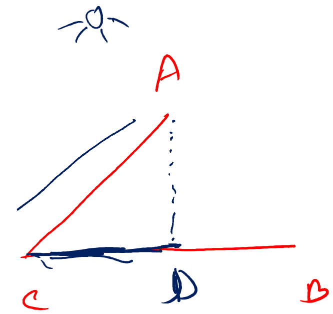
Statement: In any triangle, the area of the square drawn on the opposite side of an acute angle is equal to the squares drawn on the other two sides diminished by twice the area of the rectangle included by anyone of the other sides and the orthogonal projection of the other side on that side.

AC, BC

BC

CD

$$AB^2 = AC^2 + BC^2 - 2\cdot\boxed{BC\cdot CD}$$

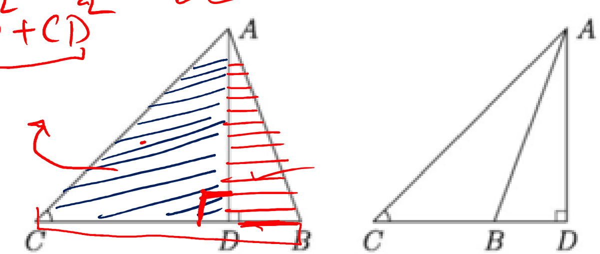


Proof

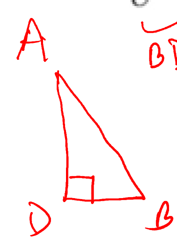
Special Nominatin: In the triangle $\Delta ABC, \angle ACD$ is an acute angle and the opposite side of the acute angle is AB. The other two sides are AC and BC. Suppose, AD is a perpendicular on the side BC (left sided figure below) and the extended side of BC (right sided figure below). So, CD is the orthogonal projection of the side AC on the side BC in the case of both triangles. It is to be proved that ,$AB^2 = AC^2 + BC^2 - 2.BC.CD$ ।

[It is to be noted that here the perpendicular is drawn from A to BC. But similarly, the theorem can be proved by drawing a perpendicular from the point B to AC.]

$AC^2 = AD^2 + CD^2$



$BD = BC - CD$



Proof: $\angle ADB$ is a right angled triangle and $\angle ADB$ is right angle.

$\therefore$ According to the theorem of Pythagoras, $AB^2 = AD^2 + BD^2 \ldots\ldots\ldots\ldots(1)$

In the left sided figure above $BD = BC - CD$ ।

$\therefore BD^2 = (BC - CD)^2 = BC^2 + CD^2 - 2.BC.CD$

In the right sided figure above $BD = CD - BC$ ।

$\therefore BD^2 = (CD - BC)^2 = CD^2 + BC^2 - 2.CD.BC$

Therefore, in both figures, $BD^2 = BC^2 + CD^2 - 2.BC.CD \ldots\ldots\ldots\ldots(2)$

Now from equation (1) and (2) we get,

$AB^2 = AD^2 + BC^2 + CD^2 - 2.BC.CD$

Or, $AB^2 = AD^2 + CD^2 + BC^2 - 2.BC.CD \ldots\ldots\ldots\ldots\ldots(3)$

Again, $\Delta\mathrm{ADC}$ is a right angled triangle and $\angle ADC$ is a right angle.

$\therefore$ According to the theorem of Pythagoras $AC^2 = AD^2 + CD^2 \ldots\ldots\ldots\ldots\ldots\ldots(4)$

from equation (3) and (4) we get,

$AB^2 = AC^2 + BC^2 - 2.BC.CD$ [Proved]

$AB^2 = AD^2 + BD^2$
$= AD^2 + (BC - CD)^2$
$= AD^2 + BC^2 + CD^2 - 2.BC.CD$

$\Rightarrow AB^2 = AC^2 + BC^2 - 2.BC.CD$

Poll Question 03

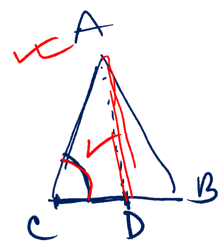
In any triangle ABC, if the angle ACB is an acute angle, then-

(a) $\mathrm{AB}^2 > \mathrm{AC}^2 + \mathrm{BC}^2$

(b) $AB^2 = AC^2 + BC^2$

(c) $AB^2 < AC^2 + BC^2$

a) (d) None



$$AB^2 = AC^2 + BC^2 - 2BC.CD$$

$$AB^2 + 2BC.CD = AC^2 + BC^2$$

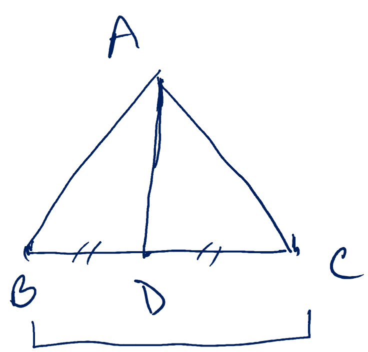
$$AB^2 < AC^2 + BC^2$$

Theorem 5 (Theorem of Apolloneus)

AB, AC

Statement: The sum of the areas of the squares drawn on any two sides of a triangle is equal to twice the sum of area of the squares drawn on the median of the third side and on either half of that side

ABC

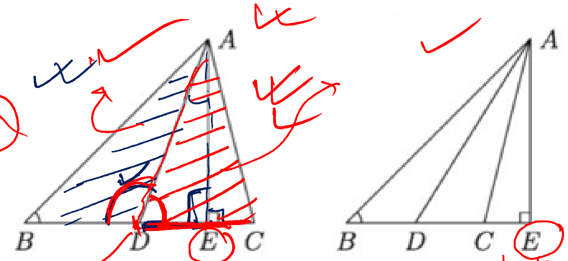


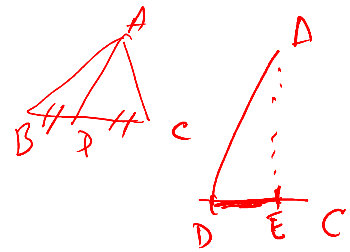
$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

Proof:

Special Nomination: AB, Median of triangle $\Delta ABC,$ bisects the side BC. It is to be proved that,
$AB^2 + AC^2 = 2(AD^2 + BD^2).$

BD = CD





Proof: We draw a perpendicular AE on the side BC (left sided figure above) and on the extended side of BC (right sided figure above). In both figures$\angle$ADB is obtuse angle of ΔABD and DE is the orthogonal projection of the line AD on the extended BD.

As per the extension of the theorem of Pythagoras in the case of the obtuse angle [Theorem 3] we get,

$AB^2 = AD^2 + BD^2 + 2.BD.DE \ldots\ldots\ldots\ldots(1)$

Here, $\angle$ADC is an acute angle ofΔACD and DC (left sided figure above) and DE is the orthogonal projection of the line AD on extended DC (right sided figure above).

As per the extension of the theorem of Pythagoras in the case of the acute angle [Theorem 4] we get,

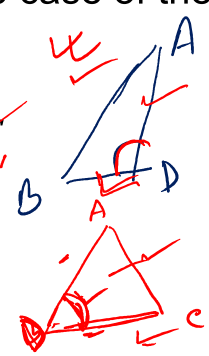
$AC^2 = AD^2 + CD^2 - 2.CD.DE \ldots\ldots\ldots\ldots\ldots(2)$

Now adding the equations (1) and (2) we get,

$AB^2 + AC^2 = 2AD^2 + BD^2 + CD^2 + 2.BD.DE - 2.CD.DE$

$= 2AD^2 + 2BD^2$ [$\therefore BD = CD$]

$\therefore AB^2 + AC^2 = 2\,(AD^2 + BD^2)$ [Proved]



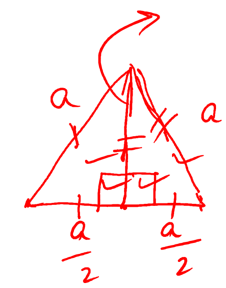
$AB^2 = AD^2 + BD^2 + 2.BD.DE$ — (i)

$AC^2 = AD^2 + CD^2 - 2.CD.DE$ — (ii)

Poll Question 04

367

In an equilateral triangle -

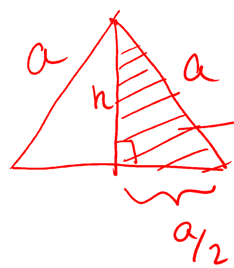


(a) The ratio between median and side $\boxed{\text{is } \sqrt{3}:1}$

(b) Angles are supplementary

(c) Medians are perpendicular bisectors

(d) None



$$a^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$h^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

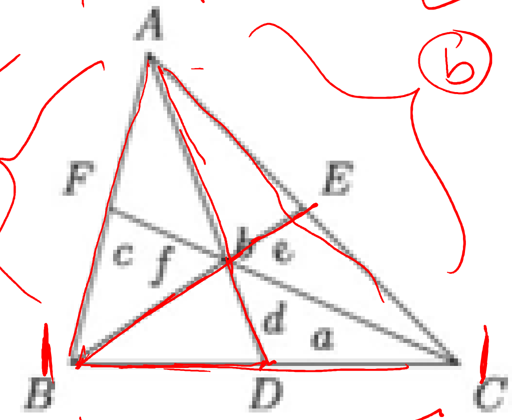
$$h = \frac{\sqrt{3}a}{2}$$

$$\frac{h}{a} = \frac{\frac{\sqrt{3}\cancel{a}}{2}}{\cancel{a}} = \frac{\sqrt{3}}{2}$$

Fun Fact #2

Let, length of the sides of BC, CA and AB of the, ΔABC are a, b, and c respectively. AD, BE and CF are the medians drawn on sides BC, CA and AB and their lengths are d, e and f respectively.

AB, AC



c

b

$AD = d$

$BE = e$

$CF = f$

a

$BD = \frac{1}{2} BC$

$BD = \frac{1}{2} a$

Then from Apollonius's Theorem we get,

$AB^2 + AC^2 = 2(AD^2 + BD^2)$

Or, $c^2 + b^2 = 2\left(d^2 + \left(\frac{a}{2}\right)^2\right)$ $\quad \left[\because BD = \frac{1}{2}a\right]$

Or, $b^2 + c^2 = 2d^2 + 2.\frac{1}{4}a^2$

Or, $b^2 + c^2 = 2d^2 + \frac{a^2}{2}$

Or, $d^2 = \frac{2(b^2+c^2)-a^2}{4}$

Similarly we can get, $e^2 = \frac{2(c^2+a^2)-b^2}{4}$ and $f^2 = \frac{2(a^2+b^2)-c^2}{4}$

Therefore we can say that if the lengths of the sides of any triangle are known, the length of the medians can be known.

Again, $d^2 + e^2 + f^2 = \frac{2(b^2+c^2)-a^2}{4} + \frac{2(c^2+a^2)-b^2}{4} + \frac{2(a^2+b^2)-c^2}{4}$

So, $d^2 + e^2 + f^2 = \frac{3}{4}(a^2 + b^2 + c^2)$

$\therefore 3(a^2 + b^2 + c^2) = 4\,(d^2 + c^2 + f^2)$

$b^2 + c^2 = 2d^2 + \frac{a^2}{2} \Rightarrow 2d^2 = b^2 + c^2 - \frac{a^2}{2}$

$2d^2 = \frac{2(b^2+c^2)-a^2}{2}$

না বুঝে
মুখস্থ করার
অভ্যাস প্রতিভাকে
ধ্বংস করে