CLASS 9 ACADEMIC PROGRAM-2020

HIGHER MATH

Lecture : H.M-27

Chapter 9.1 : Exponential and logarithmic functions

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Exercise 9.1

4(b) prove that, $\frac{a^3+a^{-3}+1}{a^{\frac{3}{2}}+a^{-\frac{3}{2}}+1} = a^{\frac{3}{2}} + a^{-\frac{3}{2}} - 1$

$$L.H.S = \frac{a^3+a^{-3}+1}{a^{3/2}+a^{-3/2}+1}$$

$$= \frac{(a^{3/2})^2+(a^{-3/2})^2+1}{a^{3/2}+a^{-3/2}+1}$$

$$= \frac{(a^{3/2})^2+(a^{-3/2})^2+2-1}{a^{3/2}+a^{-3/2}+1}$$

$$= \frac{(a^{3/2})^2+(a^{-3/2})^2+2.a^{3/2}.a^{-3/2}-1}{a^{3/2}+a^{-3/2}+1}$$

$$= \frac{(a^{3/2}+a^{-3/2})^2-1}{a^{3/2}+a^{-3/2}+1}$$

$$= \frac{(a^{3/2}+a^{-3/2}+1)(a^{3/2}+a^{-3/2}-1)}{a^{3/2}+a^{-3/2}+1}$$

$$= a^{3/2}+a^{-3/2}-1$$

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Exercise 9.1

5 (b) Simplify, $\frac{a^{\frac{3}{2}}+ab}{ab-b^3} - \frac{\sqrt{a}}{\sqrt{a-b}}$ $\quad \sqrt{a}-b$

$$= \frac{a^{1+\frac{1}{2}}+ab}{ab-b^3} - \frac{\sqrt{a}}{\sqrt{a}-b}$$

$$= \frac{a^{1}\cdot a^{1/2}+ab}{ab-b^3} - \frac{\sqrt{a}}{\sqrt{a}-b}$$

$$= \frac{a\sqrt{a}+ab}{ab-b^3} - \frac{\sqrt{a}}{\sqrt{a}-b}$$

$$= \frac{a(\sqrt{a}+b)}{b(a-b^2)} - \frac{\sqrt{a}}{\sqrt{a}-b}$$

$$= \frac{a(\sqrt{a}+b)}{b(\sqrt{a}+b)(\sqrt{a}-b)} - \frac{\sqrt{a}}{\sqrt{a}-b}$$

$$= \frac{a}{b(\sqrt{a}-b)} - \frac{\sqrt{a}}{\sqrt{a}-b}$$

$$= \frac{a-b\sqrt{a}}{b(\sqrt{a}-b)}$$

$$= \frac{\sqrt{a}(\sqrt{a}-b)}{b(\sqrt{a}-b)} = \frac{\sqrt{a}}{b}$$

Exercise 9.1

$a^m \cdot a^{-m} = 1$

$a^m = \frac{1}{a^{-m}}$

5 (d) Simplify, $\frac{1}{1+a^{-m}b^n+a^{-m}c^p} + \frac{1}{1+b^{-n}c^p+b^{-n}a^m} + \frac{1}{1+c^{-p}a^m+c^{-p}b^n}$

$$\frac{1}{1+a^{-m}b^n+a^{-m}c^p} = \frac{1}{a^m \cdot a^{-m}+a^{-m}b^n+a^{-m}\cdot c^p} = \frac{1}{a^{-m}(a^m+b^n+c^p)} = \frac{a^m}{a^m+b^n+c^p} \quad \text{--- (i)}$$

$$\frac{1}{1+b^{-n}c^p+b^{-n}a^m} = \frac{1}{b^n\cdot b^{-n}+b^{-n}c^p+b^{-n}a^m} = \frac{1}{b^{-n}(a^m+b^n+c^p)} = \frac{b^n}{a^m+b^n+c^p} \quad \text{--- (ii)}$$

$$\frac{1}{1+c^{-p}a^m+c^{-p}b^n} = \frac{c^p}{a^m+b^n+c^p} \quad \text{--- (iii)}$$

$$(i)+(ii)+(iii) \Rightarrow \frac{1}{1+a^{-m}b^n+a^{-m}c^p} + \frac{1}{1+b^{-n}c^p+b^{-n}a^m} + \frac{1}{1+c^{-p}a^m+c^{-p}b^n} = \frac{a^m+b^n+c^p}{a^m+b^n+c^p}$$

$$= 1$$

Exercise 9.1

5(f) Simplify, $\frac{\left(a^2-b^{-2}\right)^a\left(a-b^{-1}\right)^{b-a}}{(b^2-a^{-2})b\ (b+a^{-1})^{a-b}}$

Poll Question 01

which one is correct?

