بِسْمِ اللّٰهِ الرَّحْمٰنِ الرَّحِيْمِ

বিস্‌মিল্লাহির রাহ্‌মানির রাহীম

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

Class Nine

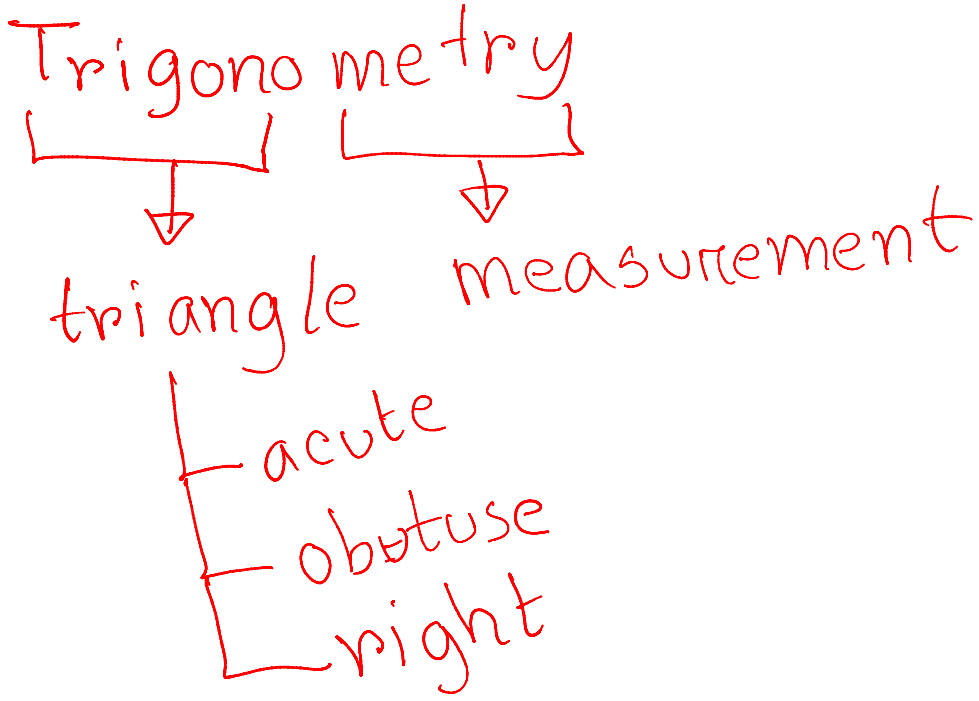
Higher Math

Unit Eight: 8.1 (Trigonometry)

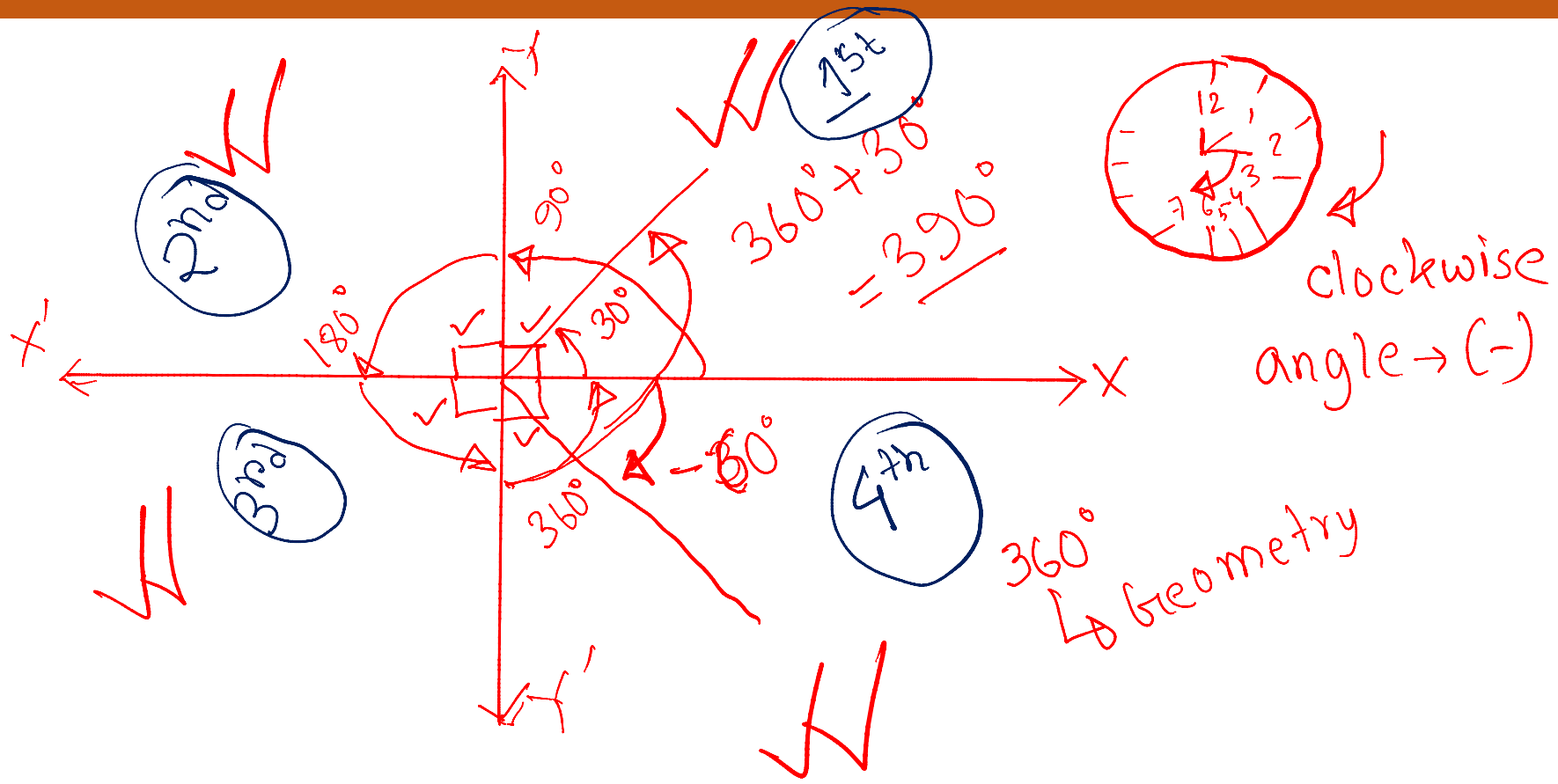
LECTURE-HM 12

Mashrur Ahmed

'Trigonometry'!!

