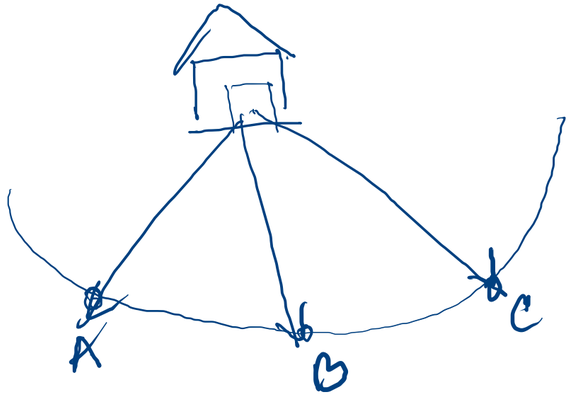
Engineering Admission Program-2020

# PHYSICS

Lecture : P-01

Chapter 2 : Vector

# Physical Quantities




## Scalar

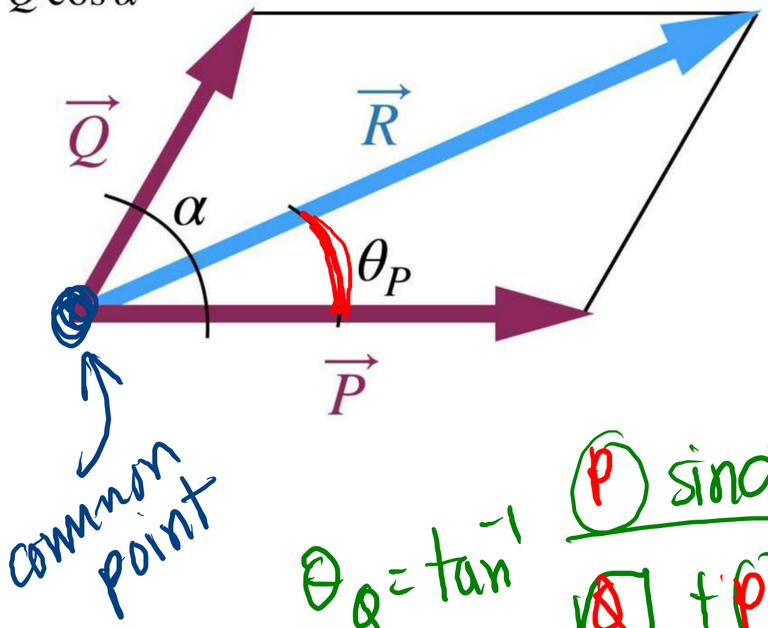
- magnitude is sufficient
- Ex: mass, time

## Vector

- both magnitude and direction are required
- Ex: velocity, displacement

# Vector Addition: Parallelogram law

- $\vec{R} = \vec{P} + \vec{Q}$
- **Magnitude of Resultant** $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$
- **Angle with** $\vec{P}$ , $\theta_P = \tan^{-1}\dfrac{Q\sin\alpha}{P + Q\cos\alpha}$
- **Angle with** $\vec{Q}$ , $\theta_Q =$
- $R_{max} =$
- $R_{min} =$




Resultant

$\vec{P} + \vec{Q} = \vec{R}$

Combined effect

Resultant

only one alone

$P + Q \rightarrow \alpha = 0°$

$P \sim Q \rightarrow \alpha = 180°$

$\theta_Q = \tan^{-1}\dfrac{P\sin\alpha}{Q + P\cos\alpha}$

$\vec{P}, \vec{Q}$

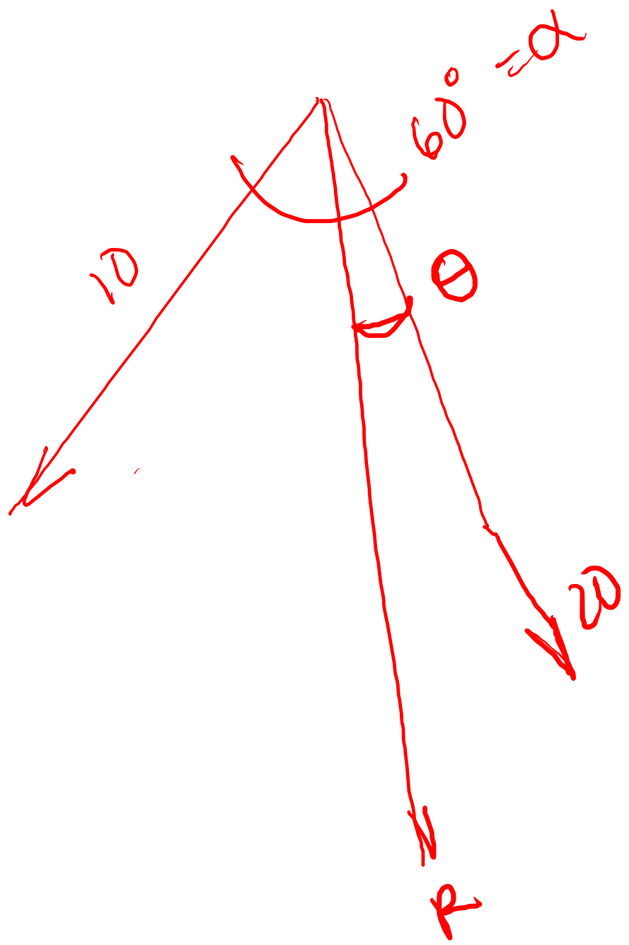
P, Q, $\alpha$

Find

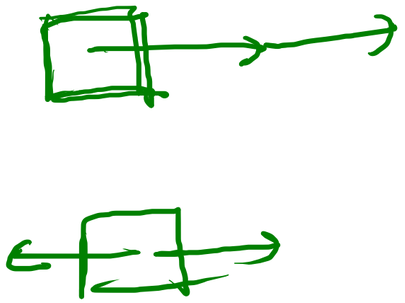
$\vec{R}$ → magnitude (R)

↘ direction ($\theta$)

Note: P & Q must originate from same point




$$\theta = \tan^{-1}\frac{10\sin 60^\circ}{20 + 10\cos 60^\circ}$$



# Problem

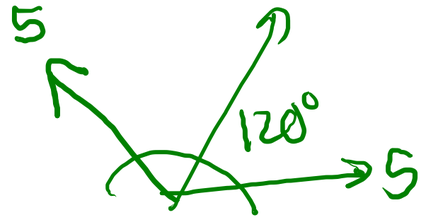
The value of two vectors is 5 units. They act at the same point at an angle of 120 degrees to each other. Determine the value and direction of their resultant. [RUET '12-13]

$$R = \sqrt{5^2 + 5^2 + 2 \times 5 \times 5 \cos 120^\circ}$$

$$= 5$$

$$\theta = \tan^{-1} \frac{5 \sin 120^\circ}{5 + 5 \cos 120^\circ}$$

$$= 60^\circ$$

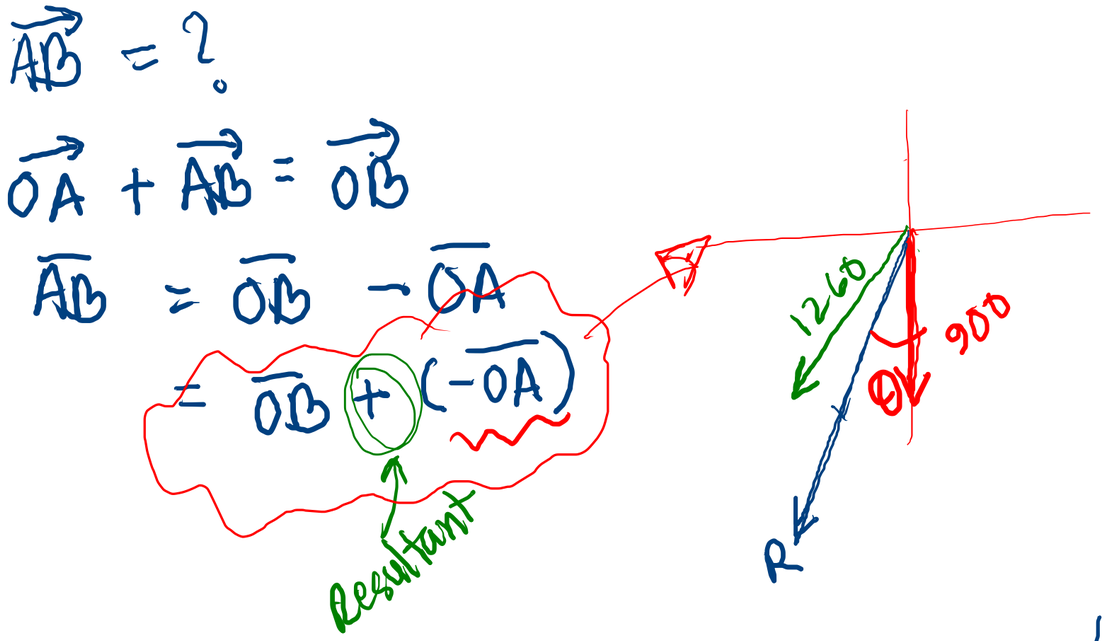


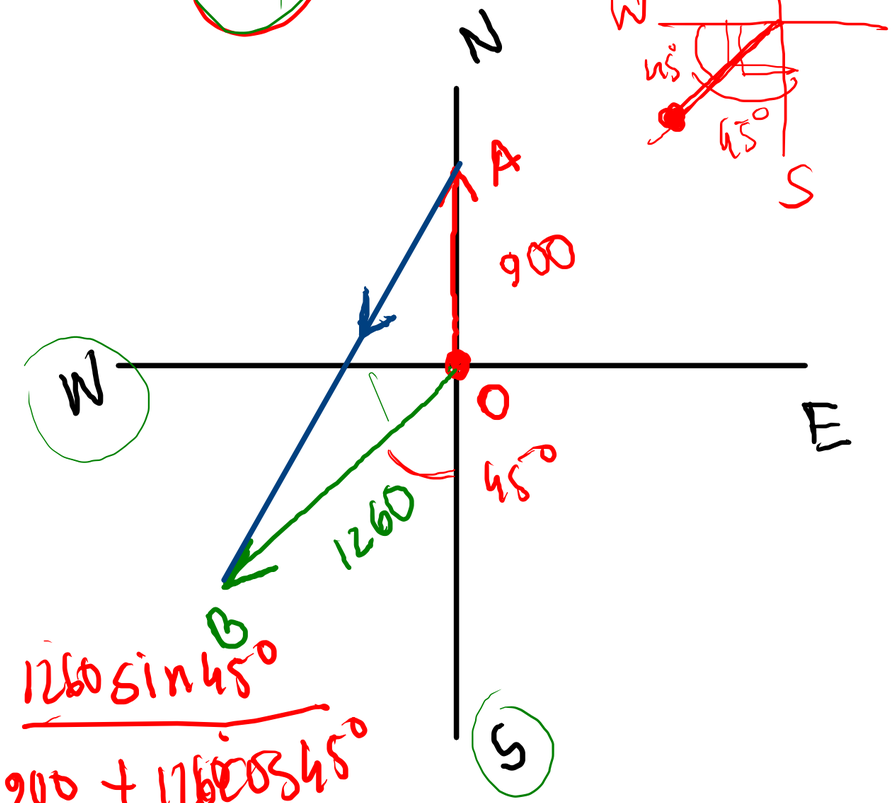

$P = Q$

$\theta = \frac{\alpha}{2}$

# Problem

Two cars A and B start their journey from the same point to the north and southwest, respectively, at speeds of 5m/s and 7m/s. Determine the position of car B relative to A after 3 minutes.




$R = \sqrt{900^2 + 1260^2 + \dots} = 2000.33$

$\theta = \tan^{-1}\frac{1260\sin 45^\circ}{900 + 1260\cos 45^\circ}$

$= 26.45^\circ$ west of south (S-W)

# Problem

The value of the resultant of two vectors A and B is 10 units and they produce angles of 30 and 60 degrees on both sides of the resultant, respectively. Determine the values of A and B.

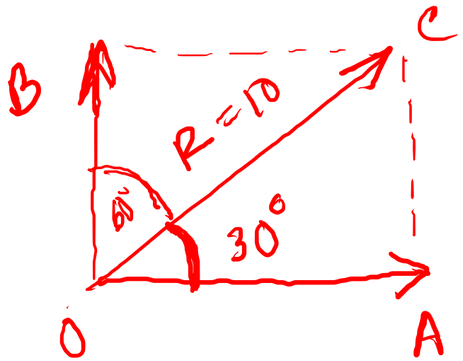
$\triangle OAC$

$\cos 30^\circ = \frac{OA}{10}$

$OA = 10 \cos 30^\circ =$

$\sin 30^\circ = \frac{AC}{10}$

$AC = 10 \sin 30^\circ =$

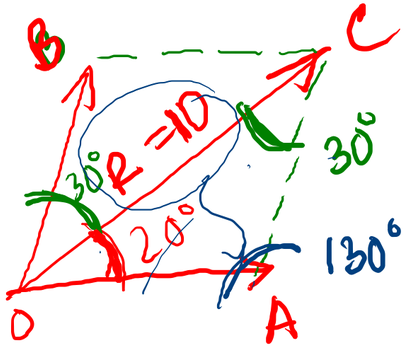


# Problem

In the previous problem, what would be the value of A and B if they generate 20 and 30 degree angles on both sides of the resultant, respectively?

OA = ?,
OB = ?,

$\Delta OAC$




$$\frac{10}{\sin 130^\circ} = \frac{OA}{\sin(30^\circ)} = \frac{AC}{\sin(20^\circ)}$$

$$OA = \frac{10}{\sin 130^\circ} \times \sin 30^\circ = 6.5$$

$$AC = \frac{10}{\sin 130} \times \sin 20^\circ = 4.46 = OB$$

Perpendicular

# Components of a Vector

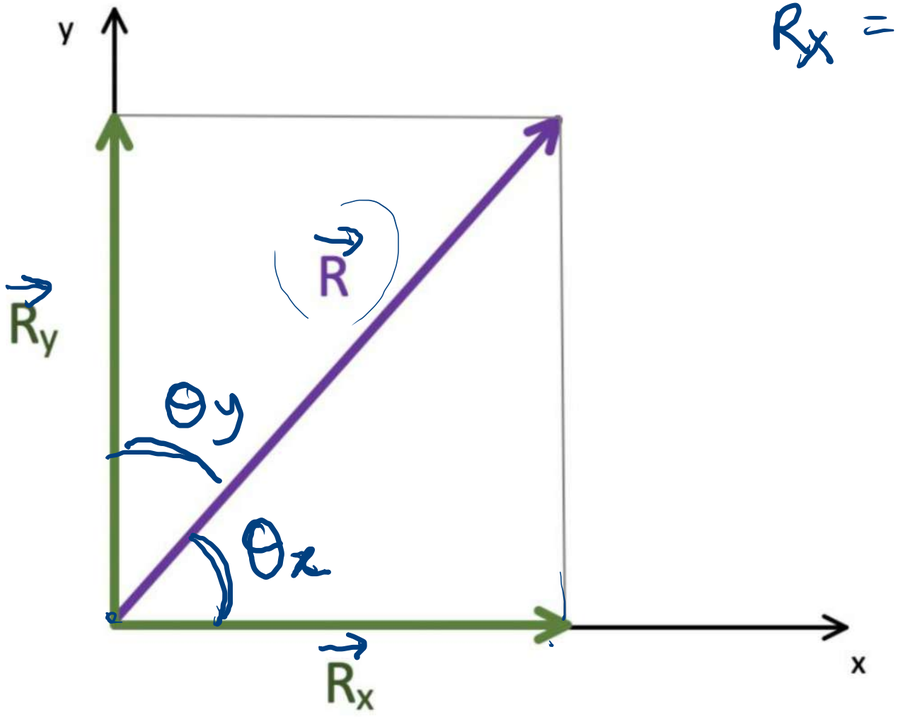
$\vec{R} = \vec{R_x} + \vec{R_y}$

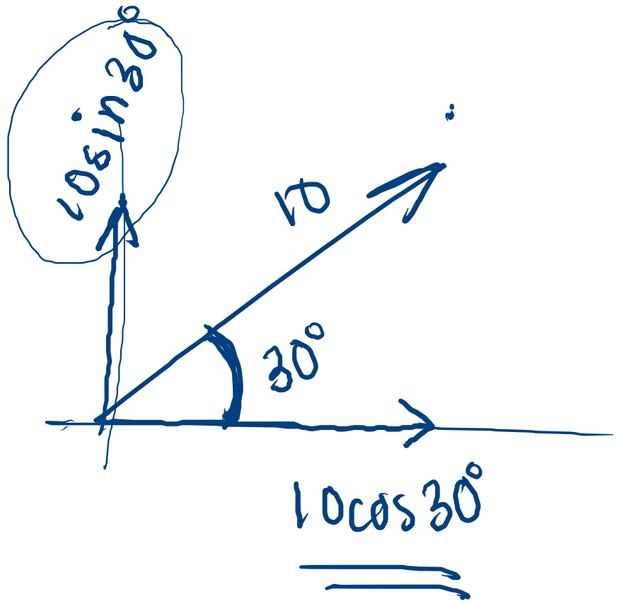
components

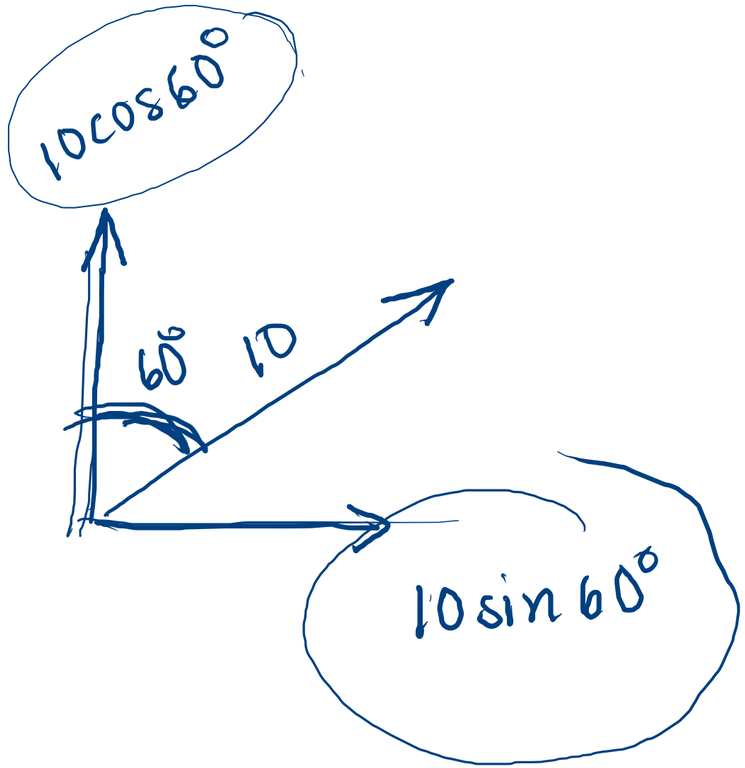
$\theta_x + \theta_y = 90^\circ$

$\vec{R_x} = (R_x)\hat{i}$

- $R_x = R\cos\theta_x$
- $R_y = R\cos\theta_y = R\cos(90^0 - \theta_x) = R\sin\theta_x$
- $\vec{R} = \vec{R_x} + \vec{R_y}$
- $\vec{R} = R_x\hat{i} + R_y\hat{j}$




10sin30°
10
30°
10cos30°

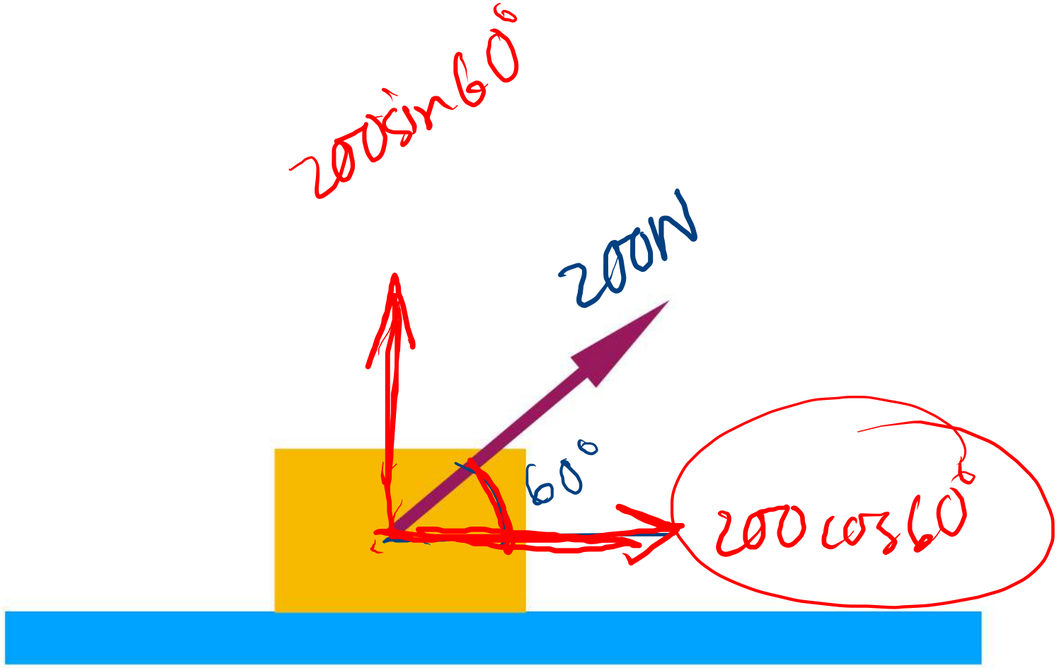

10cos60°
60°
10
10sin60°

# Problem

A piece of wood is being pulled by a 200 N force at a 60 degree angle with the horizontal. What is the horizontal effective force on the object? [DU ’13-14]

$200\cos 60^\circ$

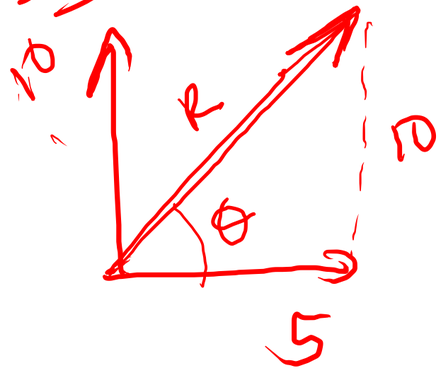
$= 100$



$\alpha = 0°$

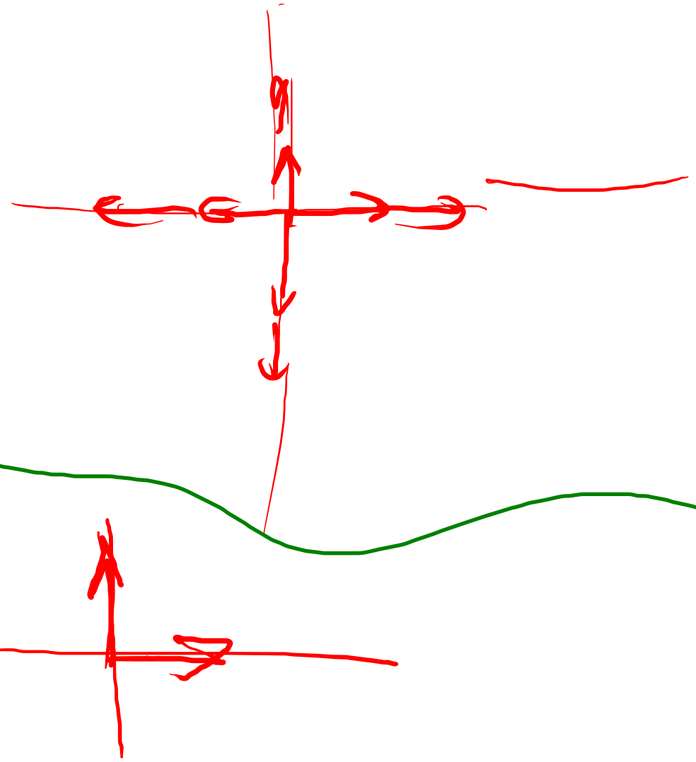

$\alpha = 180°$


$\alpha = 90°$

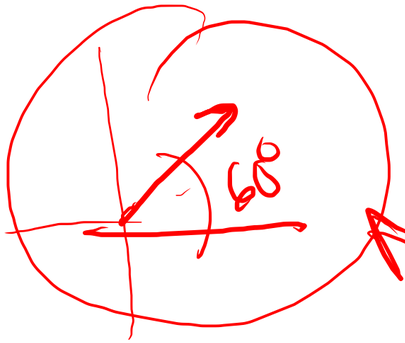



$\sqrt{10^2 + 5^2}$

$\theta = \tan^{-1}\frac{10}{5}$



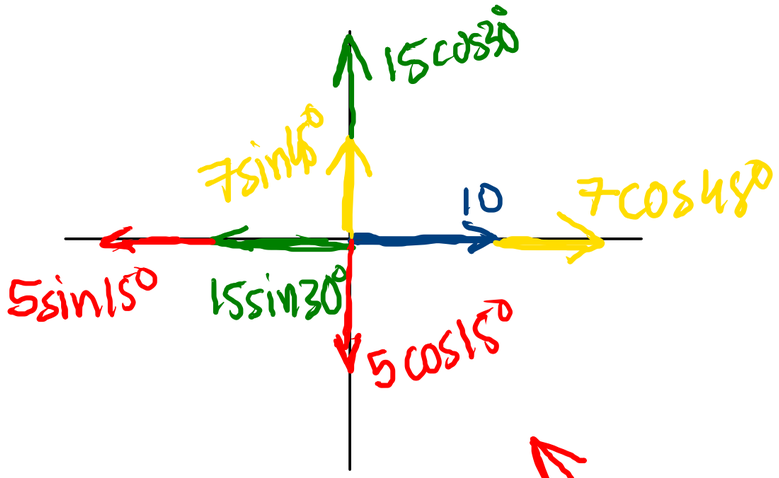
GOOD!




BAD!

# Problem

Find the resultant of the vectors in the following figure.

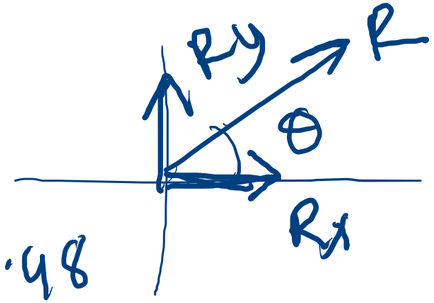



$$R_x = 10 + 7\cos 45^\circ - 5\sin 15^\circ - 15\sin 30^\circ$$

$$= 6.165$$

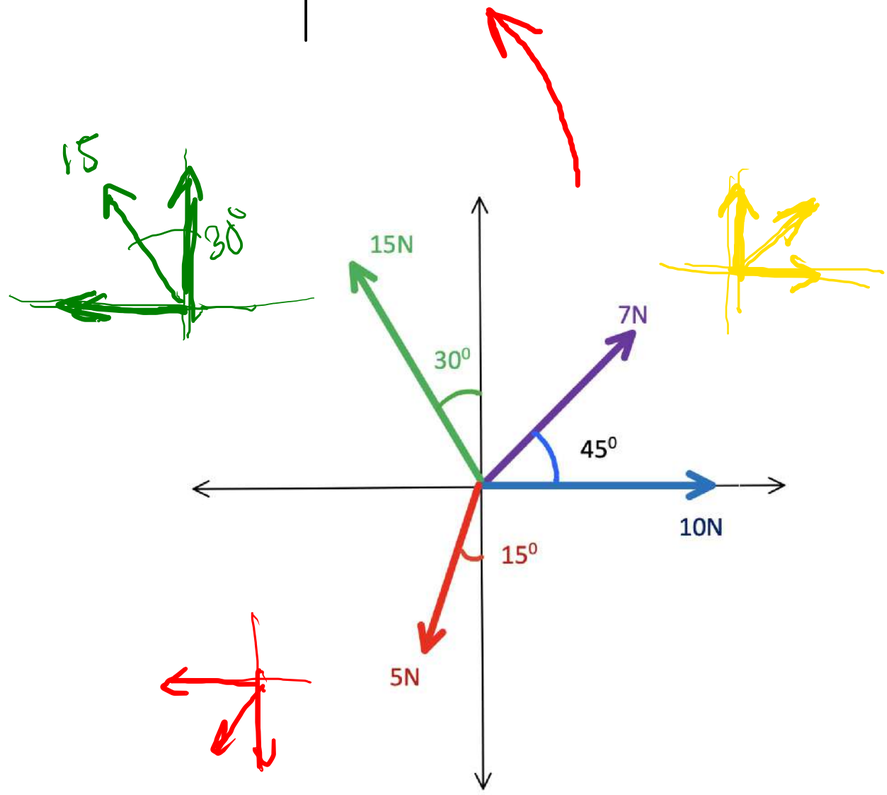
$$R_y = 15\cos 30^\circ + 7\sin 45^\circ - 5\cos 15^\circ$$

$$= 13.11$$

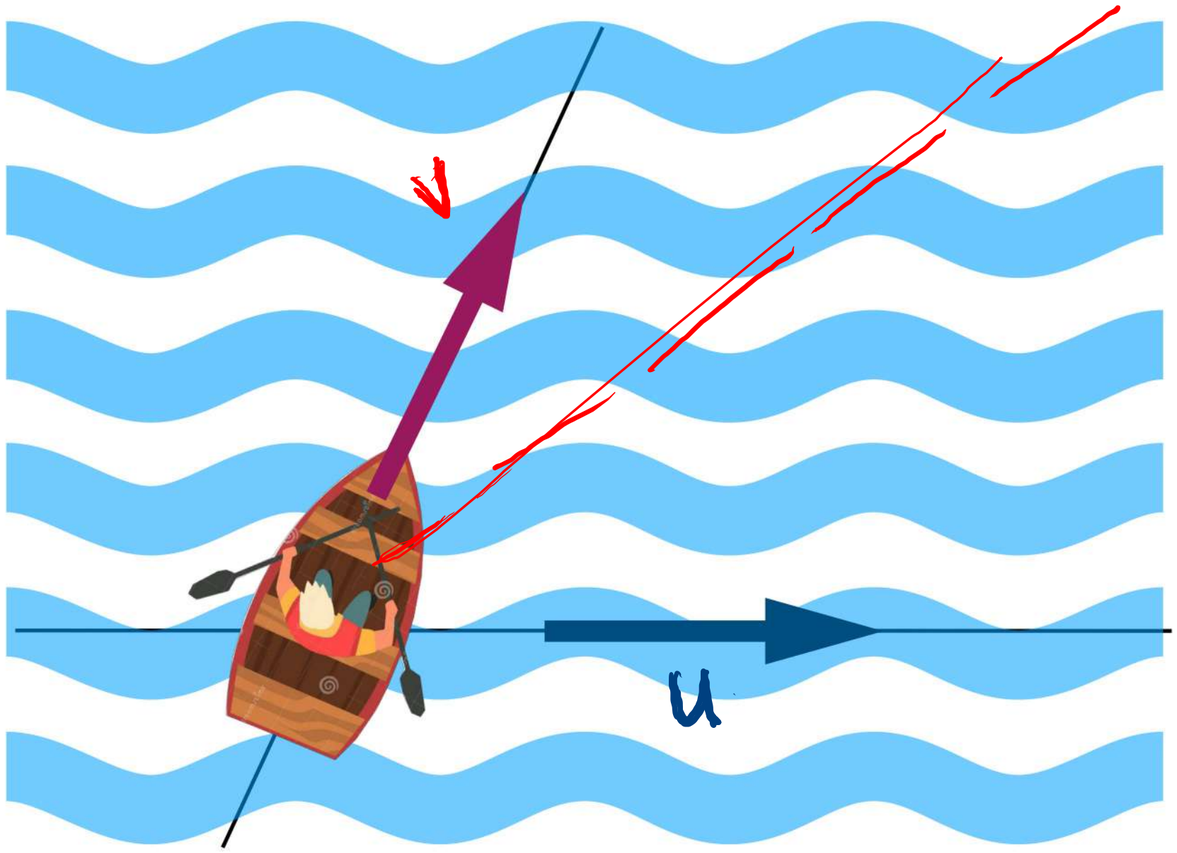



$$R = \sqrt{R_x^2 + R_y^2} = 14.48$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = 64.85$$



# River and Boat



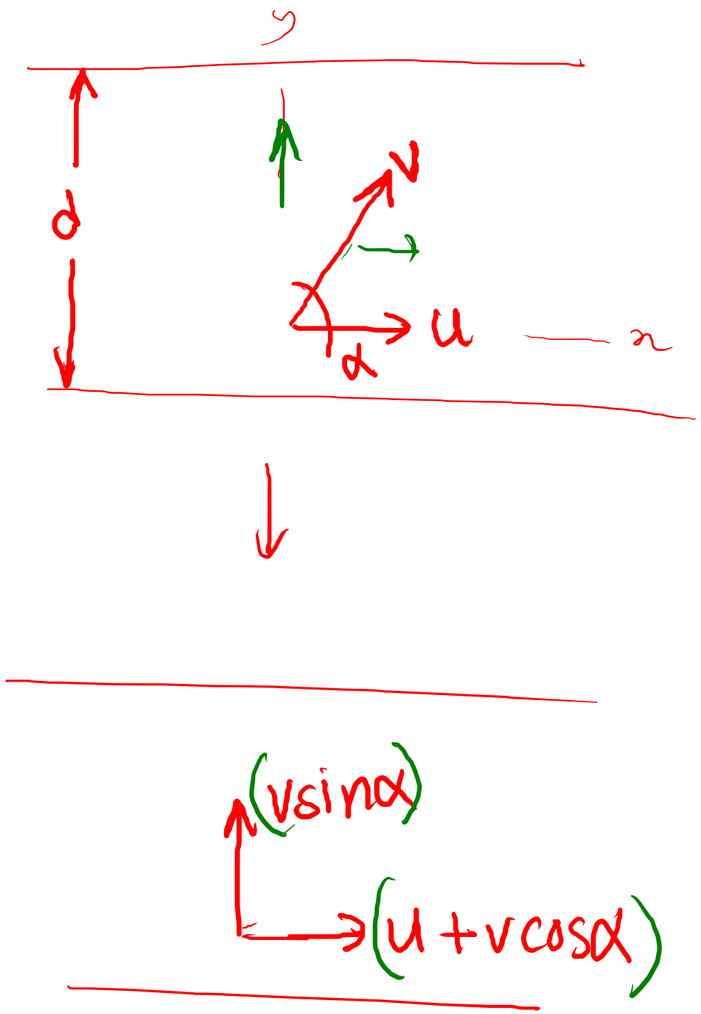
# River Crossing

↳ d - distance along y axis

distance = speed × time

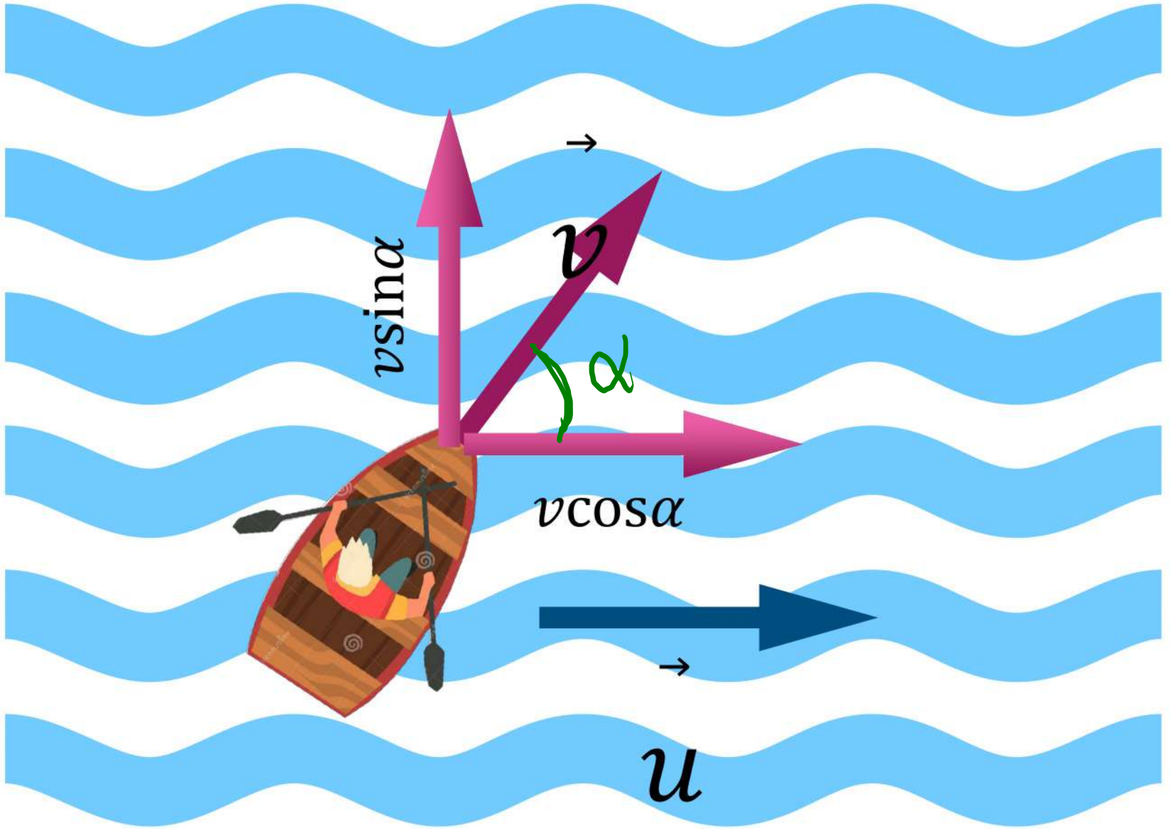
$d = (v \sin\alpha) \times t$

$$\boxed{t = \frac{d}{v \sin\alpha}}$$



# River and Boat

- Velocity of current = u
- Velocity of the boat = v
- The angle of the velocity of boat = $\alpha$




**Total velocity along x axis =** $u + v\cos\alpha$

**Total velocity along y axis =** $v\sin\alpha$

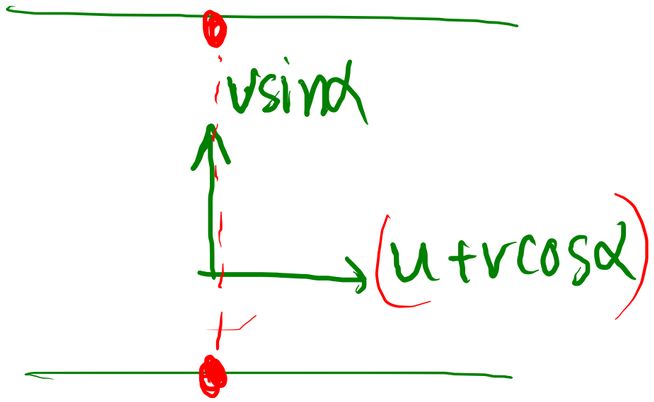
# Problem

The speed of current in a river is 6 km/hr and the speed of a boat is 12 km / hr. The width of the river is 10km. How long will it take to cross the river along the shortest distance path?