(a) $(a+b)^n = a^n + b^n$

(b) $(a^m)^n = a^{m+n}$

(c) $a^0 = 1$

(d) all

Exercise 9.1

6 (a) If $x = a^{q+r} b^p, y = a^{r+p}\ b^q, z = a^{p+q} b^r$, then show that, $x^{q-r} . y^{r-r} . z^{p-q} = 1$

Exercise 9.1

6 (b) If $a^p = b, b^q = c, c^r = a,$ then show that, $pqr = 1$

$a^p = b$

$\Rightarrow (a^p)^q = b^q$

$\Rightarrow a^{pq} = b^q$

$\Rightarrow a^{pq} = c$

$\Rightarrow (a^{pq})^r = c^r$

$\Rightarrow a^{pqr} = a^1 \Rightarrow pqr = 1$

Exercise 9.1

7 (a) If $x^3\sqrt{a} + y^3\sqrt{b} + z^3\sqrt{c} = 0, and\ a^2 = bc,$ then show that, $ax^3 + by^3 + cz^3 = 3axyz$

Exercise 9.1

7 (b) If $x = (a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}$, *and* $a^2 - b^2 = c^3$, *then show that,* $x^3 - 3cx - 2a = 0$

$\Rightarrow x = (a+b)^{1/3} + (a-b)^{1/3}$

$\Rightarrow x^3 = \left\{(a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}\right\}^3$; $[(a+b)^3 = a^3 + b^3 + 3ab(a+b)]$

$\Rightarrow x^3 = \left\{(a+b)^{\frac{1}{3}}\right\}^3 + \left\{(a-b)^{\frac{1}{3}}\right\}^3 + 3(a+b)^{\frac{1}{3}} \cdot (a-b)^{\frac{1}{3}}\left\{(a+b)^{\frac{1}{3}} + (a-b)^{\frac{1}{3}}\right\}$

$\Rightarrow x^3 = (a+b) + (a-b) + 3\{(a+b)(a-b)\}^{\frac{1}{3}} x$

$\Rightarrow x^3 = 2a + 3(a^2-b^2)^{\frac{1}{3}} x$

$\Rightarrow x^3 = 2a + 3(c^3)^{\frac{1}{3}} x$

$\Rightarrow x^3 = 2a + 3cx$

$\Rightarrow x^3 - 3cx - 2a = 0$

Poll Question 02

If $2^x = (2^3)^y$, then which one is correct?

(a) $x = 3y$

(b) $x = 3+y$

(c) $x = 3^y$

(d) $x = y^3$

$2^x = (2^3)^y$

$\Rightarrow 2^x = 2^{3y}$

$x = 3y$

Exercise 9.1

7 (c) $If\ a = 2^{\frac{1}{3}} + 2^{-\frac{1}{3}}, then\ show\ that, 2a^3 - 6a = 5$

Exercise 9.1

$(a^m)^n = a^{mn}$

$\left(3^{\frac{1}{3}}\right)^2 = 3^{2/3}$

$3^{1/3} \cdot 3^{-1/3} = 1$

$a^m \cdot a^{-m} = 1$

7 (d) $If\ a^2 + 2 = 3^{\frac{2}{3}} + 3^{-\frac{2}{3}}, then\ show\ that, 3a^3 + 9a = 8$

$a^2 = 3^{2/3} + 3^{-2/3} - 2.1$

$\Rightarrow a^2 = \left(3^{1/3}\right)^2 + \left(3^{-1/3}\right)^2 - 2 \cdot 3^{1/3} \cdot 3^{-1/3}$

$\Rightarrow a^2 = \left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)^2$

$\Rightarrow a = 3^{1/3} - 3^{-1/3}$

$\Rightarrow a^3 = \left\{3^{1/3} - 3^{-1/3}\right\}^3$

$\Rightarrow a^3 = \left(3^{1/3}\right)^3 - \left(3^{-1/3}\right)^3 - 3 \cdot 3^{1/3} \cdot 3^{-1/3}\left(3^{\frac{1}{3}} - 3^{-\frac{1}{3}}\right)$

$\Rightarrow a^3 = 3 - 3^{-1} - 3 \cdot 1 \cdot a$

$\Rightarrow a^3 = 3 - \frac{1}{3} - 3a$

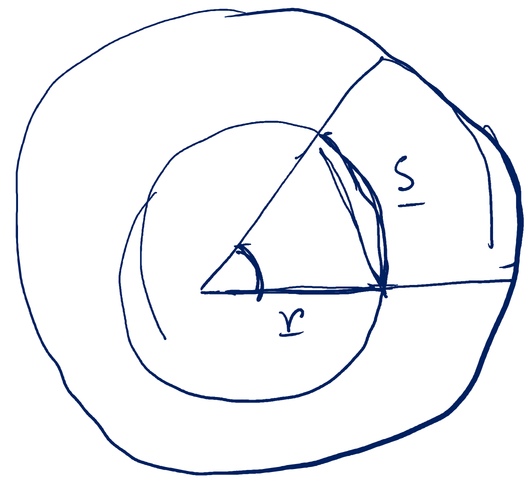
$\Rightarrow a^3 = \frac{9 - 1 - 9a}{3}$

$\Rightarrow 3a^3 = 8 - 9a$

$\Rightarrow 3a^3 + 9a = 8$

Exercise 9.1

7(e) $If\ a^2 = b^3$ than show that, $\left(\frac{a}{b}\right)^{\frac{3}{2}} + \left(\frac{b}{a}\right)^{\frac{2}{3}} = a^{\frac{1}{2}} + a^{-\frac{1}{3}}$



Exercise 9.1

7 (g) $If\ a + b + c = 0,$ then show that, $\frac{1}{x^b+x^{-c}+1} + \frac{1}{x^c+x^{-a}+1} + \frac{1}{x^a+x^{-b}+1}$

Exercise 9.1

8 (b) $If x^a = y^b = z^c$ and $xyz = 1, then$ $find$ the $value$ of $ab + bc + ca$

Poll Question 03

$$\left(\left(a^{\frac{x}{y}}\right)^{\frac{y}{z}}\right)^{\frac{z}{x}} = ?$$

(a) 0
(b) 1
(c) xyz
(d) a ✓

Let, $a^{x/y} = b$

$$\left(\left(a^{\frac{x}{y}}\right)^{\frac{y}{z}}\right)^{\frac{z}{x}} = \left(b^{y/z}\right)^{z/x}$$

$$= b^{\frac{y}{x}}$$

$$= \left(a^{\frac{x}{y}}\right)^{\frac{y}{x}}$$

$$= a^1 = a$$

Exercise 9.1

8 (c) *If* $9^x = 27^y$*, then find the value of* $\frac{x}{y}$

$9^x = 27^y$

$\Rightarrow (3^2)^x = (3^3)^y$

$\Rightarrow 3^{2x} = 3^{3y}$

$\therefore 2x = 3y$

$\frac{x}{y} = \frac{3}{2}$

Exercise 9.1

$a^{m+n} = a^m \cdot a^n$

$3^{2x+2} = 3^{2x} \cdot 3^2$

9 (a) *Solve,* $3^{2x+2} + 27^{x+1} = 36$

$\Rightarrow 3^{2x} \cdot 3^2 + (3^3)^{x+1} = 36$

$\Rightarrow 3^{2x} \cdot 9 + 3^{3x+3} = 36$

$\Rightarrow 9 \cdot 3^{2x} + 3^{3x} \cdot 3^3 = 36$

$\Rightarrow 9 \cdot 3^{2x} + 27 \cdot 3^{3x} = 36$

$\Rightarrow 9 \cdot (3^x)^2 + 27 \cdot (3^x)^3 = 36$

Let, $3^x = a$

$\Rightarrow 9a^2 + 27a^3 = 36$

$\Rightarrow a^2 + 3a^3 = 4$

$\Rightarrow 3a^3 + a^2 - 4 = 0$

$\Rightarrow 3a^3 - 3a^2 + 4a^2 - 4 = 0$

$\Rightarrow 3a^2(a-1) + 4(a^2-1) = 0$

$\Rightarrow 3a^2(a-1) + 4(a+1)(a-1) = 0$

$\Rightarrow (a-1)(3a^2 + 4a + 4) = 0$

if, $(a-1) = 0$

$\Rightarrow a = 1$

$\Rightarrow 3^x = 1$

$\Rightarrow x = 0$

Exercise 9.1

9 (b) $Solve, 5^{x} + 3y = 8, 5^{x-1} + 3^{y-1} = 2$

Exercise 9.1

9 (c) $Solve, 4^{3y-2} = 16^{x+2y} = 9^{2x+1}$

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