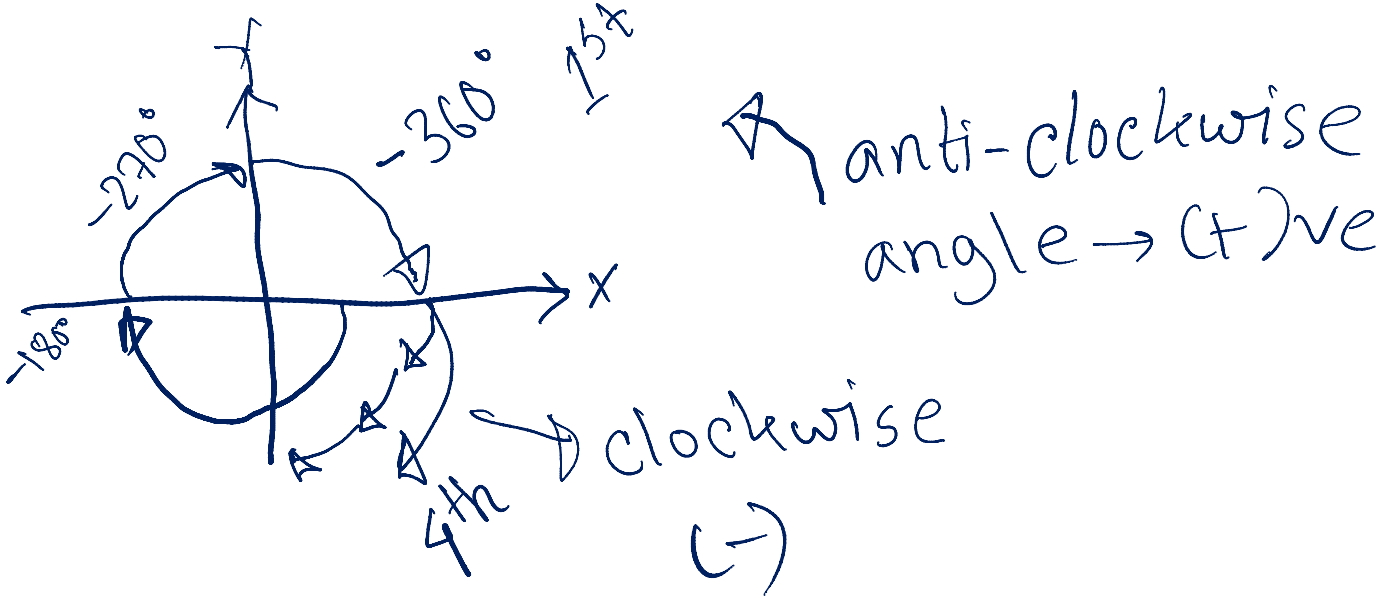


Trigonometric angles & Geometric Quadrant



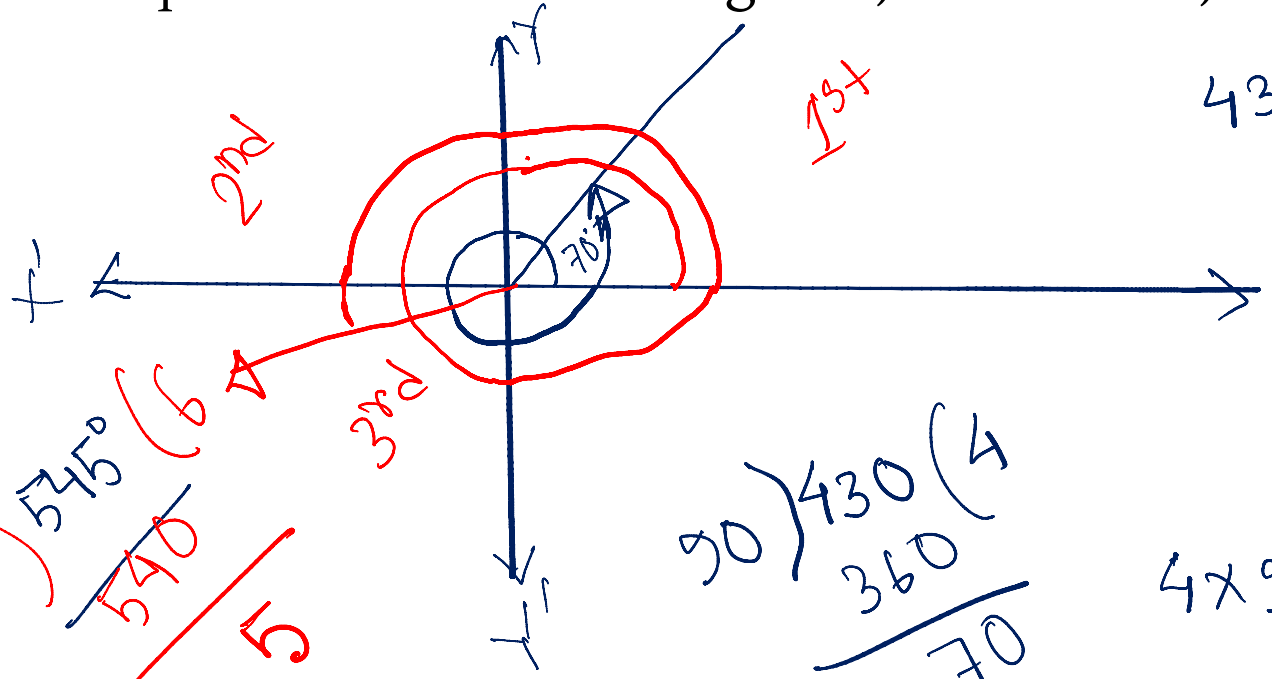
"Positive & Negative Angles"