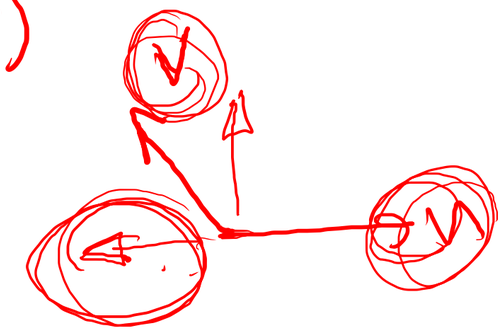
$u + v\cos\alpha = 0$

$\cos\alpha = -\frac{u}{v} = -\frac{6}{12}$

$\alpha = 120^\circ$




$t = \frac{d}{v\sin\alpha} = \frac{10\text{ km}}{12\text{km/hr}\ \sin(120^\circ)} = 0.96\text{hr}$



# Problem

The speed of current in a river is 6km/hr and the speed of a boat is 12km/hr. The width of the river is 10km. Find the shortest possible time for the boat to cross the river. In this case, what is the distance traveled by the boat along the river bank?

$$t_{min} = \frac{d}{v(\sin\alpha)_{max}}$$

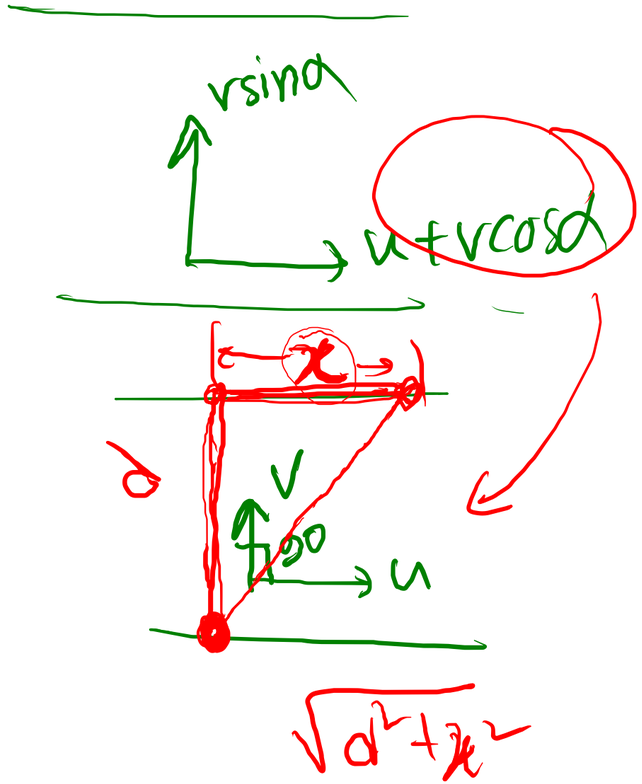
$(\sin\alpha)_{max} = 1$

$\alpha = 90^\circ$

$$t_{min} = \frac{d}{v} = \frac{10\,km}{12\,km/hr}$$

$$= 0.86\,hr$$

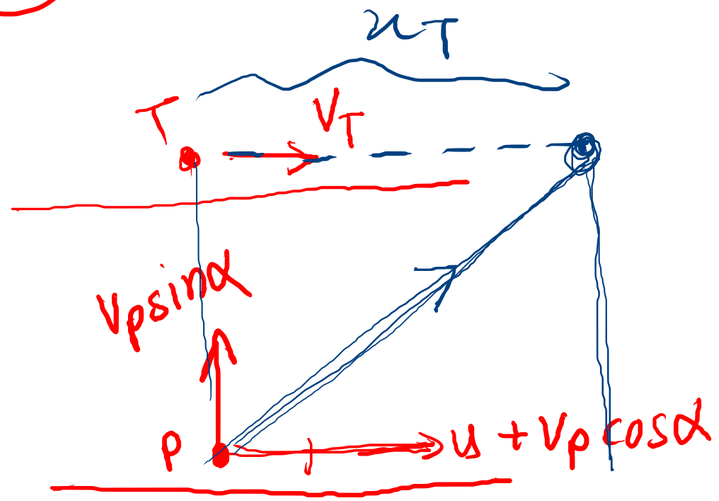
$$x = 6 \times \frac{5}{6} = 5\,km$$



# Problem

A thief standing on one side of a river saw a police boat on the opposite side and ran in the direction of current with a speed of 6m/s along the bank of the river. If the speed of the boat is 5m/s, determine the angle of the boat velocity in order to catch the thief. Speed of current is 4m/s.

$\alpha = ?$




$$x_p = (u + v_p \cos\alpha)\, t$$

$$x_T = V_T\, t$$

$$x_p = x_T$$

$$(u + v_p \cos\alpha)\, t = V_T\, t$$

$\alpha = ?$

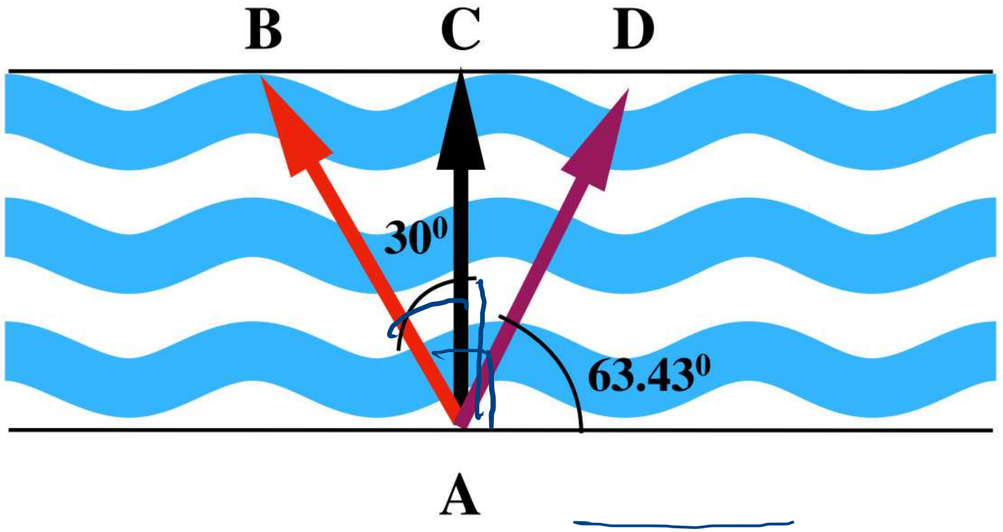
$$\cos\alpha =$$

# Problem

The width of the river is 31km. Two engine boats start traveling at the same speed from point A, one along AB and the other along AC. The first one reaches the point C, but the other reaches the point D. The velocity of the current is 9km/hr.

(1) Determine the speed of both boats.

(2) Will the two boats reach the other side at the same time? [HSC - 2017]




Boat-1

$u + v\cos\alpha = 0$

$9 + v\cos(120^\circ) = 0$

$v = 18\ km/hr$ (1)

Boat-1

$t_1 = \frac{d}{v\sin\alpha_1} = \frac{31}{18\sin 120^\circ}$

$t_2 = \frac{d}{v\sin\alpha_2} = \frac{31}{18\sin 30^\circ}$

$\rightarrow t_2 < t_1$

# Problem

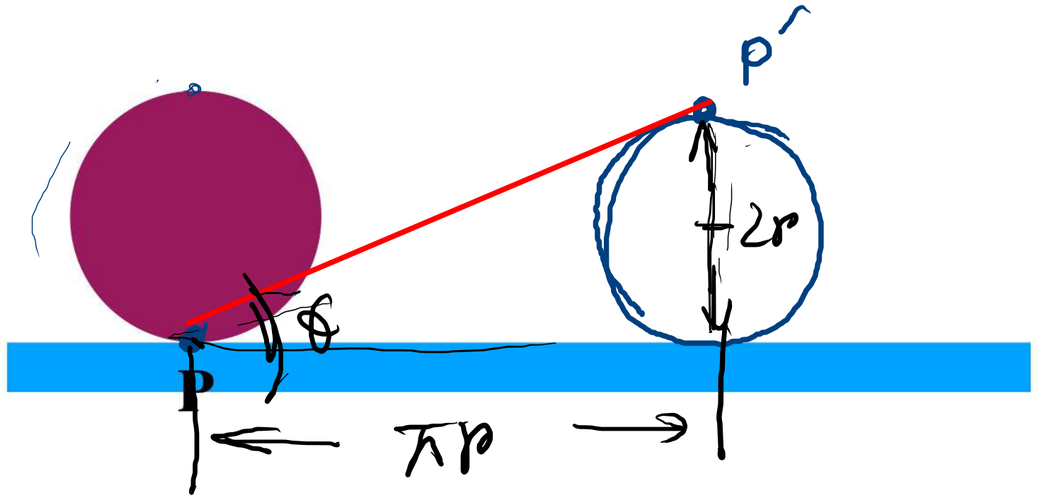
Determine the displacement of the point P when the wheel rotates 180 degrees. Radius of the wheel is 100cm.

$360^\circ \rightarrow 2\pi r$

$180^\circ \rightarrow \pi r$

$PP' = \sqrt{(\pi r)^2 + (2r)^2} =$

$\theta = \tan^{-1} \frac{2r}{\pi r} =$



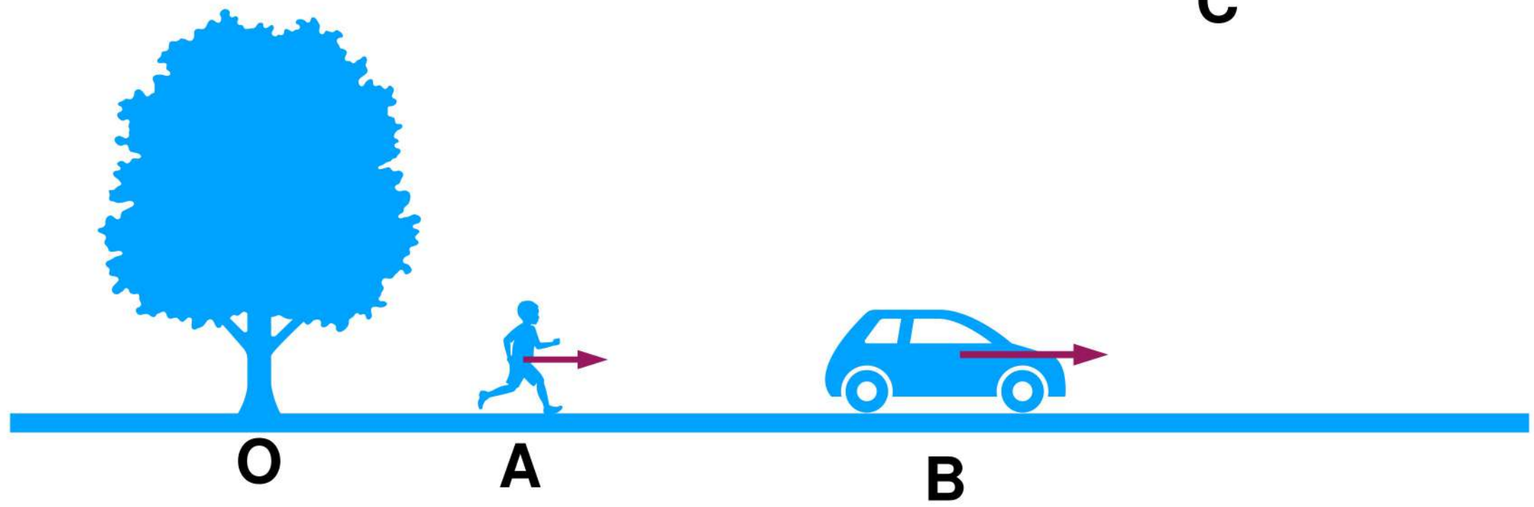
# Relative Velocity

$\vec{V}_{BC}$

whose velocity? w.r.t whom?

$\vec{V}_{B}$

whose velocity? w.r.t to the ground

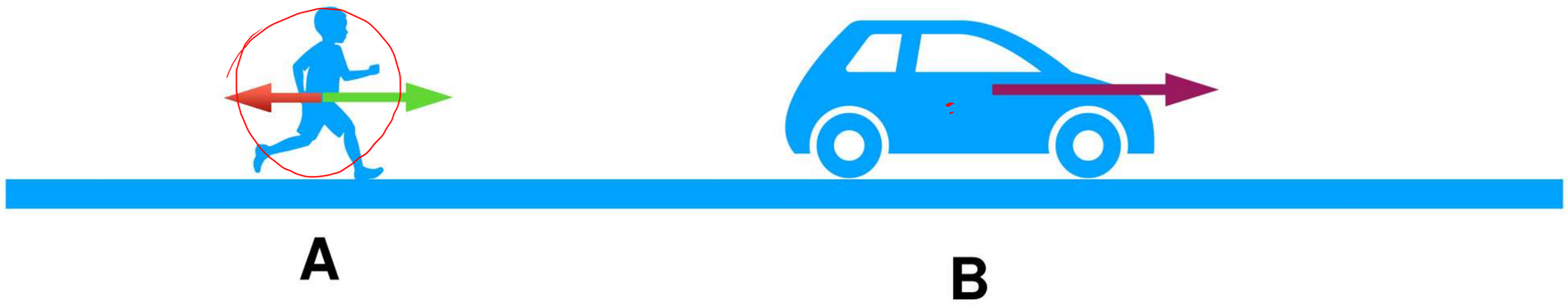


# Relative Velocity

$$\bar{V}_{AB} = \bar{V}_A - \bar{V}_B$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$\vec{V}_{AB} = \vec{V}_A + (-\vec{V}_B)$$



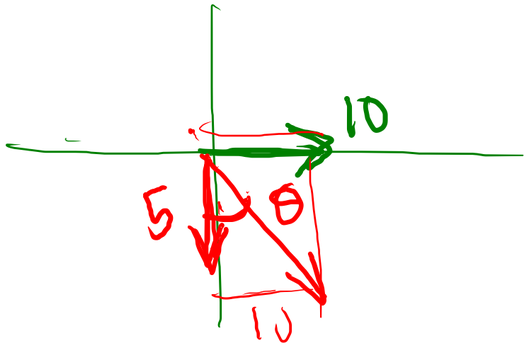
# Problem

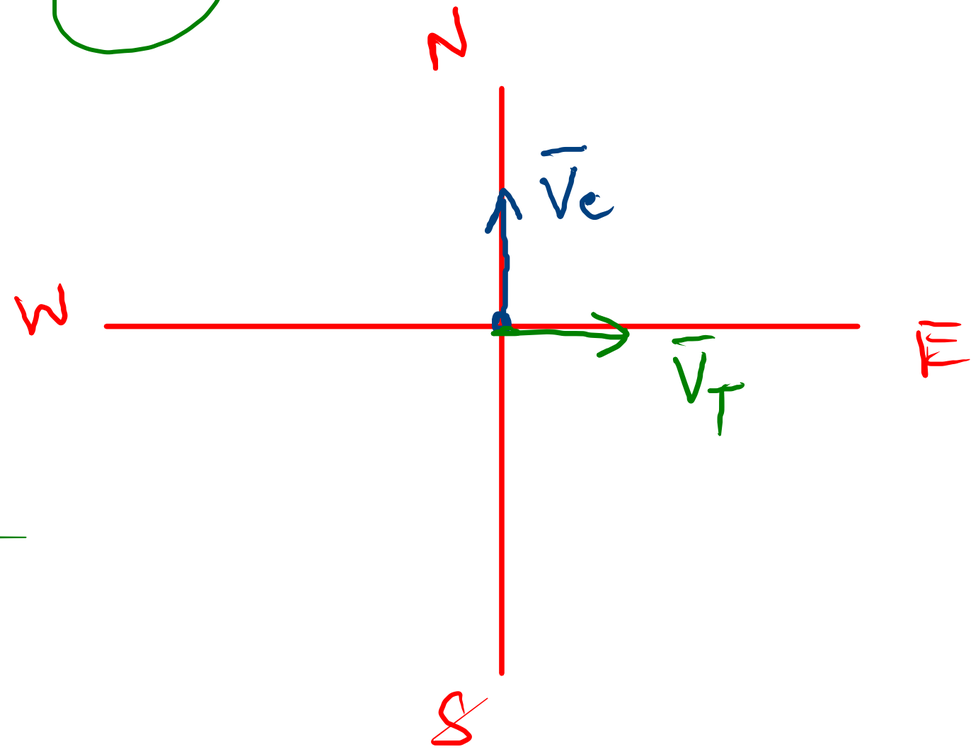
A car is moving north at 5m/s and a truck is moving east at 10m/s. Determine the velocity of the truck relative to the car.

$\bar{V}_{TC} = \bar{V}_T + (-\bar{V}_C)$

$V_{TC} = \sqrt{10^2 + 5^2} =$

$\theta = \tan^{-1}\frac{10}{5} =$

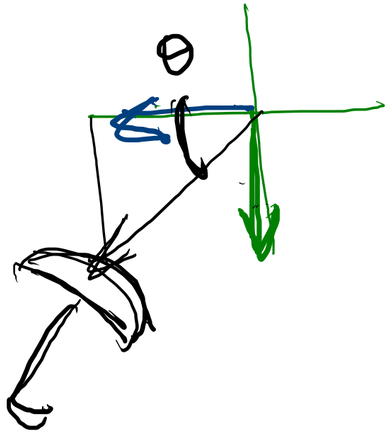


# Problem

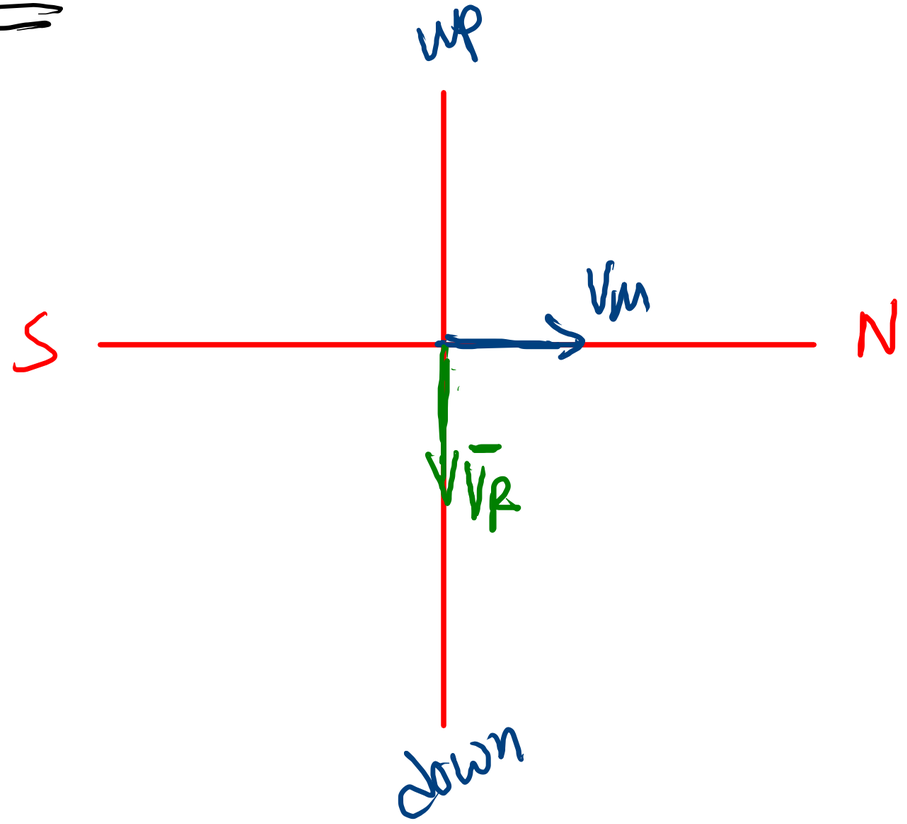
When a man is moving north at a speed of 5m/s, it starts raining vertically at a speed of 3m/s. To avoid the rain, at what angle he has to hold the umbrella with the horizontal? In which direction?

$\bar{V}_{RM} = \bar{V}_R + (-\bar{V}_m)$




$\theta = \tan^{-1}\frac{3}{5}$

$= 30.9^{\circ}$



# Problem

It is raining vertically at 10km/hr and wind is blowing from east to west at 60km/hr. Determine the speed of the vehicle moving from east to west so that - (a) both the front and rear windows of the vehicle are wet (b) only the rear glass is wet (c) only the front glass is wet.

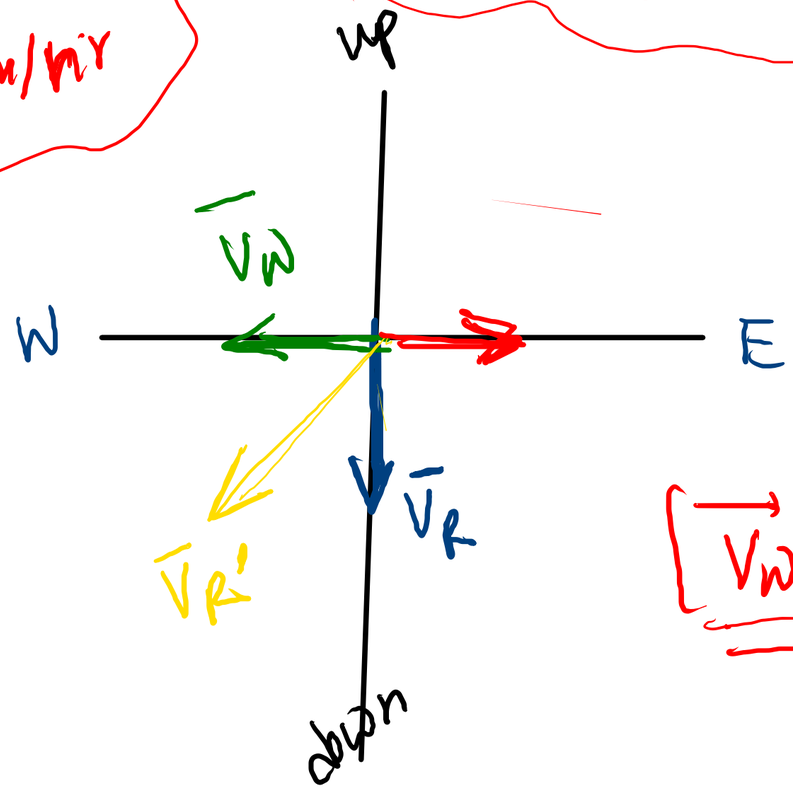
$\vec{V}_{R'} = \vec{V}_R + \vec{V}_W$

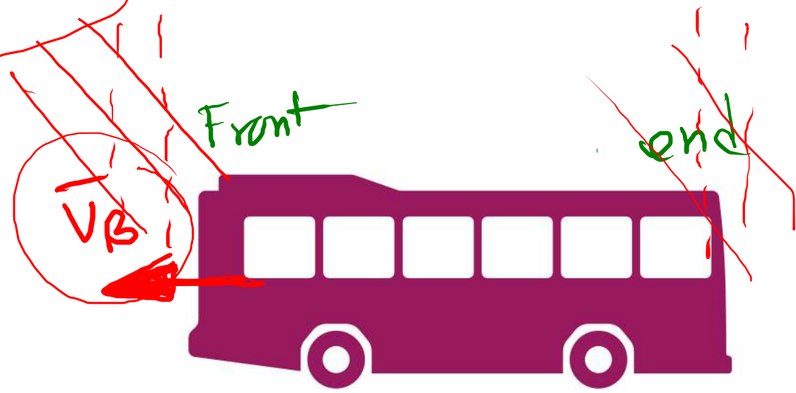
$\vec{V}_{R'B} = \vec{V}_{R'} + (-\vec{V}_B)$

$= \vec{V}_R + \vec{V}_W + (-\vec{V}_B)$

$V_B < 60\ km/hr$

$V_B > 60\ km/hr$

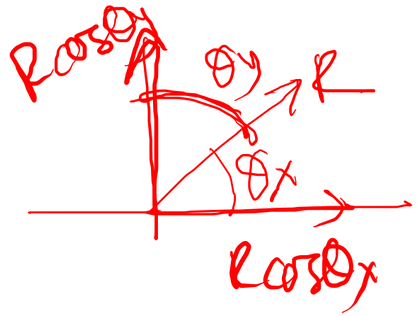



$|\vec{V}_W| = |-\vec{V}_B|$

$V_B = 60\ km/hr$

# Vector in 3D

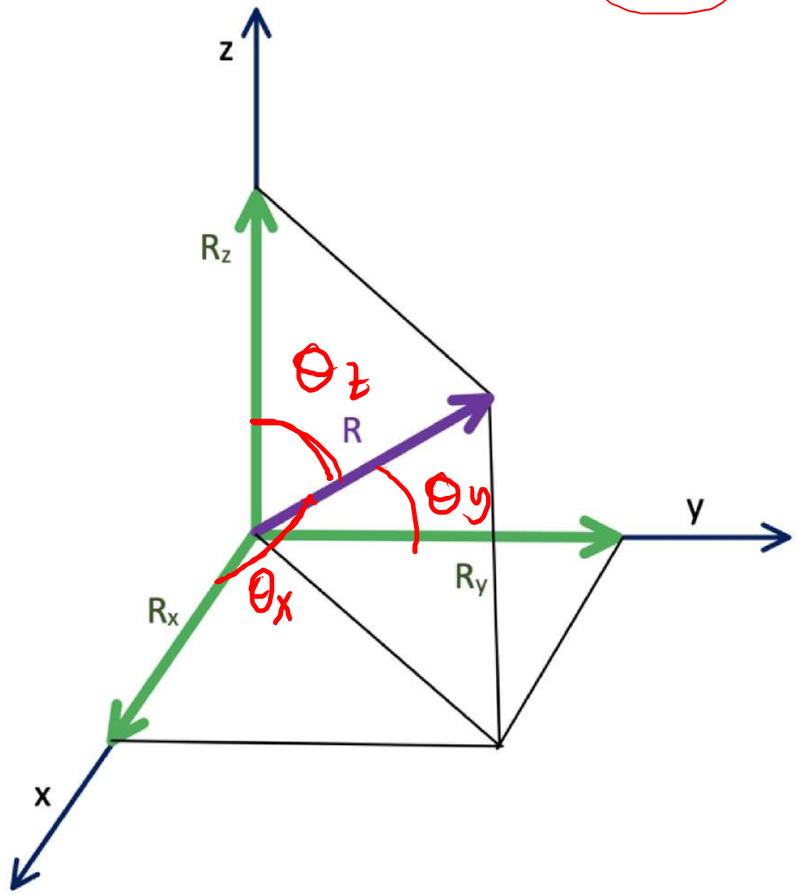



Direction cosine:

$\cos\theta_x = \frac{R_x}{R}$

- $R_x = R\cos\theta_x$
- $R_y = R\cos\theta_y$
- $R_z = R\cos\theta_z$
- $\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z$
- $\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$
- $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$



# Problem

$\sqrt{5^2+6^2+10^2} = \sqrt{161}$

Find the direction cosines and angles with the axes of the vector $\vec{A} = 5\hat{i} - 6\hat{j} + 10\hat{k}$

$\cos\theta_x = \frac{A_x}{A} = \frac{5}{\sqrt{161}} \rightarrow \theta_x = 66.8^\circ$

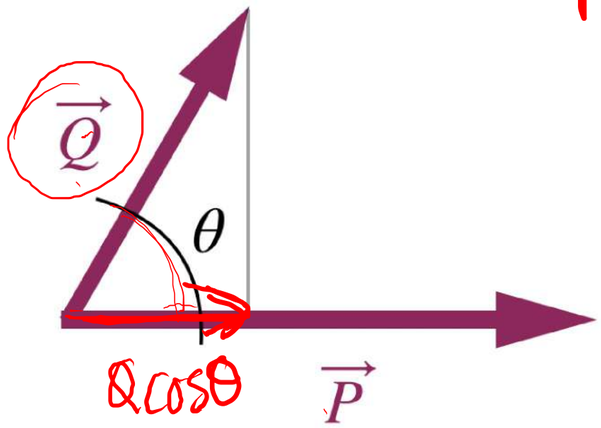
$\cos\theta_y = \frac{A_y}{A} = \frac{-6}{\sqrt{161}}$ $\theta_y =$

$\cos\theta_z = \frac{A_z}{A} = \frac{10}{\sqrt{161}}$ $\theta_z =$

# Dot Product

$$\vec{P} \cdot \vec{Q} = PQ\cos\theta$$

$$\vec{P} \cdot \vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$




$\vec{P}\cdot\vec{Q} = PQ\cos\theta$

$Q\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{P}$

**Condition for two vectors to be perpendicular:** $\vec{P} \cdot \vec{Q} = PQ\cos 90^o = 0$

**Projection of** $\vec{P}$ **onto** $\vec{Q}$ $= Q\cos\theta = \dfrac{\vec{P} \cdot \vec{Q}}{P}$

**Angle between** $\vec{P}$ **&** $\vec{Q}$ $= \theta = \cos^{-1}\dfrac{\vec{P} \cdot \vec{Q}}{PQ}$

# Problem

$\vec{A} = -7\hat{i} + 2m\hat{j} - 3\hat{k}$

$\vec{B} = m\hat{i} + m\hat{j} - 2\hat{k}$

What is the value of m if the vectors are perpendicular.

$\bar{A} \cdot \bar{B} = 0$

$-7m + 2m^2 + 6 = 0$

$2m^2 - 7m + 6 = 0 \rightarrow m = 1.5, 2$

# Problem

$\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$

On the plane formed by two vectors, find the component of B perpendicular to vector A.

$\sqrt{2^2+1^2+2^2}$

$\vec{B_1} \rightarrow B_1$

$\vec{B_1} \rightarrow \hat{a}$

Target : $\vec{B_2}$

$\hat{a} = \frac{\vec{A}}{A}$

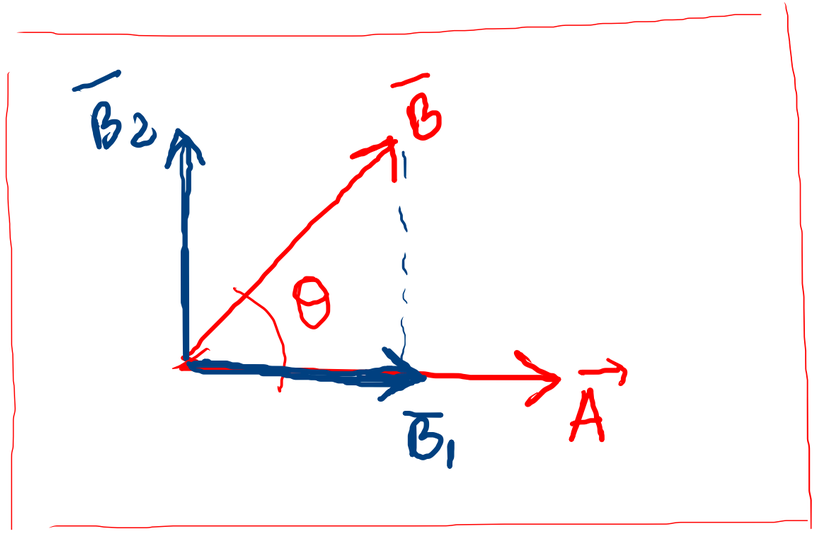
$B_1 = B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A} = (\quad)$

$\hat{a} = \frac{\vec{A}}{A} = (\quad)$

$\vec{B_1} = B_1\hat{a} = \frac{2}{9}\hat{i} - \frac{1}{9}\hat{j} + \frac{2}{9}\hat{k}$

$\vec{B} = \vec{B_1} + \vec{B_2}$

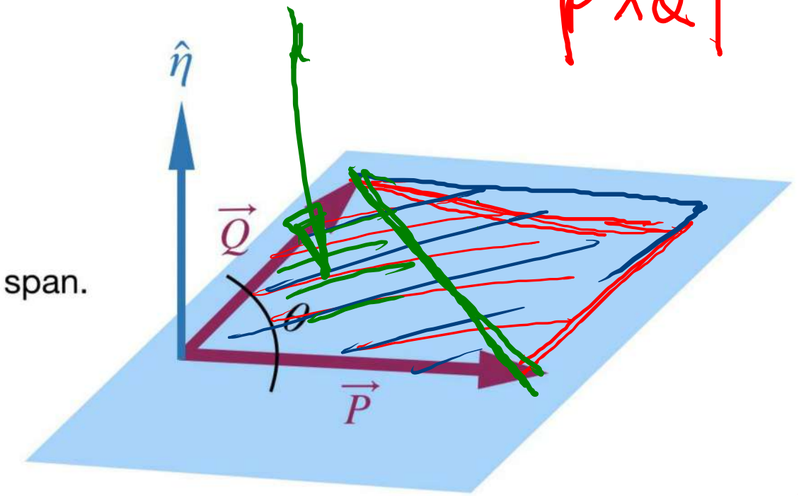
$\vec{B_2} = \vec{B} - \vec{B_1} = \frac{16}{9}\hat{i} + \frac{10}{9}\hat{j} - \frac{11}{9}\hat{k}$



# Cross Product

$$\vec{P} \times \vec{Q} = PQ \sin\theta \hat{\eta}$$

**Magnitude:** The area of the parallelogram that the vectors span.
**Direction:** Perpendicular to both P and Q, with a direction given by the right-hand rule.




**Condition for two vectors to be parallel** $\vec{P} \times \vec{Q} = PQ \sin 0^o = 0$

**or,** $\frac{P_x}{Q_x} = \frac{P_y}{Q_y} = \frac{P_z}{Q_z}$

**Unit vectors perpendicular to the plane containing** $\vec{P}$ **and** $\vec{Q}$ $\hat{\eta} = \pm \frac{\vec{P} \times \vec{Q}}{PQ \sin\theta}$

# Problem

$\vec{A} = 5\hat{i} - 6\hat{j} + 10\hat{k}$

$\vec{B} = -10\hat{i} + m\hat{j} - 20\hat{k}$

What is the value of $m$ if the vectors are parallel?

$\frac{5}{-10} = \frac{-6}{m} = \frac{10}{-20}$

$+1/2$

$m = 12$

$-1/2$

# Problem

$\overline{OA} + \overline{AB} = \overline{OB}$

$\overline{AB} = \overline{OB} - \overline{OA}$

Coordinates of 3 points A,B,C are (2,1,-1),(3,-2,4) and (1,-3,5)

(1) Check whether the position vectors of the points are in the same plane.[HSC - 2018]

(2) Find the area of a triangle formed by points.

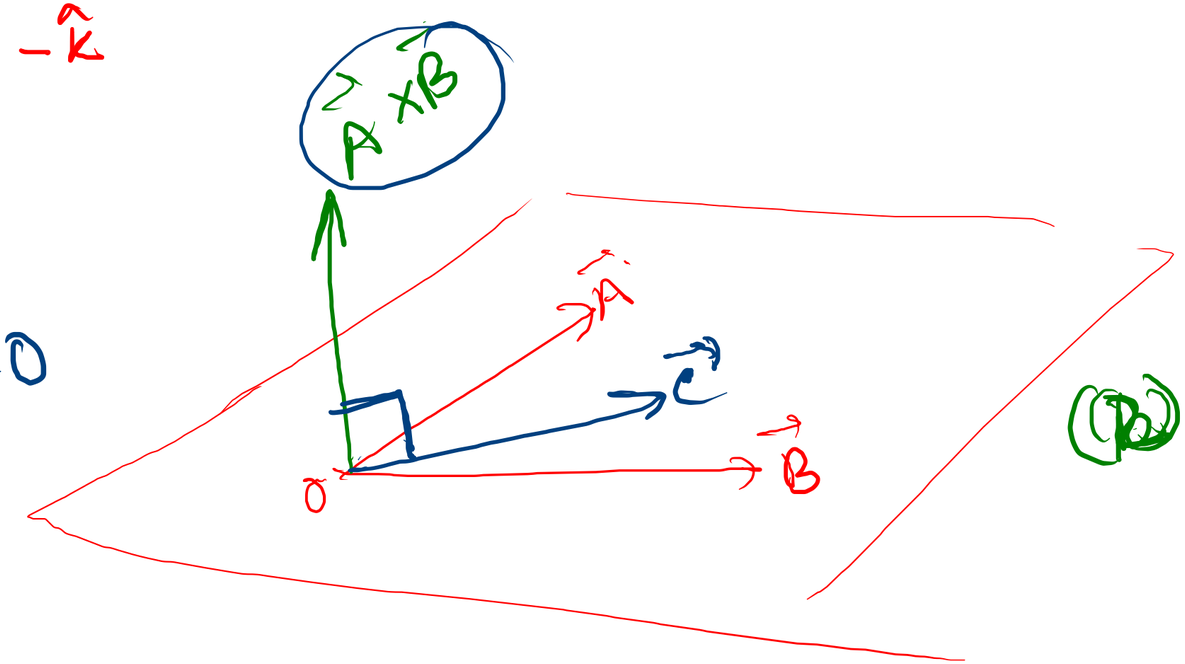
$\overline{OA} = 2\hat{i} + \hat{j} - \hat{k}$

$\overline{OB} =$

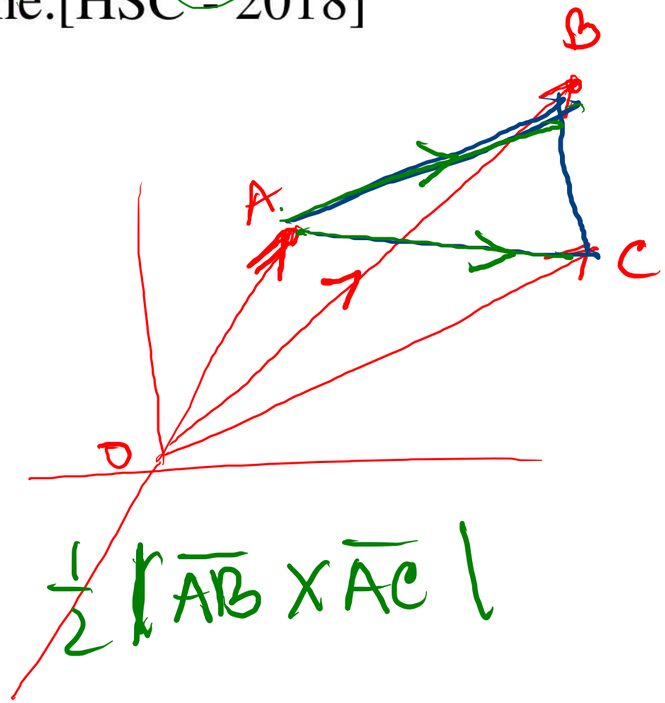
$\overline{OC} =$

If $(\overline{A} \times \overline{B}) \cdot \overline{C} = 0$

(1) → True

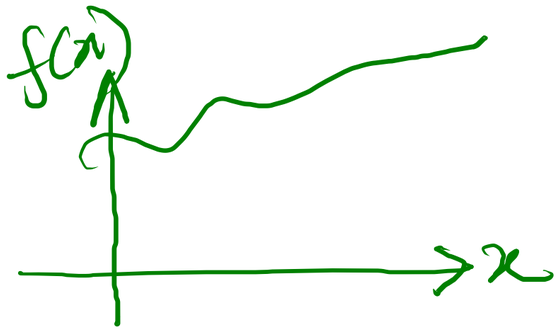



(2)

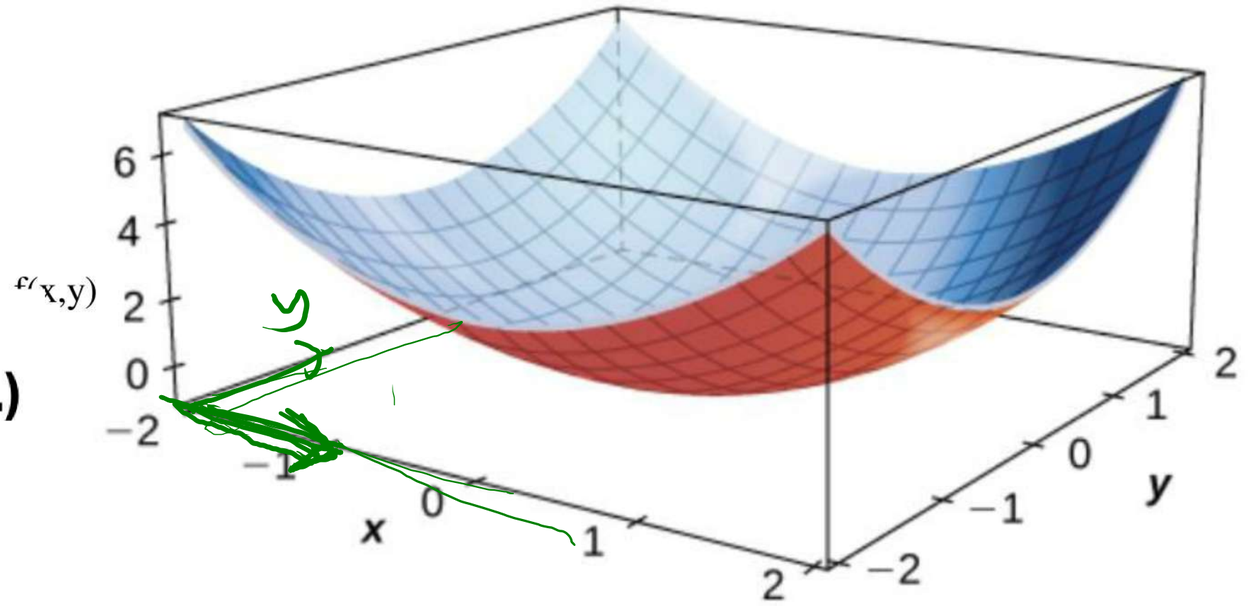



$\frac{1}{2} |\overline{AB} \times \overline{AC}|$

# Partial Derivative




$\frac{df}{dx}$




- **Function of single variable: f(x)**
- **Function of multiple variables: f(x,y,…..)**

$\frac{\partial f}{\partial x}$

$\frac{\partial f}{\partial x}$ → **The rate of change of function relative to x when other variables are constant**

# Problem

Find the partial derivatives w.r.t $x$ and $y$ of the function $f(x, y) = x^2 - 3xy + y^2$

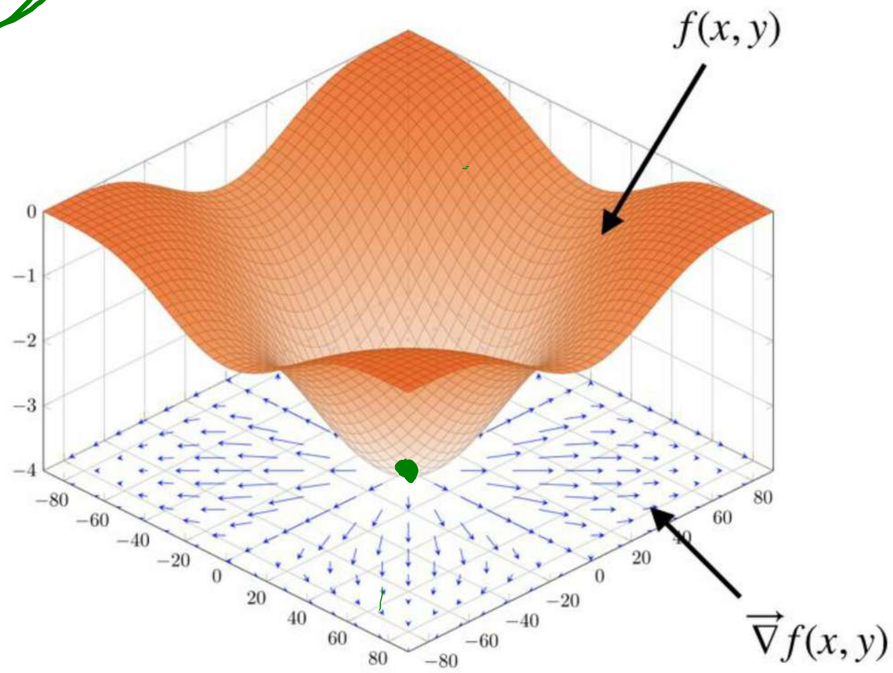
$$\frac{\partial f}{\partial x} = 2x - 3y$$

$$\frac{\partial f}{\partial y} = -3x + 2y$$

# Gradient

**Gradient of a scalar field** $f(x, y, z)$ : $grad\ f = \vec{\nabla} f \rightarrow$ **vector field**

- Gradient is only defined for scalar fields.
- Gradient points in the direction of the greatest increase of the corresponding scalar field .
- Magnitude of gradient corresponds to the maximum rate of increase of the scalar field at any point



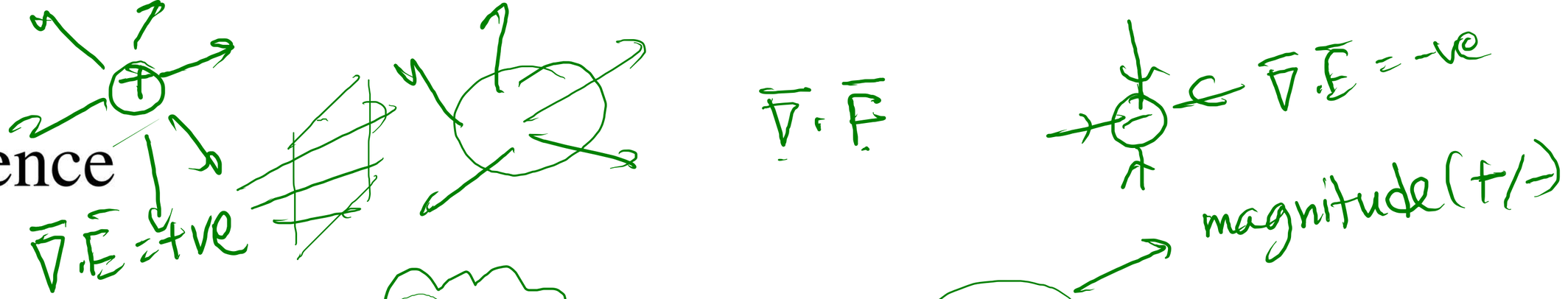
# Problem

Find the gradient of $f(x, y) = -\cos x + x.\cos y$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$= (\sin x + \cos y)\hat{i} + (-x \sin y)\hat{j} + 0$$

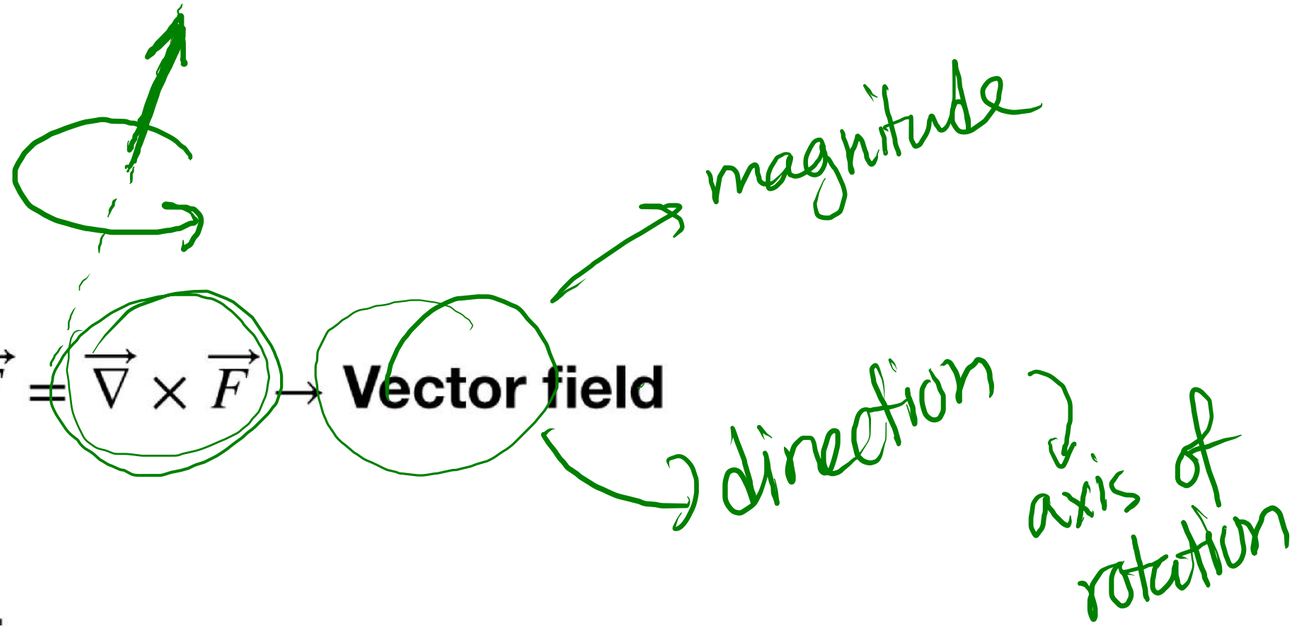
# Divergence




**Divergence of a vector field** $\vec{F}(x,y,z)$ : $div\,\vec{F} = \vec{\nabla}\cdot\vec{F}$ → **Scalar field**

- Divergence is only defined for vector fields.
- The value of divergence indicates the difference between input flux and output flux of an infinitesimal volume element around a point.
- For positive divergence : output flux > input flux
- For negative divergence : input flux > output flux
- If divergence is zero at all points then the vector field is called solenoidal

# Curl




**Curl of a vector field** $\vec{F}(x,y,z)$ : $curl\,\vec{F} = \vec{\nabla} \times \vec{F}$ → **Vector field**

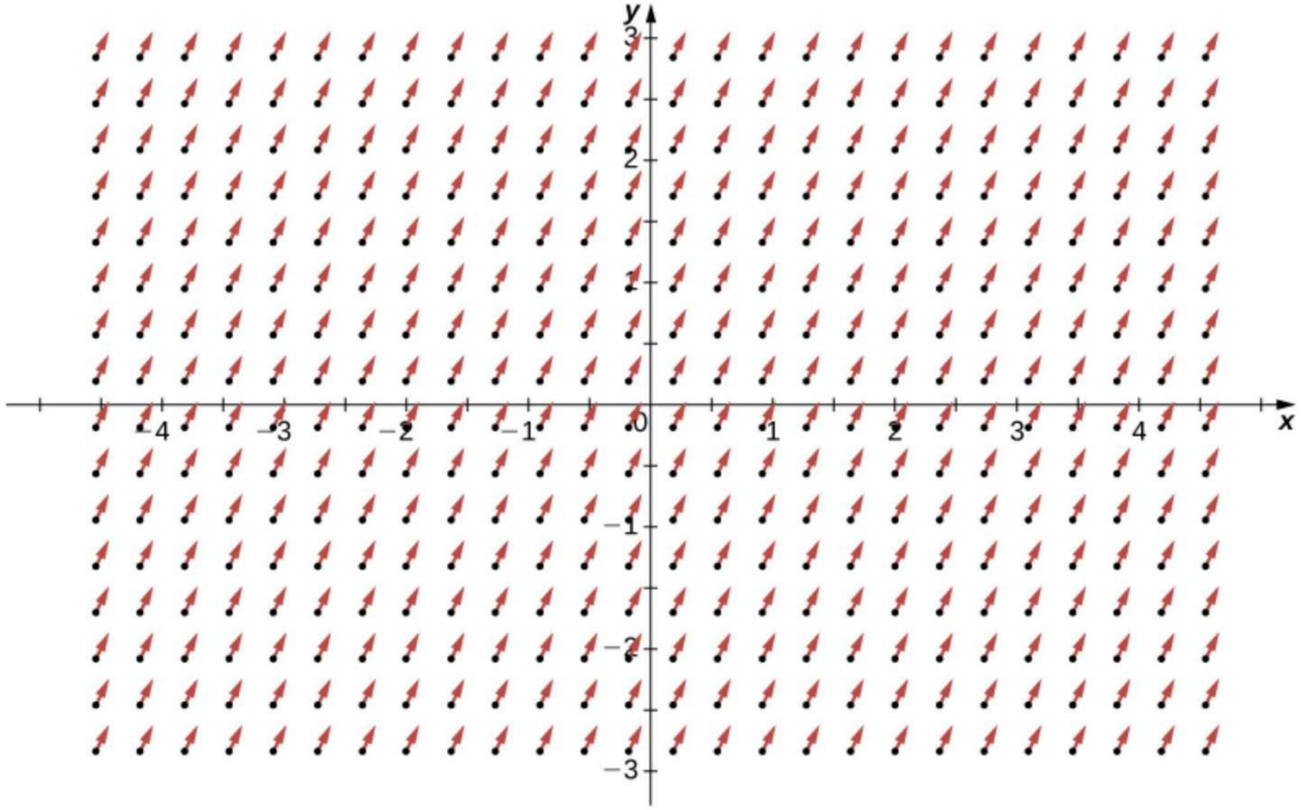
- Curl is only defined for vector fields.
- Curl of a vector field is also a vector field.
- The magnitude of curl indicates circulation of the field around a point and the direction shows the axis of circulation.
- If the curl is zero, then a velocity field is called irrotational and a force field is called conservative.
- $\vec{\nabla} \times \vec{V} = 2\vec{\omega}$
- $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{V}) = 0$

# Problem

Determine divergence and curl of $\vec{F} = \hat{i} + 2\hat{j}$

$\text{div}\,\vec{F} = 0$

$\text{curl}\,\vec{F} = 0$



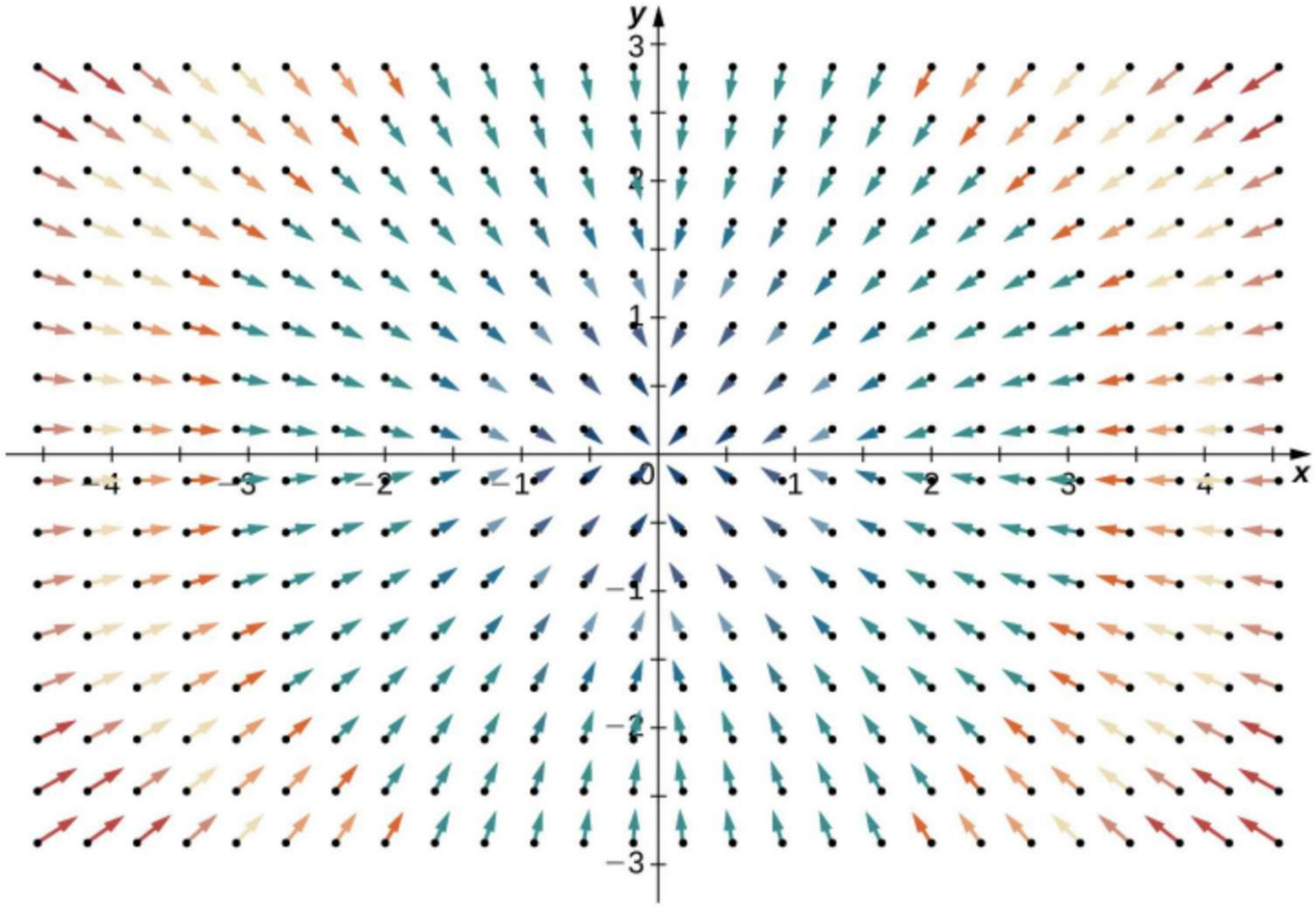

**Note:** Here at all points, input flux = output flux

# Problem

Determine divergence and curl of $\vec{F} = -x\hat{i} - y\hat{j}$

$\text{Div}\,\vec{F} = -2$

$\text{Curl}\,\vec{F} = 0$

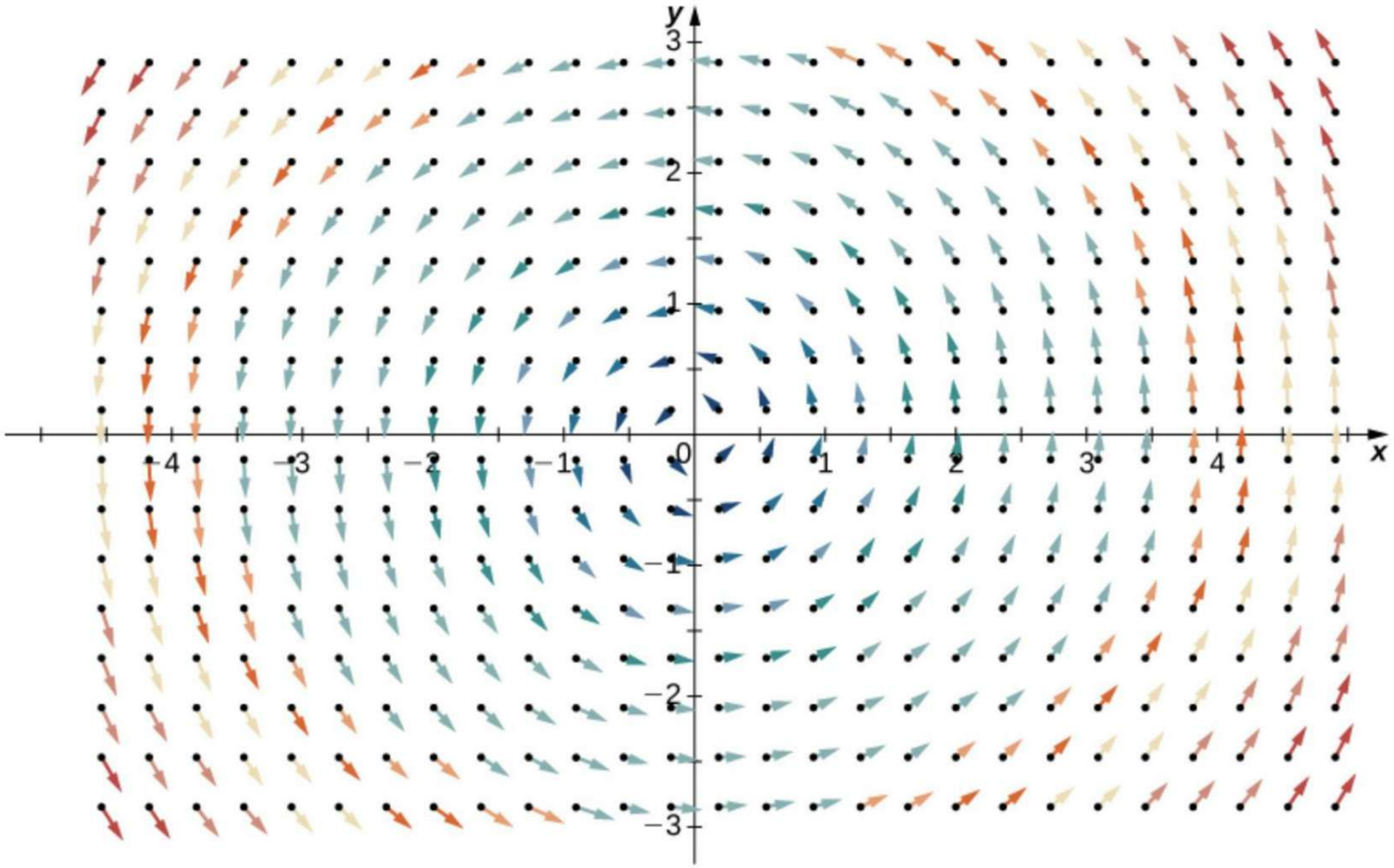


# Problem

Determine divergence and curl of $\vec{F} = -y\hat{i} + x\hat{j}$.

$\text{Div}\,\bar{F} = 0$

$\text{Curl}\,\bar{F} = +2\hat{k}$



# Image Sources

1. https://en.wikipedia.org/wiki/Gradient

2. **https://openstax.org/books/calculus-volume-3/pages/6-5-divergence-and-curl**

# Poll Question 01

Which one is a vector quantity?

(a) Area

(b) Current

# Poll Question 02

The magnitudes of two vectors are 5 and 10 units respectively. Which of the following can not be the magnitude of their resultant.

(a) 3

(b) 5

(c) 7

# Poll Question 03

With the increase in the angle between two vectors, the magnitude of their resultant ($\theta < 180°$)-

(a) Increases

(b) Decreases

(c) Stays constant

# Poll Question 04

The speed of the current in a river is $10m/s$ and a boat's speed is $5m/s$. What is the required angle of the boat with current to cross the river straight?

(a) 120

(b) 90

(c) 60

(d) None of the above

# Poll Question 05

It is raining at a 30 degree angle with the horizontal to the windshield of a car. As the speed of rain increases, the angle of the rain relative to the vehicle-

(a) Increases

(b) Decreases

(c) Stays the same

(d) Decreases

# Poll Question 06

If two vectors, A and B, are parallel then which of the following is correct?

(a) $A \times B = 0$

(b) $A.B = 0$

(c) $A = mB$ where $m$ is a scalar

(d) Both b and c

# Poll Question 07

The tow vectors $3i + 5j - 2k$ and $-6i + mj - 4k$ will be parallel if the value of $m$ is-

(a) 10

(b) $-10$

(c) Both a and b

(d) None of the above

# Poll Question 08

For two vectors A and B, $(A + B).(A \times B) = ?$

(a) 0

(b) $(AB)^2$

(c) None of the above

# Poll Question 09

Which of the following is correct if the electric field is a conservative vector field?

(a) Gradient = 0

(b) Divergence = 0

(c) Curl = 0

# Poll Question 10

For which of the following divergence can not be determined?

(a) Velocity field

(b) Magnetic field

(c) Temperature field

# না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে।