CLASS 9 ACADEMIC PROGRAM-2020

GENERAL MATH

Lecture : M-19

Chapter 8 : Circle

Exercise 8.1 (5)

(i) $AE = DE$
(ii) $BE = CE$ } Proof

Soln:

$\triangle MEO$ & $\triangle NEO$

$OM = ON$

$OE = OE$

$\angle OME = \angle ONE$; [90°]

$\triangle MEO \cong \triangle NEO$

$ME = NE$

$OM \perp AB$; $ON \perp CD$

$AM = BM$; $CN = DN$

Ques $AB = CD$

$\therefore \frac{1}{2}AB = \frac{1}{2}CD$

$\therefore AM = DN$

$\therefore AM + ME = DN + NE$

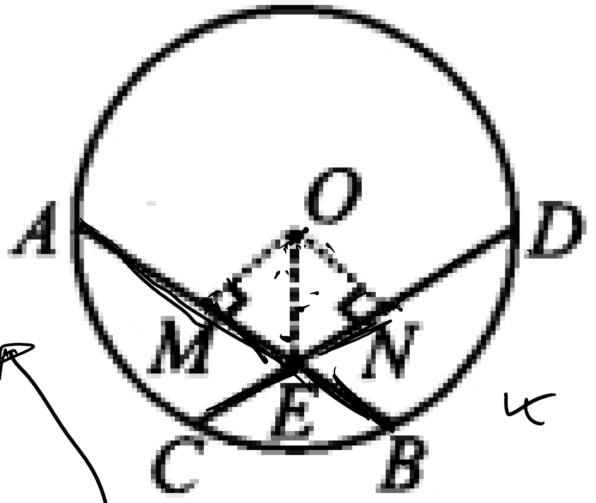
$\therefore AE = DE$

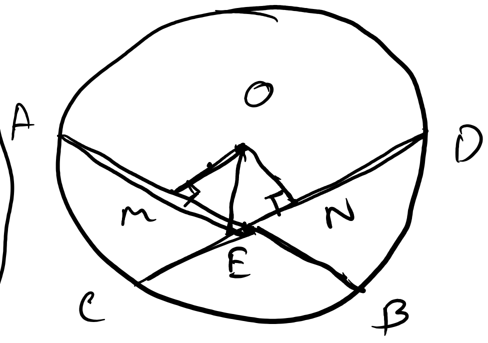
$AB = CD$

$\therefore AB - AE = CD - DE$

$\therefore BE = CE$

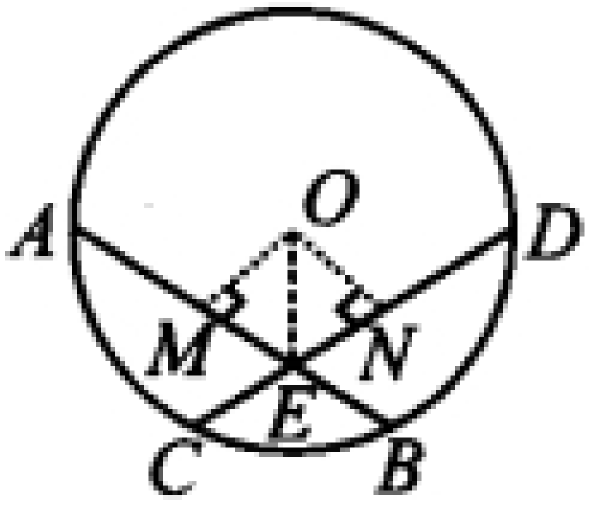
[Proved]





$AB = CD$

Exercise 8.1 (5)



Let, in circle $ABCD$, two equal chords AB & CD intersects each other at the point E. We have to prove that $AE = DE$ & $CE = BE$

Construction: Connect with E, and draw OM perpendicular on AB and ON perpendicular on CD. Proof: In the triangle ΔOME & ΔONE,

OM = ON [equal chords are equidistant from the center] and OE is a common side

& $\angle OME = \angle ONE$ [OM and ON are perpendicular bisectors]

So, ΔOME & ΔONE are congruent.

So, $ME = NE$

And we know, $AB = CD$

So, AM = DN [M, N are mid points of AB & CD respectively]

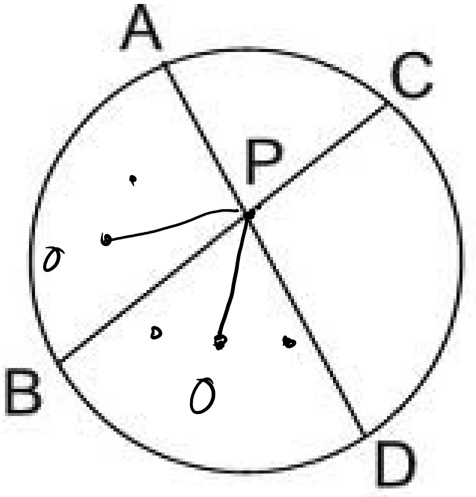
So, $AM + ME = DN + NE$

$Or, AE = DE$

And, CE = CD - DE = AB - AE = BE

Or, CE = BE

Exercise 8.1 (12)



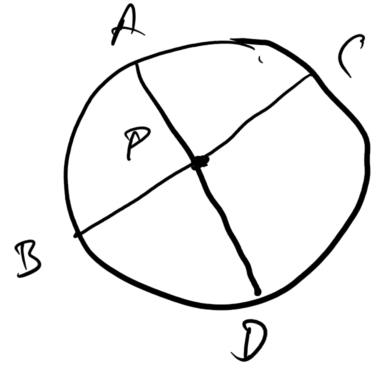
Soln: O, is the center of the circle

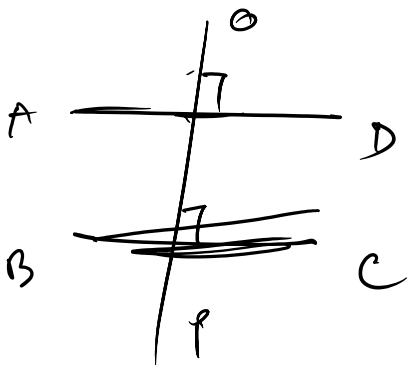
P is the middle point of AD and BC

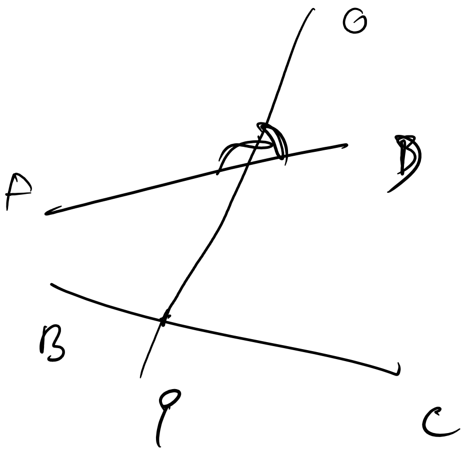
If $OP \perp AD$ and $OP \perp BC$, $AD \parallel BC$ ✗

$$OP \begin{cases} \to AD \\ \to BC \end{cases}$$

AD and BC can't be parallel because they bisect each other.







OP can't be a perpendicular on both AD and BC.

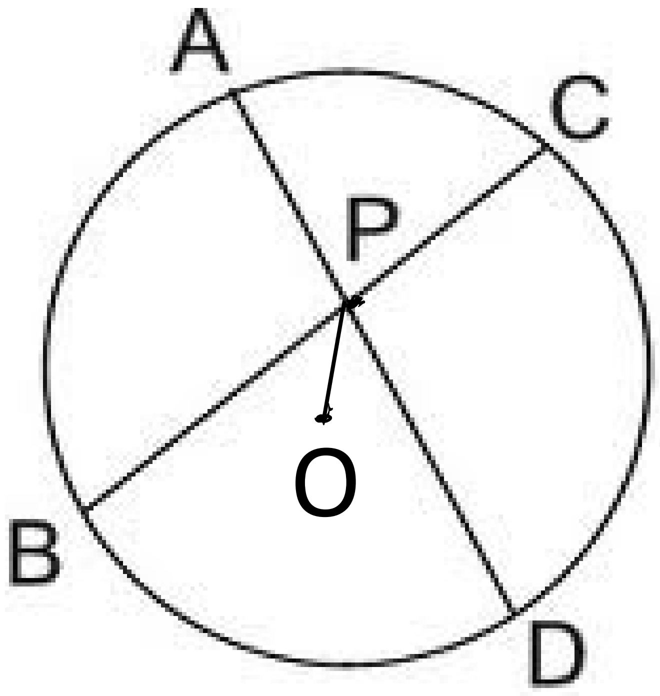
$AP = PD$

$BP = CP$

So, therefore, O is not the center of the circle.

$\therefore$ P is the centre of the circle.

Exercise 8.1 (12)



Let, in circle ABCD, two chords AD & BC bisects each other at the point P. We have to prove that, P is the center of circle ABCD

Construction: Let's assume, O is the center of the circle of the circle.

Proof: As, P is the mid point of AD, and O is the center of the circle.

Then, OP is perpendicular on AD [The perpendicular line drawn on any chord from the center bisects the chord]

Similarly, OP is perpendicular on BC also.

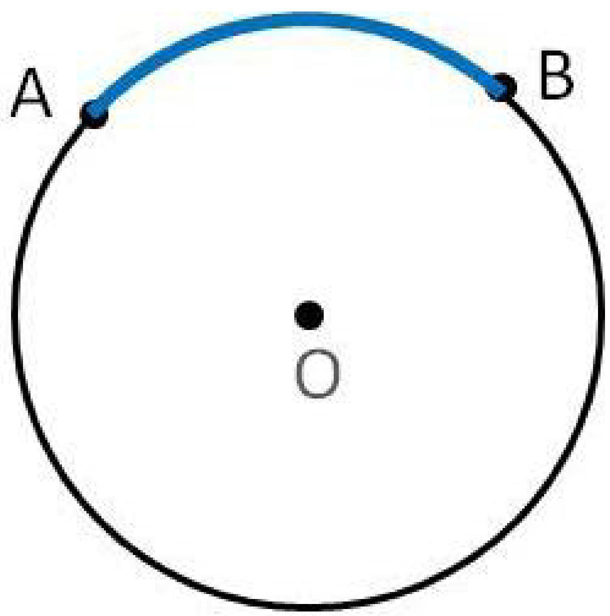
But, AD & BC are two bisecting lines, hence not parallel to each other.

So, OP cannot be perpendicular on both AD & BC.

So, O & P are not different points.

So, P is the center of the circle.

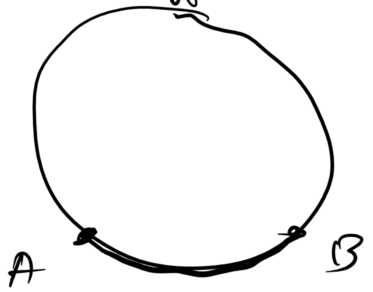
Arc Of A Circle



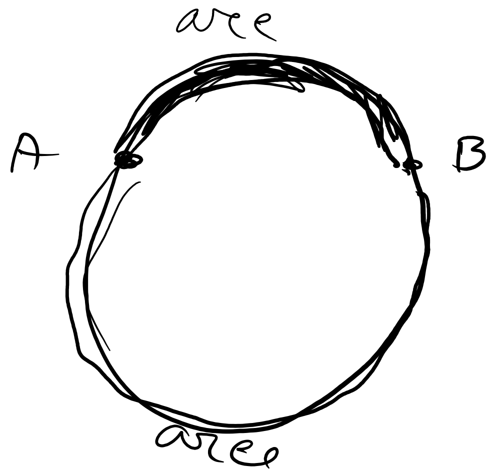
An arc is a piece of the circle between any two points of the circle. It means if the circle is like a road, then any part of that road is an arc.

By cutting, the circle is divided into two arcs. The bigger one is called the major arc, and the smaller one is called the minor arc.

bigger arc → major arc

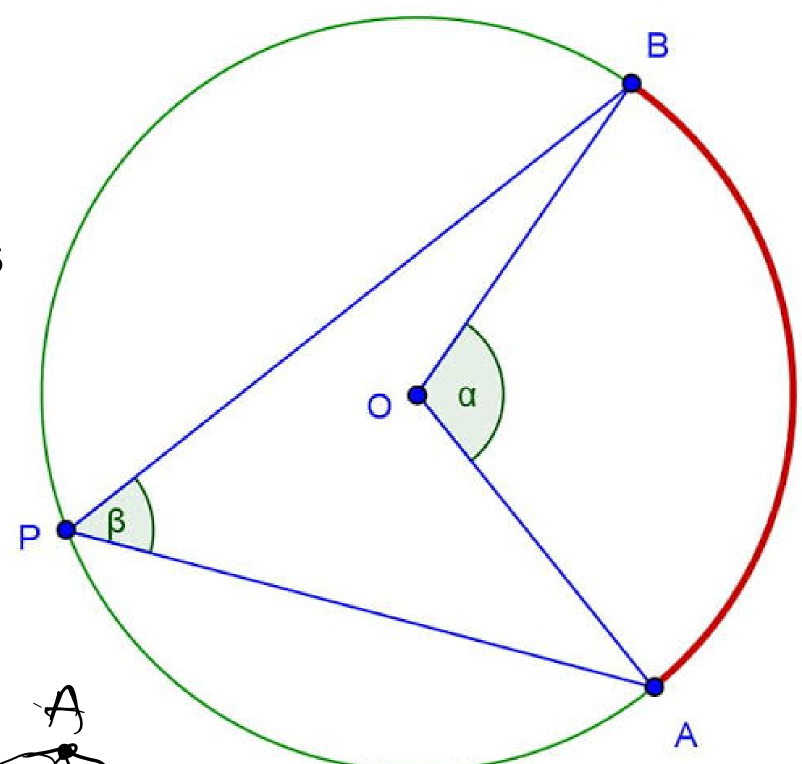


small arc → minor arc



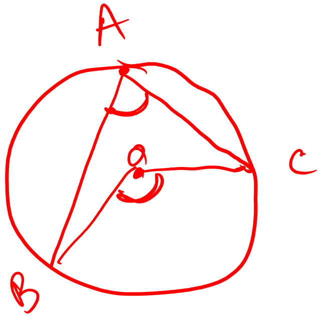
Inscribed Angle

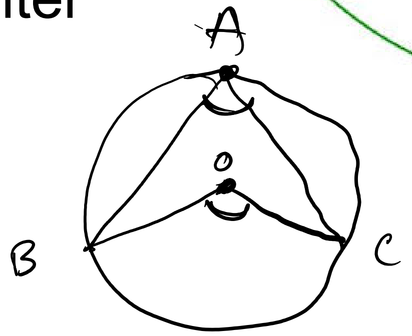
If two chords of a circle meet at a point on the circle, then the angle formed between these chords is called inscribed angle or angle inscribed in the circle



Center Angle

The angle with vertex at the center of the circle is called the center angle or the angle at the center





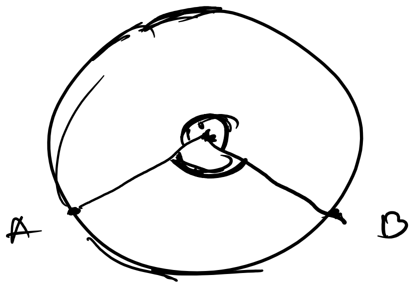
$\angle BAC \rightarrow$ inscribed angle

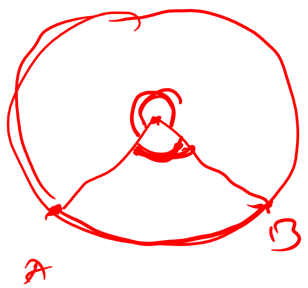
$\angle BOC \rightarrow$ centre angle

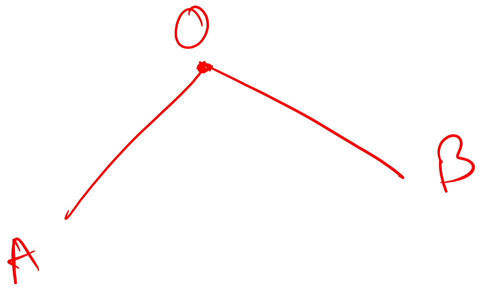
Poll Question- 1

What is the relations between the center angles on the major arc and minor arc of a circle?

(a) Complementary to each other
(b) Supplementary to each other
(c) Summation of them is 360°
(d) None







Theorem 20

The angle subtended by the same arc at the center is double of the angle subtended by it at any point on the remaining part of the circle.

Theorem 21

Inscribed

Angles in a circle standing on the same arc is equal.

Theorem 22

The angle inscribed in a semi-circle is a right angle.

Proof : Theorem 20

$$\boxed{\angle BOC = 2\angle BAC}$$

Soln:

ΔOAB,

$OA = OB$; [radius]

$\angle OBA = \angle OAB$

ΔOAB,

external $\angle BOD = \angle OAB + \angle OBA$

$= \angle OAB + \angle OAB$

$\therefore \angle BOD = 2\angle OAB$ — (I)

Similarly, $\angle COD = 2\angle OAC$ — (II)

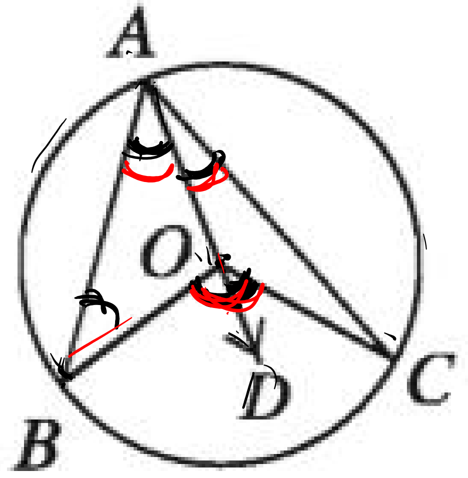
(I) + (II)

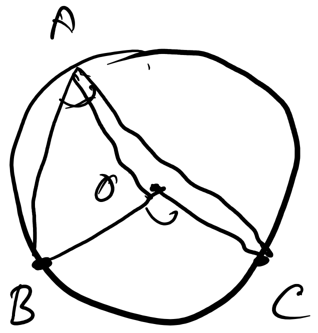
$\angle BOD + \angle COD$

$= 2\angle OAB + 2\angle OAC$

or, $\angle BOC = 2(\angle OAB + \angle OAC)$

$\therefore \angle BOC = 2\angle BAC$





$\angle BAC \rightarrow$ inscribed angle

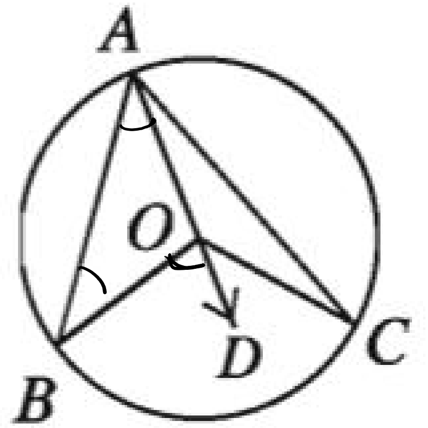
$\angle BOC \rightarrow$ centre angle

Proof (Theorem 20)

Given an arc BC of a circle subtending angles $\angle BOC$ at the center OI and $\angle BAC$ at a point A of the circle ABC.

We need to prove that $\angle BOC = 2\angle BAC$

Drawing: Suppose, the line segment AC dose not pass through the center. In the case, draw a line segment AD at A passing through the center.



Proof:

Step 1. In ΔAOB the external angle $\angle BOD = \angle BAO + \angle ABO$ [∴ An exterior angle of a triangle is equal to the sum of the two interior opposite angles]

Step 2. In $\Delta AOB, OA = OB$ [∴ Badius of a circle]

Therefore, $\angle BAO = \angle ABO$ [∴ Base angles of an isosceles triangle are equal]

Step 3. From steps (1) and (2), $\angle BOD = 2\angle BAo$

Step 4. Similarly, In ΔAOC $\angle COD = 2\angle CAO$

Step 5. From steps (3) and (4),

$\angle BOD + \angle COD = 2\angle BAO + 2\angle CAO$ [by adding]

This is the same as $\angle BOC = 2\angle BAC$ (Proved)

Proof : Theorem 21

$\angle BAD = \angle BED$

Soln:

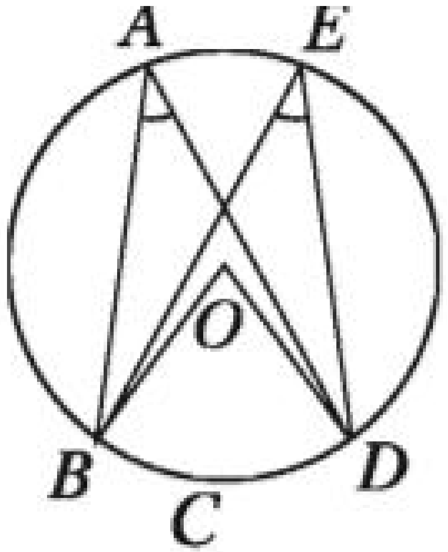
as of Theorem 20,

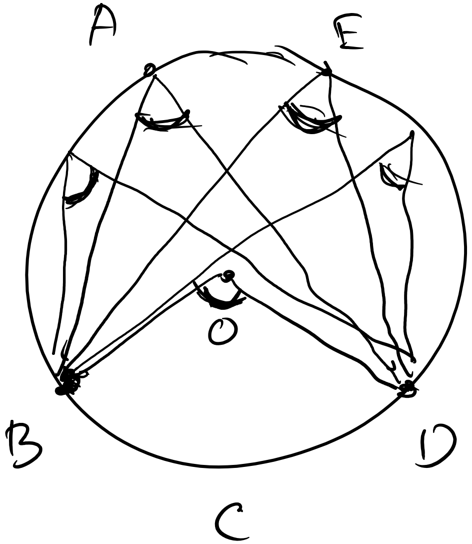
centre $\angle BOD = 2\angle BAD$

" $\angle BOD = 2\angle BED$

$\cancel{2}\angle BAD = \cancel{2}\angle BED$

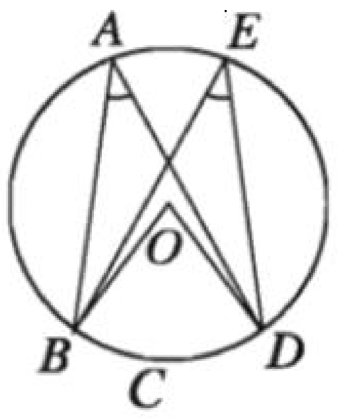
$\therefore \angle BAD = \angle BED$





Proof (Theorem 21)

Let, O be center of a circle and standing on the arc BCD, $\angle BAD$ and $\angle BED$ be the two angles is the circle. We need to prove that $\angle BAD = \angle BED$.



Drawing: Join O, B and O, D.

Proof:

Step 1. The arc BCD subtends an angle $\angle BOD$ at the center O.

Therefore, $\angle BOD = 2\angle BAD$ and $\angle BOD = 2\angle BED$ [$\because$ The angle subtended by an arc at the center is double of the angle subtended on the circle]

$\therefore 2\angle BAD = 2\angle BED$

or, $\angle BAD = \angle BED$ (Proved)

Poll Question- 2

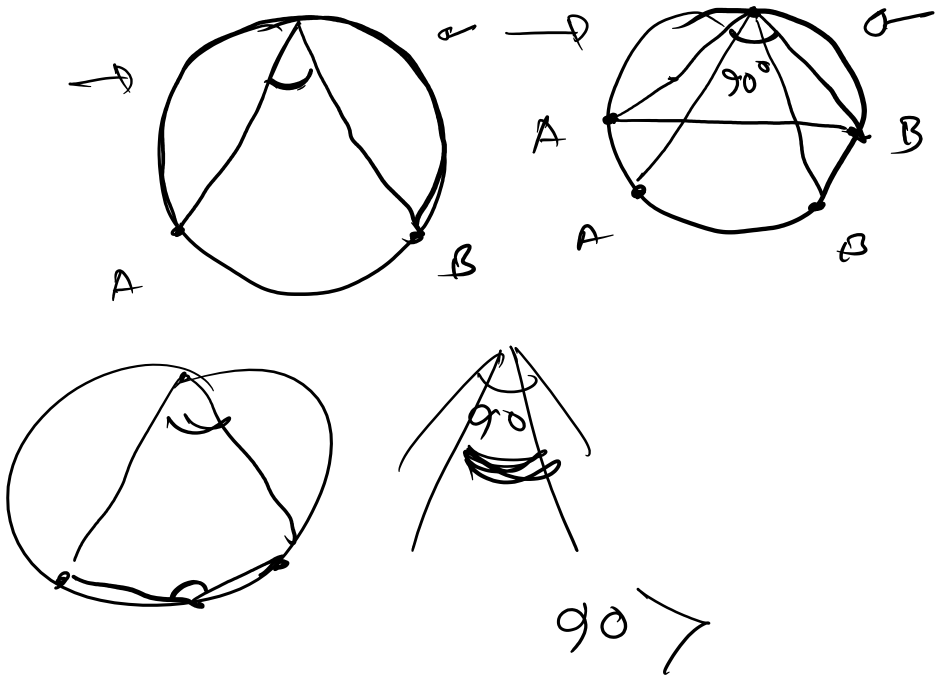
The angle inscribed in a major arc is-

(a) An acute angle

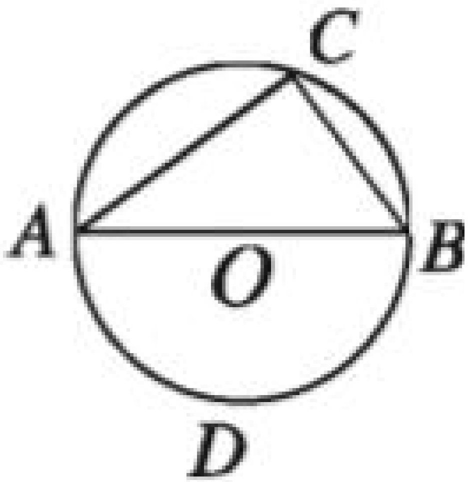
(b) An obtuse angle

(c) A right angle

(d) A complementary angle



Proof : Theorem 22

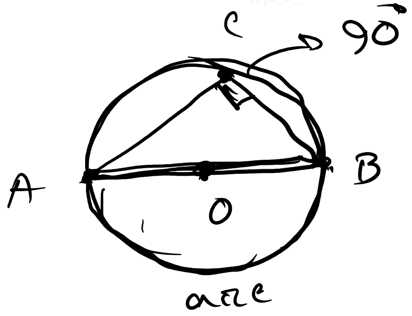


$\angle ACB = 90^\circ$

Soln: in circle ACB

the centre angle on AB arc is

$\angle AOB = 180^\circ$



So, as of theorem 20,

centre $\angle AOB = 2\angle ACB$

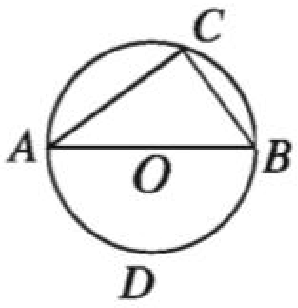
$180^\circ = 2\angle ACB$

$\therefore \boxed{\angle ACB = \frac{180}{2} = 90^\circ}$

Proof (Theorem 22)

Let, AB be a diameter of circle with center at O and $\angle ACB$ is the angle subtended by a semi-circle. It is to be proved that $\angle ACB$ is a right angle.

Drawing: Take a point D on the circle on the opposite side of AB of the circle where C is located.



Proof:

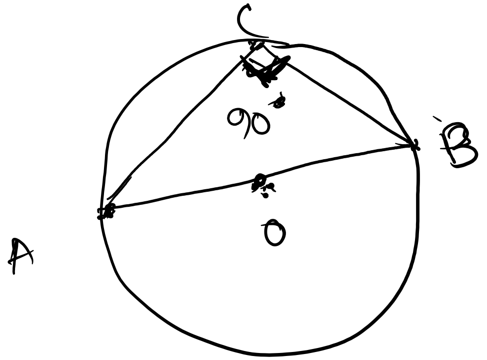
Step 1. The angle standing on the arc. ADB

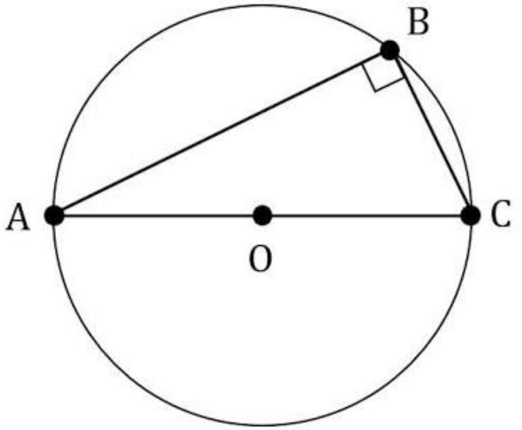
$\angle ACB = \frac{1}{2}$ (straight angle in the center $\angle AOB$) [$\therefore$ The angle standing on an arc at any point of the circle is half the angle at the center]

Step 2. But the straight angle $\angle AOB$ is equal to 2 right angles.

$\therefore \angle ACB = \frac{1}{2}$ (2 right angles) = 1 right angles (Proved)

Exercise 8.1 (3)





Soln:

$\angle ACB = 90^\circ$

ACB is a semi circle

AB is diameter of the circle,

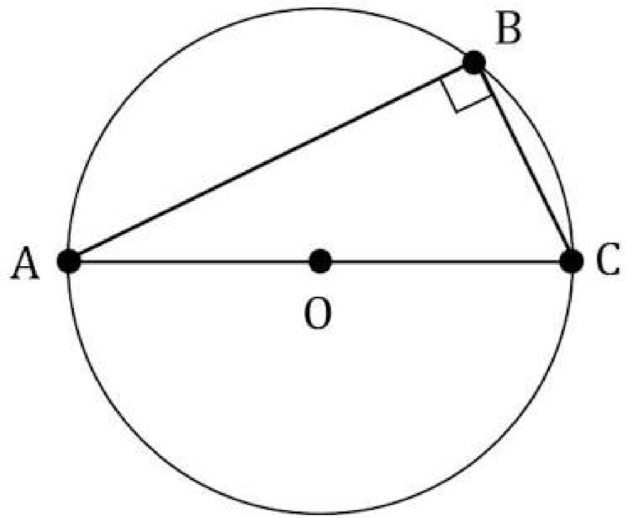
Then O is the centre.

AB is the hypotenuse.

So, O is the middle point of AB.

Exercise 8.1 (3)

Let, in the circle ABC, the points A, B, C are the vertices of a right angled triangle ABC. We have to prove that, the center of the circle is the mid point of hypotenuse AC



Construction: In $\Delta BC, \angle ABC$ is a right angle, which is subtended by the arc AC.

So, $\angle ABC$ is an angle subtended by the semi-circle, as we know, the angle subtended by a semi-circle, is right angle.

That means, AC is a diameter of circle ABC.
So, the center of the circle is the mid point of the diameter AC, or the mid point of the hypotenuse AC.

না বুঝে
মুখস্থ করার
অভ্যাস প্রতিভাকে
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