- If any ray is rotated anti-clockwise, then the angle is said to be positive
- If any ray is rotated clockwise, then the angle is said to be negative



Example-1

In which quadrants do these angles a) 430° and b) 545° lie?



$430° > 360°$

$430° - 360° = 70°$

1st quadrant

=

$4 \times 90 + 70 = 430°$

1st quadrant

Example-1

In which quadrants do these angles a) 430° and b) 545° lie?

- 430° is a positive angle & larger than 4 right angles but smaller than 5 right angles.
- So to produce a 430° angle, a ray has to travel another 70 degrees more after a full turn of 4 right angles or one complete rotation.
- So the 430° angle is located in the 1st quadrant

Example-1

In which quadrants do these angles a) 430° and b) 545° lie?

- 545° angle is positive and larger than 6 right angles but smaller than 7 right angles.
- To produce an angle 545° , a ray has to move 5 degrees more than 6 right angles or 5 degrees higher than two more right angles after the first complete rotation.
- So 545° lies on 3rd quadrant.

Poll Question-01

In which quadrant *does* $-520°$ lie?

(a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}

(1) $-520° = -450° - 70°$

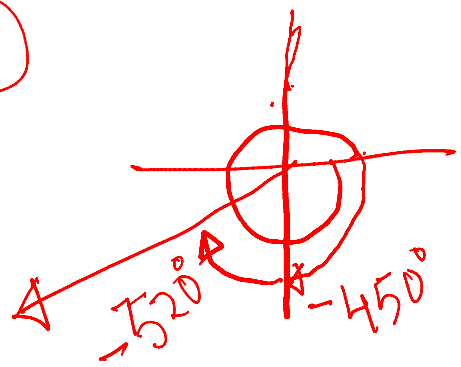
$= 5(-90°) + (-70°)$

(3) 90) -520 $(-450$

-450

-70

(1)



Units of Angle Measurement

- Usually two types of methods are used to measure an angle:
- a） Sexagesimal System
- b） Circular System

cm, m
ft, inch

"Sexagesimal System"

- In this way, the right angle is considered to be the unit of angle measurement. Each right angle is divided into equal 90 parts and each part is considered to be one degree .
- One degree is divided into 60 equal parts. Each part is called is called 1 minute.(1’ = one minute)
- One minute is divided into 60 equal parts. Each part is called is called 1 second(1” = one second)

1 right angle = 90°

1° = 60′

1′ = 60″

"Circular System"

- If any arc of length s produces an angle θ in the centre of the circle of radius r, then $s = r\theta$
  - 1 Radian $= \frac{2}{\pi}$ Right angle
  - $\therefore 1^c = \frac{2}{\pi}$ Right angle [1 Radian $= 1^c$]
  - $\therefore$ 1 Right angle $= \left(\frac{\pi}{2}\right)^c$
  - Or, $90° = \left(\frac{\pi}{2}\right)^c$
  - $\therefore 1°(180)^c$ and $1^c = \left(\frac{180}{\pi}\right)^c$

$90° \to \frac{\pi}{2}^c$

$90° \to \frac{\pi}{2}$

$360° \to 2\pi r$

$1° \to \frac{2\pi r}{360} = \frac{\pi r}{180}$

$90° \to \frac{\pi r}{180} \times 90 = \frac{\pi r}{2}$

$1^c = \frac{90}{\pi/2} = \frac{180}{\pi}$

$s = \frac{\pi r}{2}$

"Circular System"

If any arc of length s produces an angle θ in the centre of the circle of radius r, then $s = r*\theta$ → radian

Particular enunciation: Let O is the centre and OB = r unit is the radius of the circle ABC, arc AB = s unit and centred angle $\angle AOB = \theta^c$ is produced by the arc AB. It is to prove that, $s = r\theta$.

Construction: Draw the arc BP equal to radius OB at the point B so that it intersects the circle ABC at P. Join O, P.

Prof: By construction $\angle POB = 1^c$

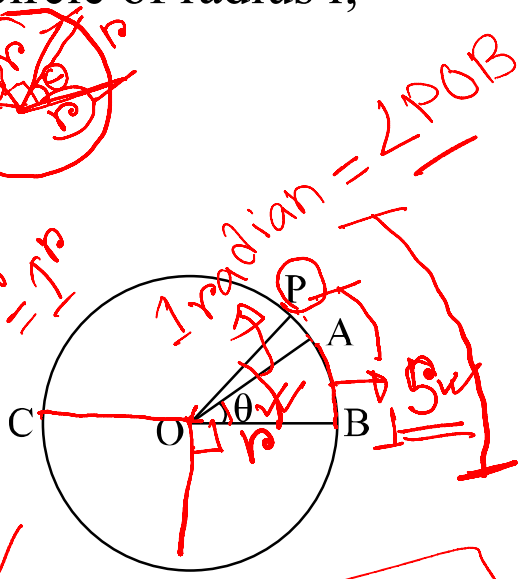
We know, centred angle produced by any arc is proportional to arc.

$$\frac{\text{Arc AB}}{\text{Arc PB}} = \frac{\angle AOB}{\angle POB}$$

or, $\frac{\text{s unit}}{\text{r unit}} = \frac{\theta^c}{1^c}$

So, $\frac{s}{r} = \theta$,

Where θ must be in radians.



Example-2

Express 30°12'36" in radians

$1'' = 60'' \therefore 1'' = \left(\frac{1}{60}\right)'$ min

$30^\circ + 12' + 36'' = 30^\circ + 12' + \left(\frac{36}{60}\right)' = 30^\circ + 12' + \left(\frac{3}{5}\right)'$

(12' = degree, 36'' = min)

$= 30^\circ + 12.6' = 30^\circ + \left(\frac{12.6}{60}\right)^\circ$

$= 30^\circ + \frac{21}{100}^\circ$

$= 30.21^\circ$

$1^\circ = 60' \therefore 1' = \left(\frac{1}{60}\right)^\circ$

$90^\circ = \frac{\pi}{2}$

$\therefore \boxed{1^\circ = \frac{\pi}{180}^c}$

$\therefore 30.21^\circ = 30.21 * \frac{\pi}{180}^c = .5237$ radian

Example-3

Express $\frac{3\pi}{13}$ in degrees, minutes & seconds

radian

$90^\circ = \frac{\pi}{2}$

$1^\circ = \frac{\pi}{180}$ r

$\therefore 1^r = \frac{180}{\pi}{}^\circ$

$\frac{180}{\pi}$

$\times \frac{180}{\pi}$

$\frac{3\pi}{13}{}^r = \frac{3\pi}{13} \times \frac{180}{\pi} = \left(\frac{540}{13}\right)^\circ$

$= 41.538\ldots = 41\frac{7}{13} = 41^\circ + \frac{7\times 60}{13}{}'$

$= 41^\circ 32' 18.46''$

ENGG

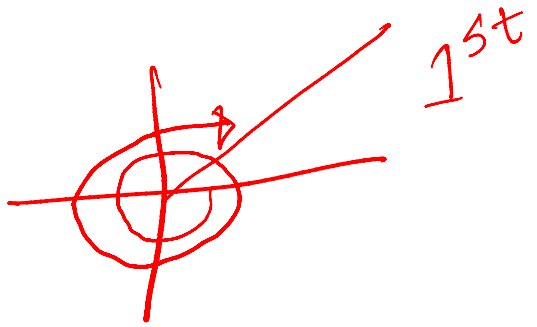
8

Poll Question-02

In which quadrant *does* $-635°$ *lie*?

(a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}

$-635° = 7 \times (-90°) - 5$



Example-4

If the ratio of three angles of a triangle is 3: 4: 5 then what is their angular measurement?

radian

$15^\circ, 20^\circ, 25^\circ$

$3 \quad 4 \quad 5$

$30^\circ, 40^\circ, 50^\circ$

$3 \quad 4 \quad 5$

$3x, 4x, 5x$

$3x + 4x + 5x = 180^\circ$

$\text{or, } 12x = 180^\circ$

$\therefore x = 15^\circ$

$1^\circ = \frac{\pi}{180}$

$\therefore 15^\circ = \frac{\pi}{180} \times 15 = \frac{\pi}{12}$

$3x\frac{\pi}{12} = \frac{\pi}{4}$

$4x\frac{\pi}{12} = \frac{\pi}{3}$

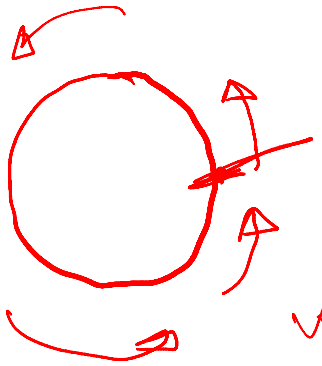
$5x\frac{\pi}{12} = \frac{5\pi}{12}$

$\therefore x = \frac{\pi}{12}$

$\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$

Example-5

A giant wheel covers a distance of 1.75km while making 40 revolutions. What is the radius of the wheel?



$1 \rightarrow 2\pi r$

$40 \text{ comp. rev} \rightarrow 2\times 40\pi r = 80\pi r$

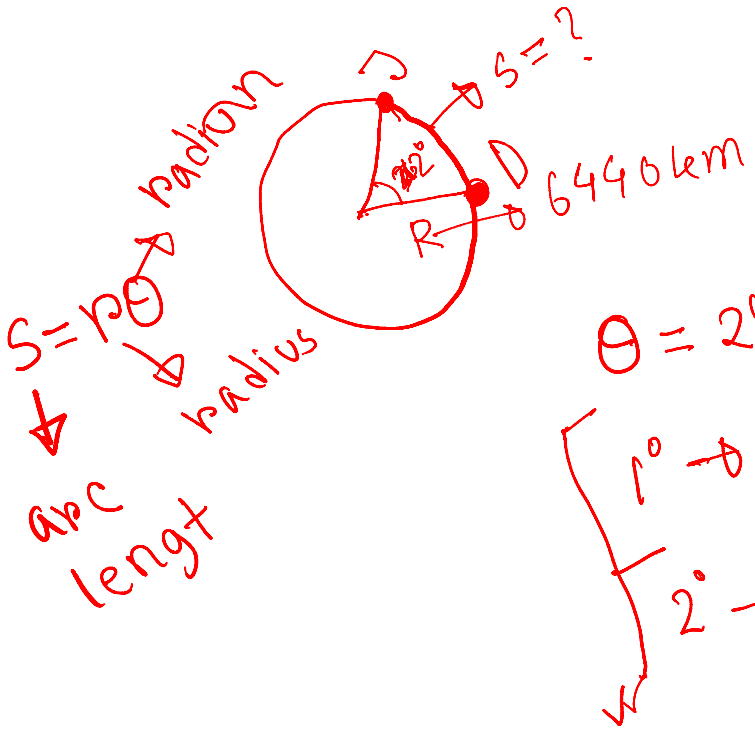
$\& \; 1.75 \text{ km}$

$80\pi r = 1.75 \text{ km} = 1750 \text{ m}$

Ans. $\therefore r = \frac{1750}{80\pi} \text{ m} = 6.963 \text{ m}$ (approx)

Example-6

Radius of earth is 6440km. If an arc joining Dhaka & Jamalpur subtends an angle 2°, then what is the distance between Dhaka & Jamalpur?



$S = r\theta$, $\theta \rightarrow$ radian

$r = 6440 \times 1000\ m$

$\theta = 2^0$

$1^0 \rightarrow \frac{\pi}{180}$

$2^0 \rightarrow \frac{2\times\pi}{180} = \frac{\pi}{90} = \theta$

$\therefore S = 6440 \times 1000 \times \frac{\pi}{90}$

$= 224.8\,km$ (app)

Poll Question-03

If the ratio of three angles of a triangle is 6: 8: 10 then what is their angular measurement? greatest angle

(a) 60° (b) 45° (c) 30° (d) 75°

$6x+8x+10x=180^\circ$

$x=$

$10x=?$

6 : 8 : 10

3 : 4 : 5

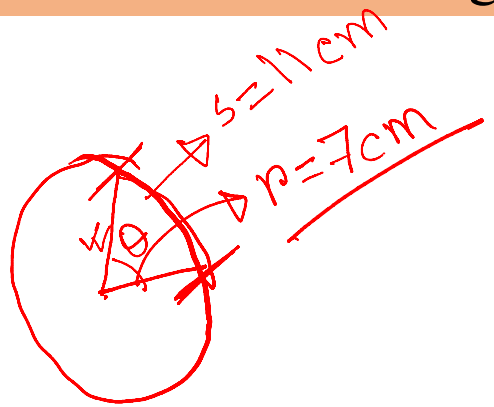
$\frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{12}$

$\frac{5\pi}{12} \times \frac{180}{\pi}$

75°

Example-7

The radius of a circle is 7 cm. Determine the angle at the center of the circle for an arc of 11 cm long.



$s = r\theta$

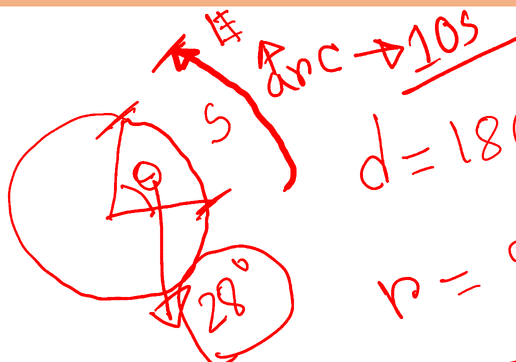
$\text{or, } \theta = \frac{s}{r} = \frac{11\text{cm}}{7\text{cm}}$

$= 1.57^{r}$

$\theta = 1.57^{r}$ (App)

Example-8

Ahsan rides a bike and crosses an arc of a circle in 10 seconds. If the arc produces a 28°angle in the center and the diameter is 180 meters then determine the speed of Ahsan.



$d = 180 m$

$r = 90 m$

$\theta = 28^\circ = 28 \times \frac{\pi}{180} r$

$[\because 1^\circ = \frac{\pi}{180} r]$

$s = 90 \times 28 \times \frac{\pi}{180} = 14\pi m$

$= 43.98 m$ (app)

$v = \frac{s}{t} = \frac{43.98}{10} = 4.398 m/s$ (app)

Poll Question-04

The radius of a circle is 21 cm. Determine the angle at the center of the circle for an arc of 33 cm long.

(a) 3.14 radian (b) 1.57 (c) 4.71 radian

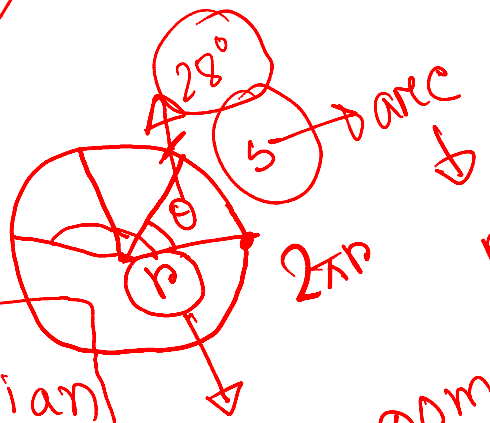
(d) 0.785 radian

$s = r\theta$

$\therefore \theta = \frac{s}{r} = \frac{33}{21} = 1.57 \text{ rad}$

$1^\circ = \frac{\pi}{180}$ radian

$1^r = \frac{180}{\pi}$ radian



$\frac{180}{2} = 90m$ part where Ahsan moved

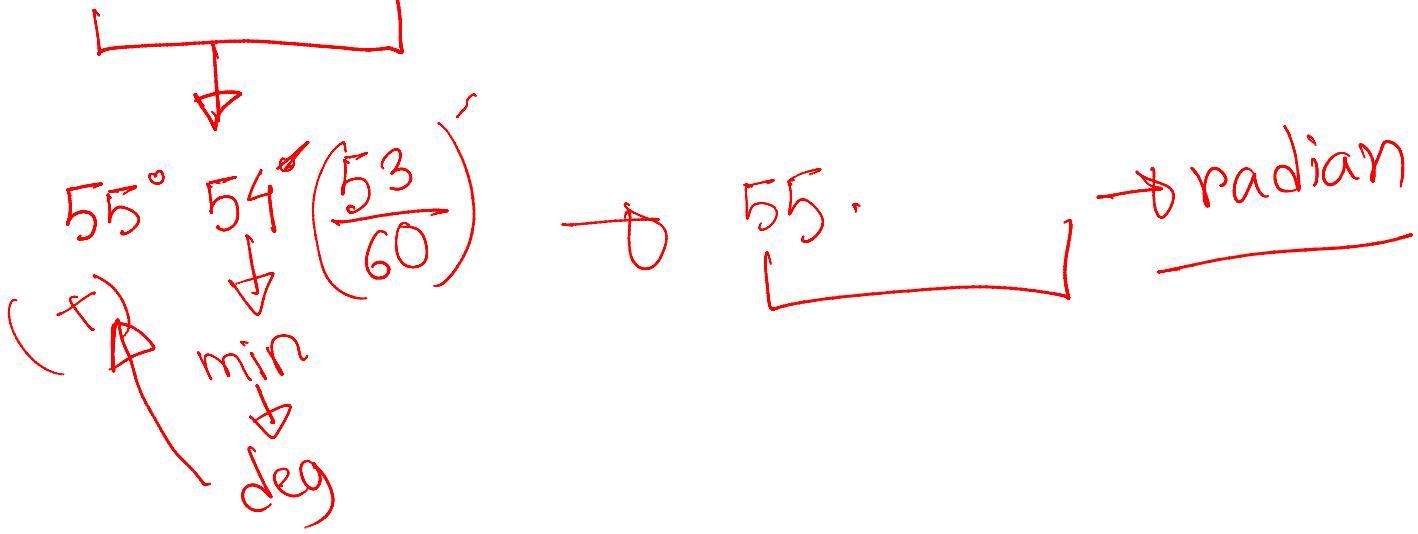
$\theta = 28^\circ = 28 \times \frac{\pi}{180}$

$V = \frac{5}{10}$ m/s $s = r\theta =$ ____ m

Exercise

1. (a) Express in radians :

(i) 75°30’ (ii) 55°54’53“ (iii) 33°22′11"



Exercise

1. (a) Express in radians :

(i) 1.3177 radian (ii) 0.9759 radian

Exercise

3. If the radius of a wheel is 2 m 3 cm, then find its circumference up to 4 decimal places

Exercise

4. The diameter of a wheel of a car is 0.84 m and it revolves 6 times per second. Find the speed of the car.

লেগে থাকো সৎভাবে,
স্বপ্ন জয় তোমারই হবে

উদ্ভাস-উন্মেষ শিক্ষা পরিবার

Thank You