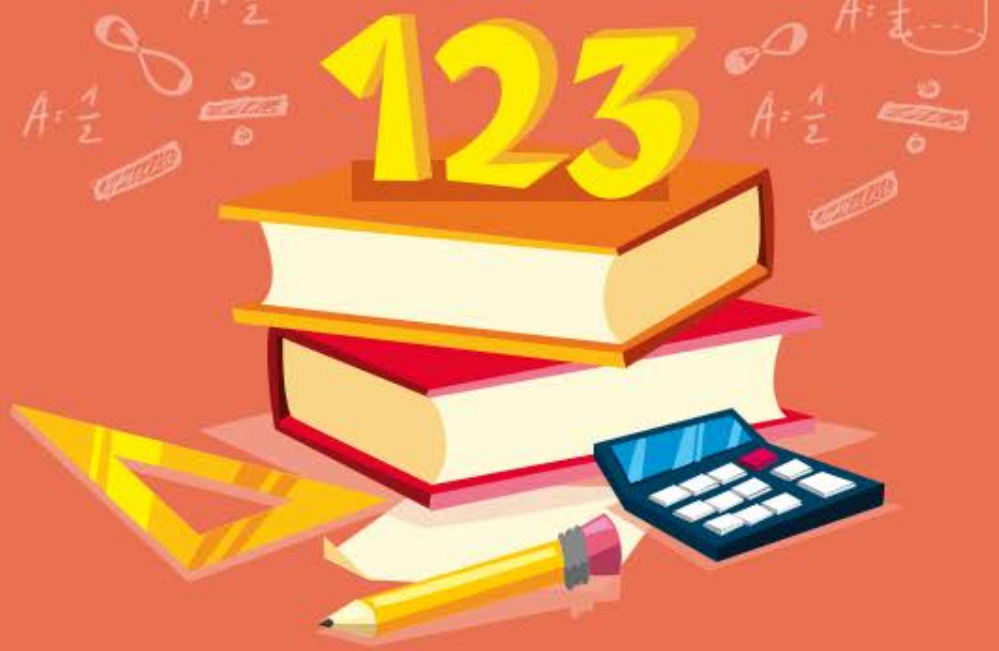
৯ম শ্রেণি একাডেমিক প্রোগ্রাম ২০২০

উচ্চতর গণিত

লেকচার : H.M-27

অধ্যায় ৯.১ : সূচকীয় ও লগারিদমীয় ফাংশন



উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

Exercise 9.1

$a^m \cdot a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

4 (b) প্রমাণ করো, $\frac{a^3+a^{-3}+1}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1} = a^{\frac{3}{2}} + a^{-\frac{3}{2}} - 1$

Sol^n:

1 → $L.H.S.[\text{বামপক্ষ}] = \frac{a^3+a^{-3}+1}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1}$

2 → $= \frac{a^3+a^{-3}+2-1}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1}$

3 → $= \frac{\left(a^{\frac{3}{2}}\right)^2+\left(a^{-\frac{3}{2}}\right)^2+2.a^{\frac{3}{2}}.a^{-\frac{3}{2}}-1}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1}$

4 → $= \frac{\left(a^{\frac{3}{2}}+a^{-\frac{3}{2}}\right)^2-1^2}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1}$

5 → $= \frac{\left(a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1\right)\left(a^{\frac{3}{2}}+a^{-\frac{3}{2}}-1\right)}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1}$

6 → $= a^{\frac{3}{2}} + a^{-\frac{3}{2}} - 1 = R.H.S.[\text{ডানপক্ষ}]$

(i) $a^3 = a^{\frac{3}{2}\times 2} = \left(a^{\frac{3}{2}}\right)^2$

(ii) $a^{\frac{3}{2}}.a^{-\frac{3}{2}} = a^{\frac{3}{2}-\frac{3}{2}} = a^0 = 1$

(iii) $a^2-b^2 = (a+b)(a-b)$

Poll Question 01

কোনটি সঠিক?

(a) $(a+b)^n = a^n + b^n$ ✗

(b) $(a^m)^n = a^{m+n}$ ✗

(c) $a^0 = 1$ ✓

(d) সবগুলো ✗

$(a \cdot b)^n = a^n \cdot b^n$

$(a^m)^n = a^{mn}$

Poll Question 02

$2^x = (2^3)^y$ হলে, নিম্নের কোনটি সঠিক?

(a) $x = 3y$

(b) $x = 3+y$

(c) $x = 3^y$

(d) $x = y^3$

$a^n = a^m$

$n = m$

$2^x = (2^3)^y$

$\Rightarrow 2^x = 2^{3y}$

$\Rightarrow x = 3y$

$a^m \cdot a^n = a^{m+n}$

Exercise 9.1

5 (b) সরল করো, $\frac{a^{\frac{3}{2}}\, a\, b}{ab-b^3} - \frac{\sqrt{a}}{\sqrt{a}-b}$

Sol^n:

$a = \sqrt{a} \cdot \sqrt{a}$

(i) $a^{\frac{3}{2}} = a^{1+\frac{1}{2}}$

$= a^1 \cdot a^{\frac{1}{2}}$

$= a \cdot \sqrt{a}$

(ii) $a - b^2$

$= (\sqrt{a})^2 - b^2$

$= (\sqrt{a}+b)(\sqrt{a}-b)$

(1) $\frac{a^{\frac{3}{2}} + ab}{ab - b^3} - \frac{\sqrt{a}}{\sqrt{a} - b}$

(2) $= \frac{a^{1+\frac{1}{2}} + ab}{b(a - b^2)} - \frac{\sqrt{a}}{\sqrt{a} - b}$

(3) $= \frac{a\sqrt{a} + ab}{b(a - b^2)} - \frac{\sqrt{a}}{\sqrt{a} - b}$

(4) $= \frac{a(\sqrt{a} + b)}{b\{(\sqrt{a})^2 - b^2\}} - \frac{\sqrt{a}}{\sqrt{a} - b}$

(5) $= \frac{a(\sqrt{a} + b)}{b(\sqrt{a} + b)(\sqrt{a} - b)} - \frac{\sqrt{a}}{\sqrt{a} - b}$

(6) $= \frac{a}{b(\sqrt{a} - b)} - \frac{\sqrt{a}}{\sqrt{a} - b}$

(7) $= \frac{a - \sqrt{a}b}{b(\sqrt{a} - b)}$

(8) $= \frac{\sqrt{a}.\sqrt{a} - \sqrt{a}b}{b(\sqrt{a} - b)}$

(9) $= \frac{\sqrt{a}(\sqrt{a} - b)}{b(\sqrt{a} - b)} = \frac{\sqrt{a}}{b}$

5 (d) সরল করো, $\frac{1}{1+a^{-m}b^{n}+a^{-m}c^{p}}+\frac{1}{1+b^{-n}c^{p}+b^{-n}a^{m}}+\frac{1}{1+c^{-p}a^{m}+c^{-p}b^{n}}$

$a^{-n}=\frac{1}{a^{n}}$

$$= \frac{1}{1+\frac{b^n}{a^m}+\frac{c^p}{a^m}} + \frac{1}{1+\frac{c^p}{b^n}+\frac{a^m}{b^n}} + \frac{1}{1+\frac{a^m}{c^p}+\frac{b^n}{c^p}}$$

$$= \frac{1}{\frac{a^m+b^n+c^p}{a^m}} + \frac{1}{\frac{b^n+c^p+a^m}{b^n}} + \frac{1}{\frac{c^p+a^m+b^n}{c^p}}$$

$\frac{1}{\text{লব}}=\frac{\text{হর}}{\text{লব}}$

$$= \frac{a^m}{a^m+b^n+c^p} + \frac{b^n}{a^m+b^n+c^p} + \frac{c^p}{a^m+b^n+c^p}$$

$$= \frac{a^m+b^n+c^p}{a^m+b^n+c^p} = 1$$

Exercise 9.1

$(a^p \cdot b^n)^m = a^{pm} \cdot b^{nm}$

6(a) If $x = a^{q+r}b^p, y = a^{r+p}b^q, z = a^{p+q}b^r$, দেখাও যে, $x^{q-r} \cdot y^{r-p} \cdot z^{p-q} = 1$

Sol^n:

1. $x = a^{q+r}b^p$
2. $or, x^{(q-r)} = (a^{q+r}b^p)^{(q-r)}$
3. $= a^{(q+r)(q-r)}b^{p(q-r)}$
4. $= a^{q^2-r^2}b^{pq-pr}$
5. $Similarly, y^{(r-p)} = a^{r^2-p^2}b^{qr-pq}$
6. $And, z^{(p-q)} = a^{p^2-q^2}b^{pr-qr}$
7. $So, L.H.S. = x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$
   $= a^{q^2-r^2}b^{pq-pr} \cdot a^{r^2-p^2}b^{qr-pq} \cdot a^{p^2-q^2}b^{pr-qr}$
8. $= a^{q^2-r^2+r^2-p^2+p^2-q^2}b^{pq-pr+qr-pq+pr-qr}$
9. $= a^0b^0 = 1 = R.H.S.$

$(a+b)(a-b) = a^2-b^2$

$y^{(r-p)}$, $z^{(p-q)}$

Poll Question 03

$\left(\left(a^{\frac{x}{y}}\right)^{\frac{y}{z}}\right)^{\frac{z}{x}}$ এর সঠিক মান কী?

(a) 0
(b) 1
(c) xyz
(d) a

$$\left(\left(a^{\frac{x}{y}}\right)^{\frac{y}{z}}\right)^{\frac{z}{x}} = a^{\frac{x}{y}\times\frac{y}{z}\times\frac{z}{x}} = a^1 = a$$

Exercise 9.1

6 (b) If $a^p = b, b^q = c, c^r = a,$ দেখাও যে, $pqr = 1$

Sol^n:

1 —— $Now, a^p = b$

2 —— $or, (a^p)^q = b^q$

3 —— $or, a^{pq} = b^q = c$

4 —— $or, (a^{pq})^r = c^r$

5 —— $or, a^{pqr} = c^r = a = a^1$

$so, pqr = 1$

$a^{pq} = b^q$

$\Rightarrow a^{pq} = c$

$(a^{pq})^r = c^r$

$\Rightarrow a^{pqr} = c^r$

$\Rightarrow a^{pqr} = a^1$

$\Rightarrow pqr = 1$

Exercise 9.1

7(d) 7(e) same process 7(g) " "

$a^p \cdot b^p = (ab)^p$

7 (b) If $x = (a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}$, and $a^2 - b^2 = c^3$, দেখাও যে, $x^3 - 3cx - 2a = 0$

Sol^n:

1. $Now, x = (a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}$
2. $or, x^3 = \left\{(a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}\right\}^3$
3. $or, x^3 = (a+b) + (a-b) + 3.(a+b)^{\frac{1}{3}}(a-b)^{\frac{1}{3}}\{(a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}\}$
4. $or, x^3 = 2a + 3.\{(a+b)(a-b)\}^{\frac{1}{3}}.x$
5. $or, x^3 = 2a + 3(a^2-b^2)^{\frac{1}{3}}.x$
6. $or, x^3 - 2a = 3(c^3)^{\frac{1}{3}}.x$
7. $or, x^3 - 2a = 3cx$
8. $or, x^3 - 3cx - 2a = 0$

$x^3 = (a+b)^3$

$\{(a+b)^{\frac{1}{3}}\}^3 = a+b$

$\{(a-b)^{\frac{1}{3}}\}^3 = a-b$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Exercise 9.1

7 (e) *If* $a^2 = b^3$ দেখাও যে, $\left(\frac{a}{b}\right)^{\frac{3}{2}} + \left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{2}} + a^{-\frac{1}{3}}$

Sol^{n}: *Now,* $a^2 = b^3$ ... (1)

$or, a^2.a = b^3.a$ ... (2)

$or, \frac{a^3}{b^3} = a$ ... (3)

$or, \left(\frac{a^3}{b^3}\right)^{\frac{1}{2}} = a^{\frac{1}{2}}$ ... (4)

$or, \left(\frac{a}{b}\right)^{\frac{3}{2}} = a^{\frac{1}{2}}$ ... (1) ... (5)

Again, $a^2 = b^3$ ... (6)

$or, \frac{a^2}{b} = \frac{b^3}{b}$ ... (7)

$or, \frac{1}{b} = \frac{b^2}{a^2}$ ... (8)

$or, \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} = \left(\frac{1}{b}\right)^{\frac{1}{3}}$ ... (9)

$or, \left(\frac{b}{a}\right)^{\frac{2}{3}} = b^{-\frac{1}{3}}$ ... (2) ... (10)

Adding (1) & (2),

$\left(\frac{a}{b}\right)^{\frac{3}{2}} + \left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{2}} + b^{-\frac{1}{3}}$ ... (11)

$\frac{a^3}{b^3} = \left(\frac{a}{b}\right)^3$

$b^{-n} = \frac{1}{b^n}$

$b^{-1} = \frac{1}{b}$

$\frac{a^2}{b} = \frac{b^3}{b} \Rightarrow \frac{a^2}{b} = \frac{b^2.b}{b} \Rightarrow \frac{a^2}{b} = b^2$

$\Rightarrow \frac{1}{b} = \frac{b^2}{a^2}$

7 (g) $If\ a+b+c=0$, দেখাও যে, $\frac{1}{x^{b}+x^{-c}+1}+\frac{1}{x^{c}+x^{-a}+1}+\frac{1}{x^{a}+x^{-b}+1}=1$

$a+b+c=0$
$\Rightarrow a+c=-b$
$a+b+c=0$
$\Rightarrow a=-b-c$

$x^{a}\cdot x^{-a}$
$=x^{a-a}$
$=x^{0}$
$=1$

(1) L.H.S. $=\frac{1}{x^{b}+x^{-c}+1}+\frac{1}{x^{c}+x^{-a}+1}+\frac{1}{x^{a}+x^{-b}+1}$

(2) $=\frac{x^{-b}}{x^{-b}\left(x^{b}+x^{-c}+1\right)}+\frac{x^{a}}{x^{a}\left(x^{c}+x^{-a}+1\right)}+\frac{1}{x^{a}+x^{-b}+1}$

(3) $=\frac{x^{-b}}{x^{-b}\cdot x^{b}+x^{-b}\cdot x^{-c}+x^{-b}}+\frac{x^{a}}{x^{a}\cdot x^{c}+x^{a}\cdot x^{-a}+x^{a}}+\frac{1}{x^{a}+x^{-b}+1}$

(4) $=\frac{x^{-b}}{x^{-b+b}+x^{-b-c}+x^{-b}}+\frac{x^{a}}{x^{a+c}+x^{a-a}+x^{a}}+\frac{1}{x^{a}+x^{-b}+1}$

(5) $=\frac{x^{-b}}{1+x^{a}+x^{-b}}+\frac{x^{a}}{x^{-b}+1+x^{a}}+\frac{1}{x^{a}+x^{-b}+1}=\frac{x^{a}+x^{-b}+1}{x^{a}+x^{-b}+1}=1$

Exercise 9.1

9 (a) সমাধান করো, $3^{2x+2} + 27^{x+1} = 36$

Sol^n:

1. $Now, 3^{2x+2} + 27^{x+1} = 36$
2. $or, 3^{2x+2} + (3^3)^{x+1} = 36$
3. $or, 3^{2x+2} + 3^{3x+3} = 36$
4. $or, 9.3^{2x} + 27.3^{3x} = 36$
5. $or, 9.(3^x)^2 + 27.(3^x)^3 - 36 = 0$
6. $or, 27a^3 + 9a^2 - 36 = 0$
7. $or, (a-1)(27a^2 + 36a + 36) = 0$

$so, a = 1,$ *because the other equation has no real value.*

$so, 3^x = 1 = 3^0$

$so, x = 0$

$3^{2x+2} = 3^{2x} \cdot 3^2 = 3^2 \cdot 3^{2x} = 9 \cdot 3^{2x}$

$3^{3x+3} = 3^{3x} \cdot 3^3 = 3^3 \cdot 3^{3x} = 27 \cdot 3^{3x}$

$3^x = 1 \Rightarrow 3^x = 3^0 \Rightarrow x = 0$

Exercise 9.1

9 (b) সমাধান করো, $5^x + 3^y = 8, 5^{x-1} + 3^{y-1} = 2$

Sol^n:

(1) $Now,\ 5^x + 3^y = 8$

$or, 5^x = 8 - 3^y$

(2) $again, 5^{x-1} + 3^{y-1} = 2$

(4) $or, \frac{5^x}{5} + \frac{3^y}{3} = 2$

$or, \frac{8 - 3^y}{5} + \frac{3^y}{3} = 2$

(5) $or, \frac{(8 - 3^y)3 + 5.3^y}{15} = 2$

(6) $or, 24 + 2.3^y = 30$

(7) $or, 2.3^y = 6$

$or, 3^y = 3 = 3^1$

$so, y = 1$

$and, 5^x = 8 - 3^1 = 5 = 5^1$

$so, x = 1, y = 1$

(3) $a^{-x} = \frac{1}{a^x}$

$5^{-1} = \frac{1}{5}$

$5^{x-1} = 5^x \cdot 5^{-1} = 5^x \cdot \frac{1}{5} = \frac{5^x}{5}$

$(8 - 3^y)3 + 5.3^y$

$\Rightarrow 8 \times 3 - 3^y.3 + 5.3^y$

$\Rightarrow 24 - 3^y(3 - 5)$

$\Rightarrow 24 + 3^y.2$

$5^x = 8 - 3^y$

$\Rightarrow 5^x = 8 - 3^1$

$5^x = 8 - 3$

$5^x = 5$

$\Rightarrow x = 1$

$2.3^y = 6$

$\Rightarrow 2.3^y = 2.3$

$\Rightarrow 3^y = 3$

$\Rightarrow 3^y = 3^1$

$\Rightarrow y = 1$

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে