Engineering Admission Program-2020

PHYSICS

Lecture : P-02

Chapter 03 : Dynamics

উদ্ভাস

একাডেমিক এন্ড এডমিশন কেয়ার

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Equations for Linear Motion

$\vec{a}$ = const.

Initial Velocity $\xrightarrow{a}$ Final Velocity

u — s — v

A — Time t — B

deceleration

a = −

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

t

$$v = u - at$$

$$s = ut - \frac{1}{2}at^2$$

$$v^2 = u^2 - 2as$$

Displacement for the 'th' second,

$$\boxed{s_{th} = u + \frac{1}{2}a(2t - 1)}$$

For uniform velocity,

$$s = vt$$

Problem-01

Starting from rest an object covers 1m in its 1st second. At the last second of motion, it covers $\frac{3}{4}th$ of the total distance travelled. Find the distance travelled and time taken.

$u=0$

s_{total}

t

$$s_1 = \frac{1}{2} a t_1^2$$

$$\therefore 1m = \frac{1}{2} \times a \times (1s)^2 \Rightarrow a = 2ms^{-2}$$

$$s_{(\text{last second} \;\; t\text{-th sec.})} = \frac{3}{4} s_{total}$$

$$\therefore u + \frac{1}{2} a(2t-1) = \frac{3}{4}\left(ut + \frac{1}{2}at^2\right)$$

$$\cancel{\tfrac{1}{2}}\cancel{g}(2t-1) = \frac{3}{4}\cdot\cancel{\tfrac{1}{2}}\cancel{g}t^2$$

$$\therefore\ 3t^2 - 8t + 4 = 0$$

$$\therefore\ t = 2,\ \frac{2}{3}$$

$0.67\,s < 1\,s$

not acceptable

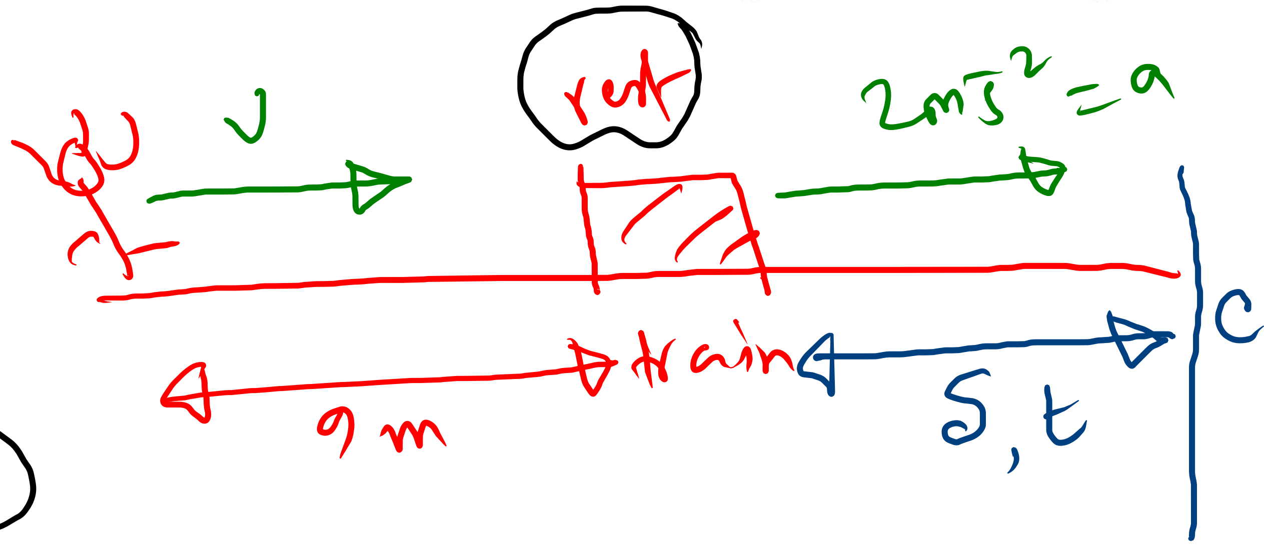
$\checkmark\ t = 2$ sec.

$$s_{total} = \frac{1}{2}at^2 = \frac{1}{2}\times 2\times 2^2 = 4\,m$$

Problem-02

A person saw a train starting from rest at $2ms^{-2}$ acceleration when he was 9m away from the train. At once he starts to run at uniform velocity. What is the minimum velocity required to catch the train? [BUET '09-'10]

$v_{min} = ?$



train,

$$s = \frac{1}{2}at^2 = \frac{1}{2} \cdot 2 \cdot t^2$$

$$\therefore s = t^2 \quad \text{.......} (1)$$

person,

$$s + 9 = vt \quad \text{.......} (ii)$$

from, ① & ② $\Rightarrow$ $t^2 - vt + 9 = 0$

$ax^2 + bx + c = 0$

$D = b^2 - 4ac$

$\therefore$ $D \geq 0$ (for real values of t)

$(-v)^2 - 4 \cdot 1 \cdot 9 \geq 0$

$v^2 \geq 36$

$v \geq 6$

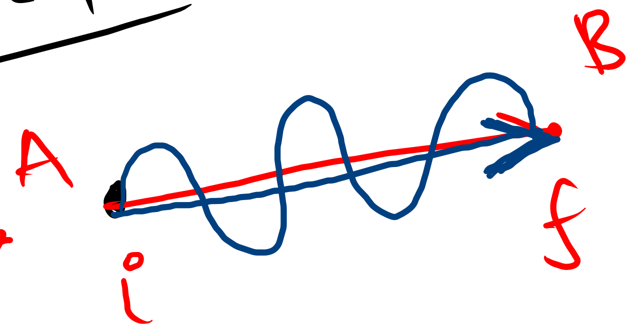
$\boxed{v_{min} = 6\,ms^{-1}}$

Average Velocity

Avg. velocity of an object,

vector

$$\bar{v} = \frac{Total\ Displacement}{Total\ Time}$$



Avg. Speed,

scaler

$$\bar{v} = \frac{Total\ Distance\ Travelled}{Total\ Time}$$

If the object is in uniform acceleration,

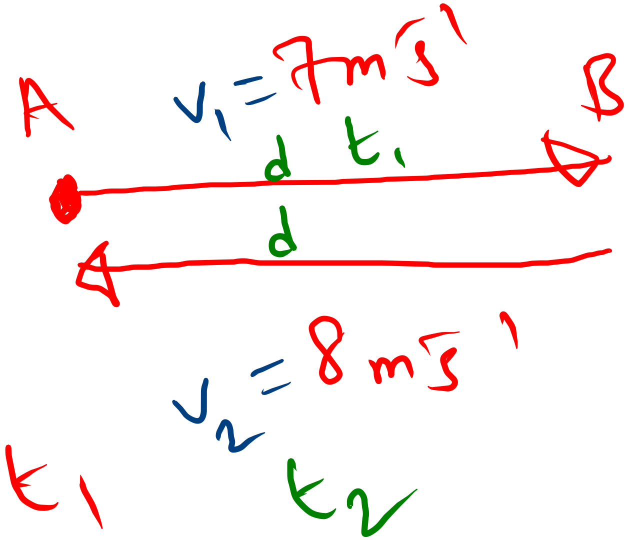
$a = \text{const.}$

$$\bar{v} = \frac{Initial\ Velocity + Final\ Velocity}{2}$$

$= \frac{u+v}{2} = v_{avg}$

Problem-04

A person travels from A to B with $7ms^{-1}$ velocity and returns to A with $8\,ms^{-1}$ velocity. What is his avg velocity and avg speed?



displacement $= 0$

avg. velocity $= 0$

$\therefore$ avg. speed $= \dfrac{d_{total}}{t_{total}}$

$= \dfrac{2d}{t_1 + t_2}$

$d = v_1 t_1$

$\therefore t_1 = \dfrac{d}{v_1}$

$t_2 = \dfrac{d}{v_2}$

$\therefore$ avg. speed $= \dfrac{2d}{d/v_1 + d/v_2} = \dfrac{2v_1v_2}{v_1+v_2}$

$= \dfrac{2 \times 7 \times 8}{7+8}\ ms^{-1}$

$= 7.467\ ms^{-1}$

Instantaneous Velocity and Acceleration

$V = \frac{ds}{dt} \Rightarrow , \int ds = \int v\,dt$

If s(t) indicates the displacement of any object then its velocity is given by,

$$v(t) = \frac{ds(t)}{dt}$$

$$s(t) = \int v(t)\,dt$$

$$\boxed{s = \int v\,dt}$$

Area under the v vs t curve gives the displacement

s = v-t curve area

If the velocity at any time is $v(t)$, then acceleration is given by,

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2 s(t)}{dt}$$

$$\Rightarrow v(t) = \int a(t)\,dt$$

Problem- 05

The equation of motion of a body is, $v = 3t^2 + 4t$

i. What is the acceleration at t = 2 sec?

ii. Find the displacement for t = 2s to t = 4s.

iii. What is the average velocity for t = 2s to t = 4s?

iv. What is the displacement at 5[th] sec?

i) $a = \frac{dv}{dt} = 6t + 4$

$t = 2 sec,$

$\therefore a = 6 \times 2 + 4 = 16\ ms^{-2}$

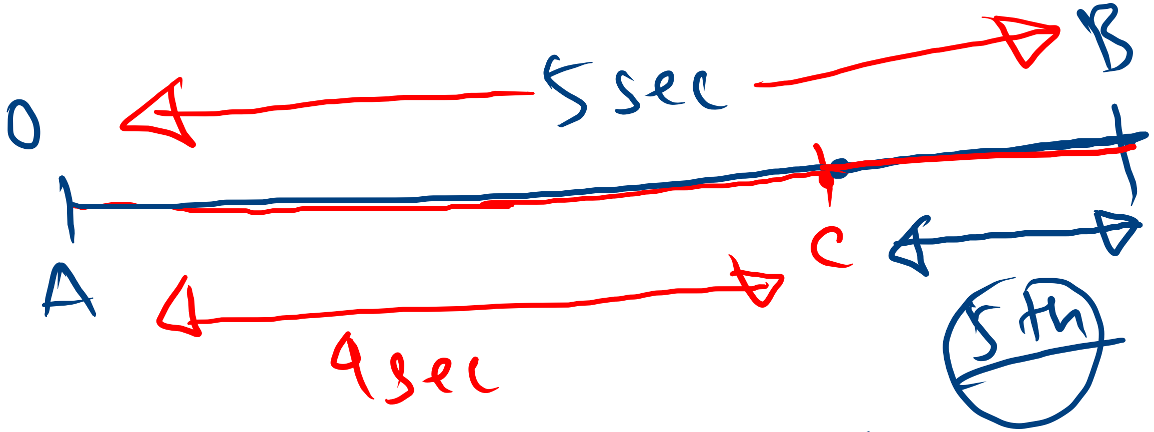
ii) $S = \int_{t=2s}^{t=4s} v\,dt = \int_{2}^{4} (3t^2 + 4t)\,dt$

$= \left[3 \cdot \frac{t^3}{3} + 4 \cdot \frac{t^2}{2}\right]_2^4 = 80\ m$

$t = 4 sec$

(iii) $v_{avg} = \dfrac{s_{total}}{t_{total}} = \dfrac{\int_2^4 v\,dt}{2\text{ sec.}} = ?$

(iv)



$$s_{(5th)} = s_5 - s_4 = \int^{t=5s.} v\,dt = ?$$

Graph

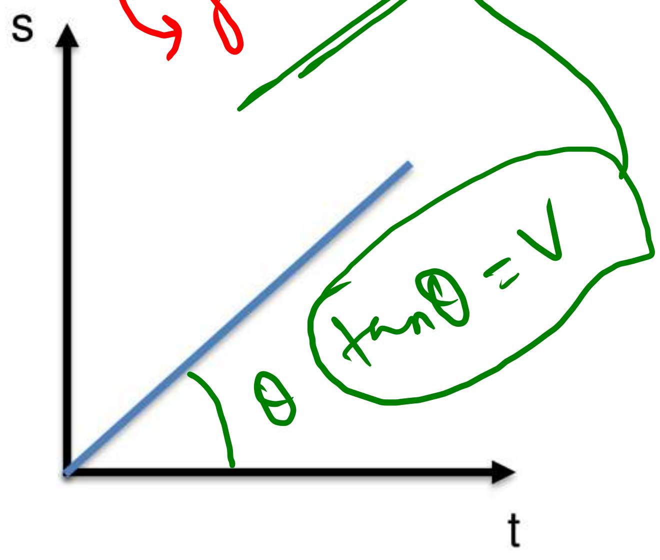
Uniform Velocity

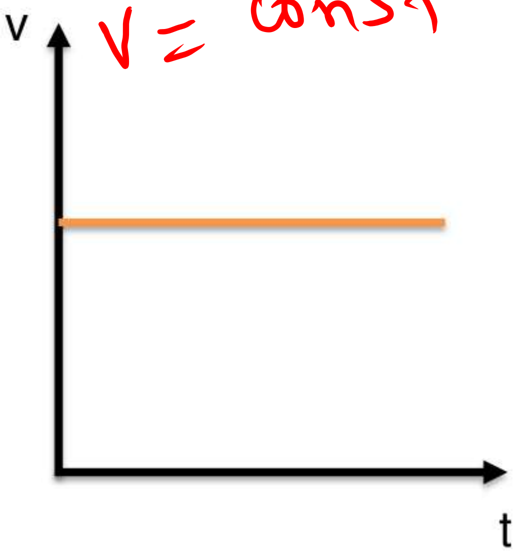
$\boxed{s = vt}$

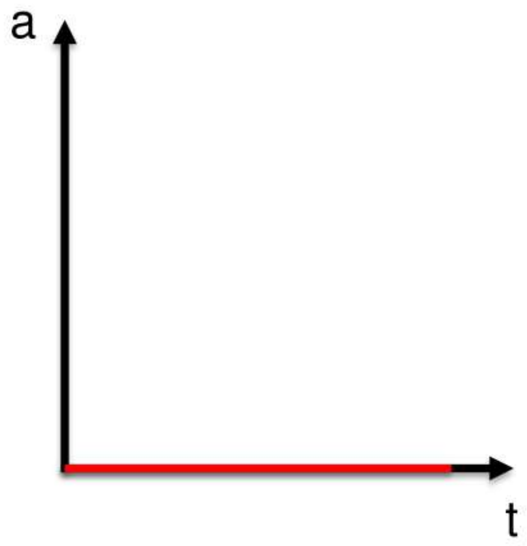
$y = mx$

$\frac{dv}{dt} = a = 0$

$v = \text{constant}$







Graph

Uniform Acceleration

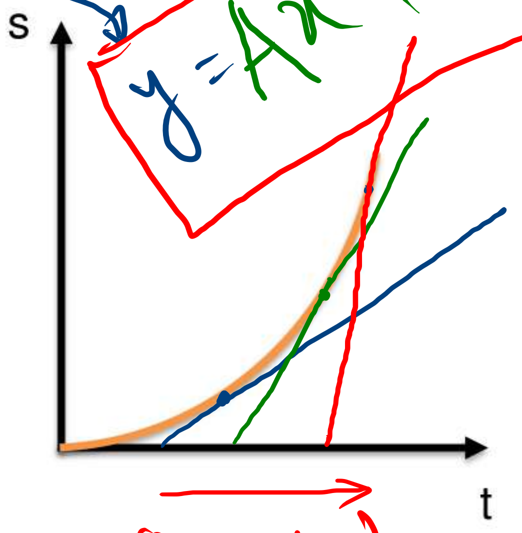
$s = ut + \frac{1}{2}at^2$

$\boxed{y = Ax + Bx^2}$

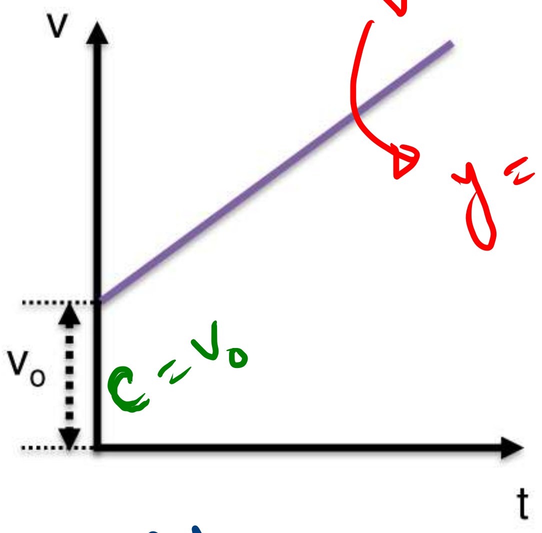
$v = v_0 + at$

$y = c + mx$

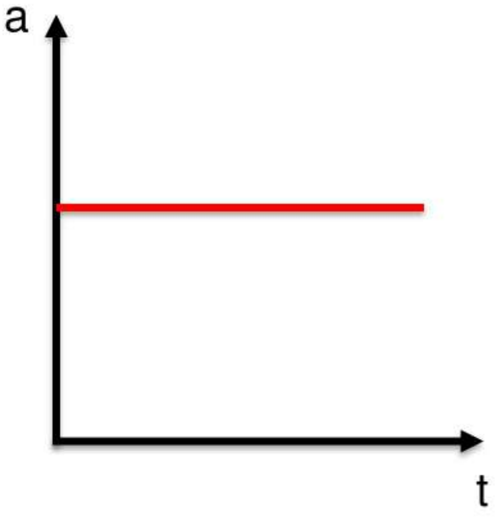
$a = const.$



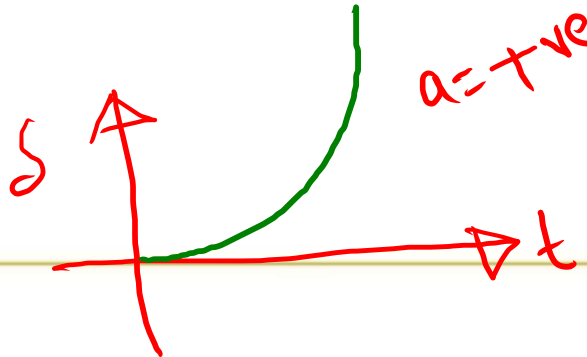
parabola



Slope = a



$v = \frac{ds}{dt}$

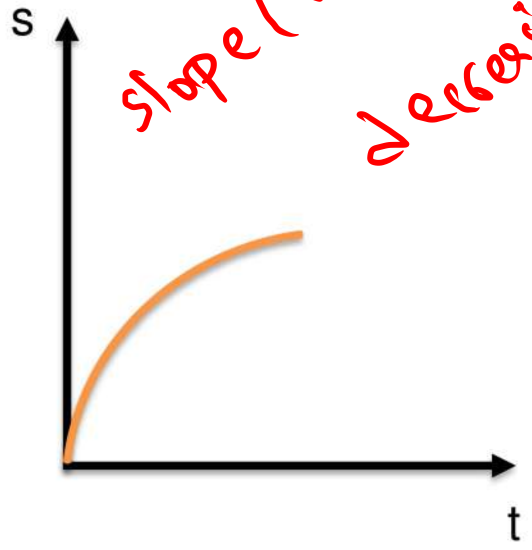


Graph

Uniform Deceleration

s vs t parabola ✓

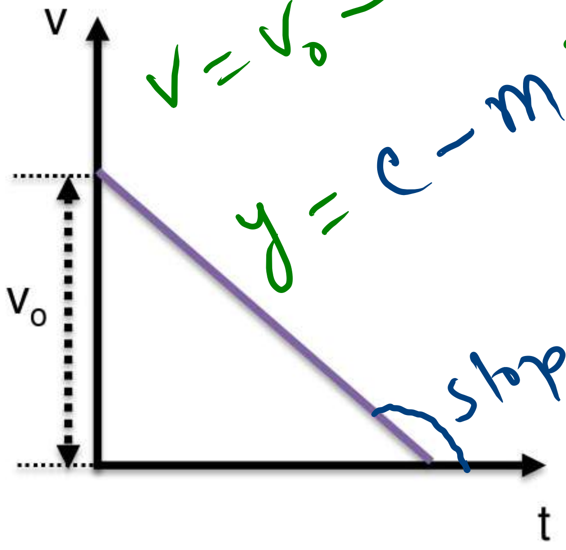
slope (v) continuously decreasing



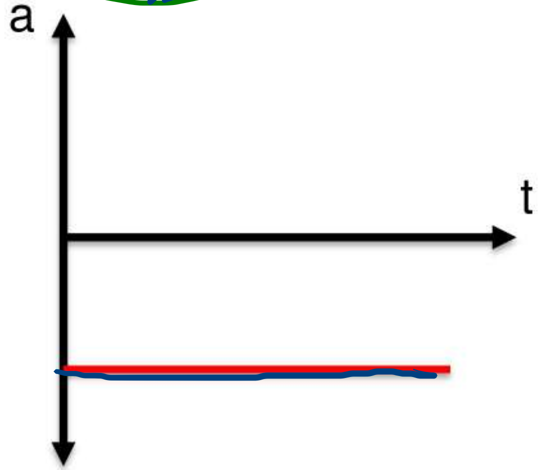
$v = v_0 - at$

$y = c - mx$

slope = −



a = − constant



Problem-06

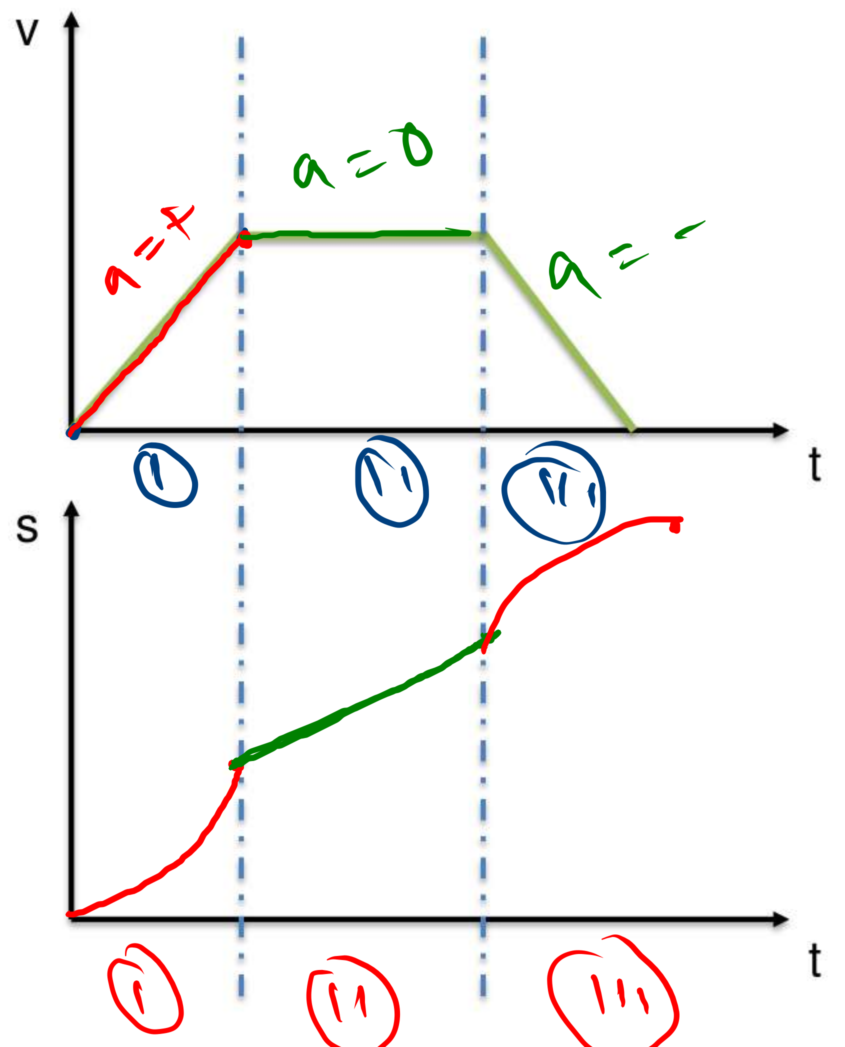
v vs **t** graph is given

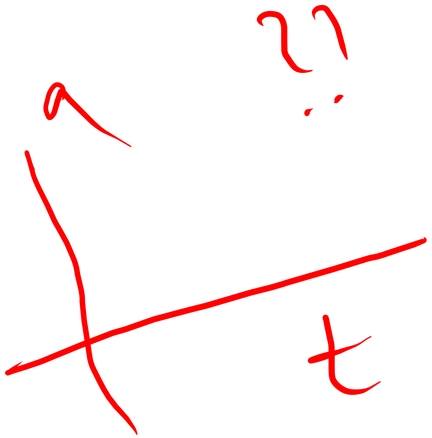
Draw the s vs t

(i) parabola

(ii) straight line

(iii) parabola



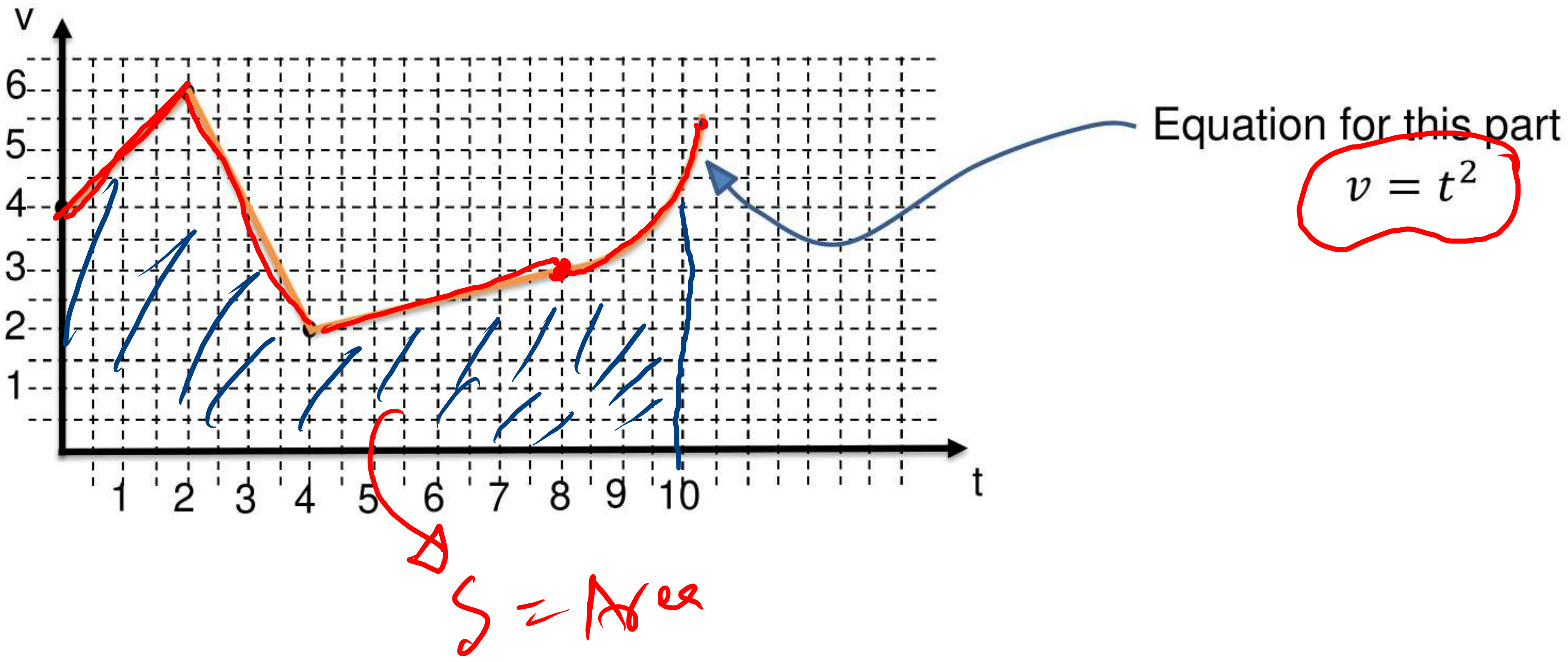


~~HW~~

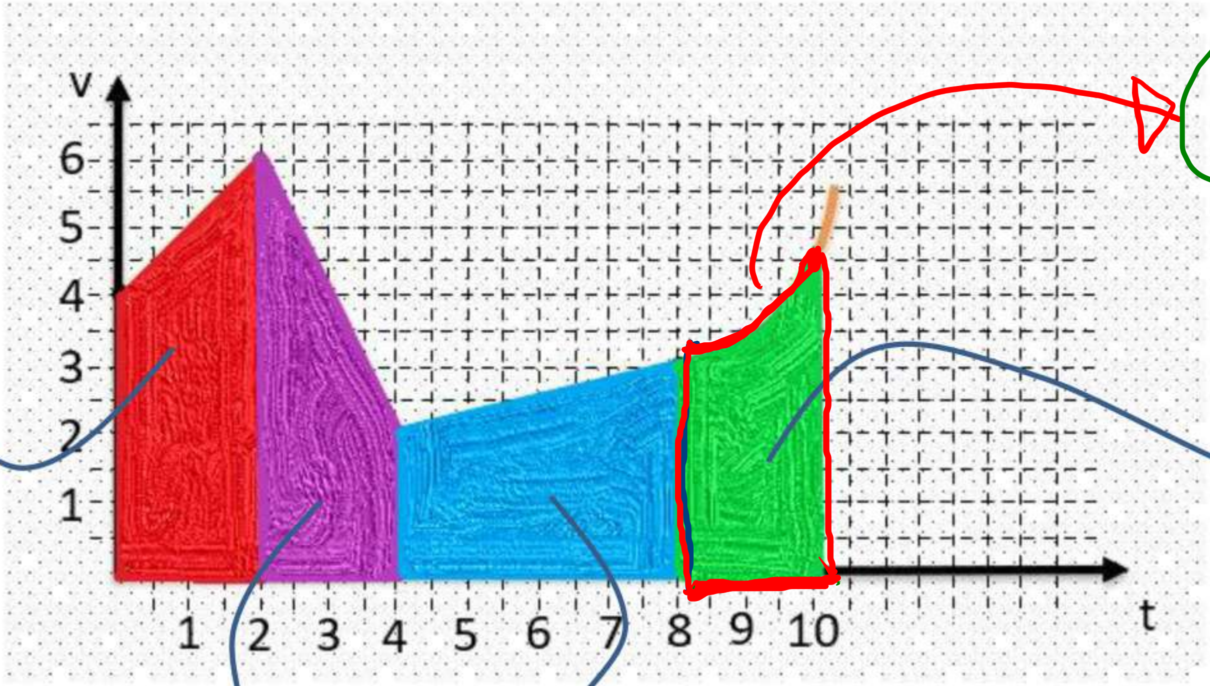
Problem-07

$S = \int v\,dt = \text{Area}$

What is the displacement for first 10 seconds?



Solution



$v = t^2$

$s = \int_8^{10} v\,dt$

$$s = \int_8^{10} t^2\, dt = 162.67$$

$$\frac{1}{2} \times 2 \times (4+6) = 10$$

$$\frac{1}{2} \times 2 \times (6+2) = 8$$

$$\frac{1}{2} \times 4 \times (2+3) = 10$$

$s = 190.67$ unit

total

$10 + 8 + 10 + 162.67 =$

Practice Problem

Two cars start a competition with 6 ms^{-1} and 2 ms^{-1} initial velocity and 3 ms^{-2} and 4 ms^{-2} uniform acceleration from the same point. They reach the destination at the same time.

i. Duration of the competition ?
ii. Distance covered by the cars?

Let, they cover s distance in t time.
For the 1st car,

$$s = 6t + \frac{1}{2} \times 3 \times t^2$$

For the 2nd car,

$$s = 2t + \frac{1}{2} \times 4 \times t^2$$

From the equations,

$$6t + \frac{3}{2}t^2 = 2t + 2t^2$$

$$\Rightarrow \frac{1}{2}t^2 - 4t = 0$$

$$\Rightarrow t(t-8) = 0$$

$$\Rightarrow t = 0, 8$$

Here, t = 8 sec is the duration.

Distance,

$$s = 6 \times 8 + \frac{3}{2} \times 64$$

$$\Rightarrow s = 144 \text{ m}$$

Practice Problem

An object moving in uniform acceleration crosses 18m and 26m distance in 3rd and 7th second respectively. Find the displacement for 10th second.

Let, Initial velocity = u
acceleration = a
For 3rd second,

$$18 = u + \frac{1}{2}a(2 \times 3 - 1)$$
$$\Rightarrow 36 = 2u + 5a$$

For 7th second,

$$26 = u + \frac{1}{2}a(2 \times 7 - 1)$$
$$\Rightarrow 52 = 2u + 13a$$

From the two equations,

$$52 - 36 = 13a - 5a$$
$$\Rightarrow 16 = 8a$$
$$\Rightarrow a = 2 \ ms^{-2}$$

Putting the value of a,

$$36 = 2u + 5 \times 2$$
$$\Rightarrow u = 13 \ ms^{-1}$$

For 10th second,

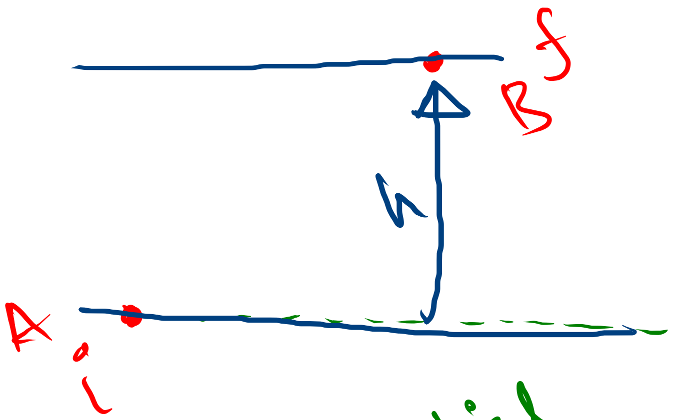
$$s_{10} = 13 + \frac{1}{2} \times 2 \times (2 \times 10 - 1)$$
$$\Rightarrow s_{10} = 32 \ ms^{-1}$$

Falling Bodies

vertical $\vec{h}$ (displacement) vector , $\vec{u}$, $\vec{g}$ (নিচে)

initial velocity

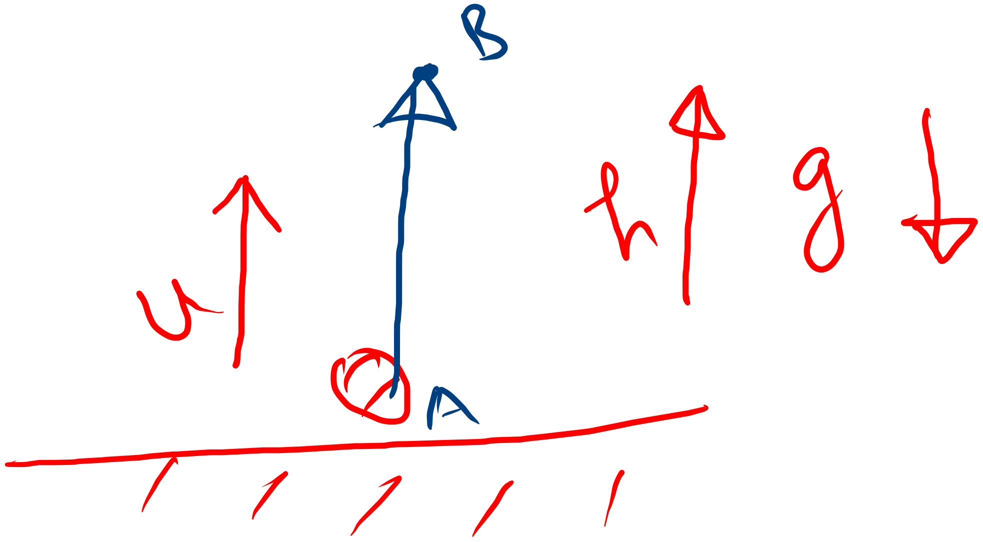
{ object initially (ঐ direction throw



$\vec{h}$ always initial to final

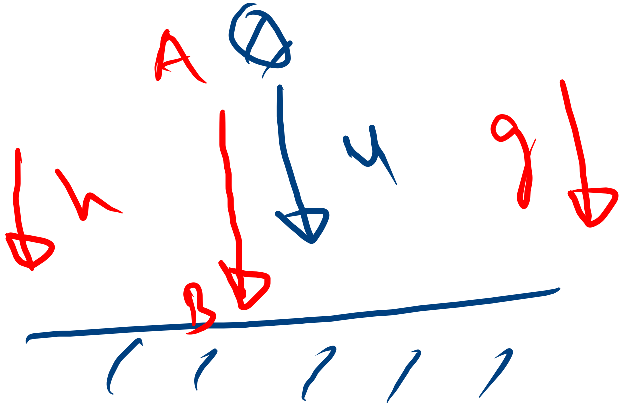
{ Similar direction, → +
opposite direction → −

Case 01:



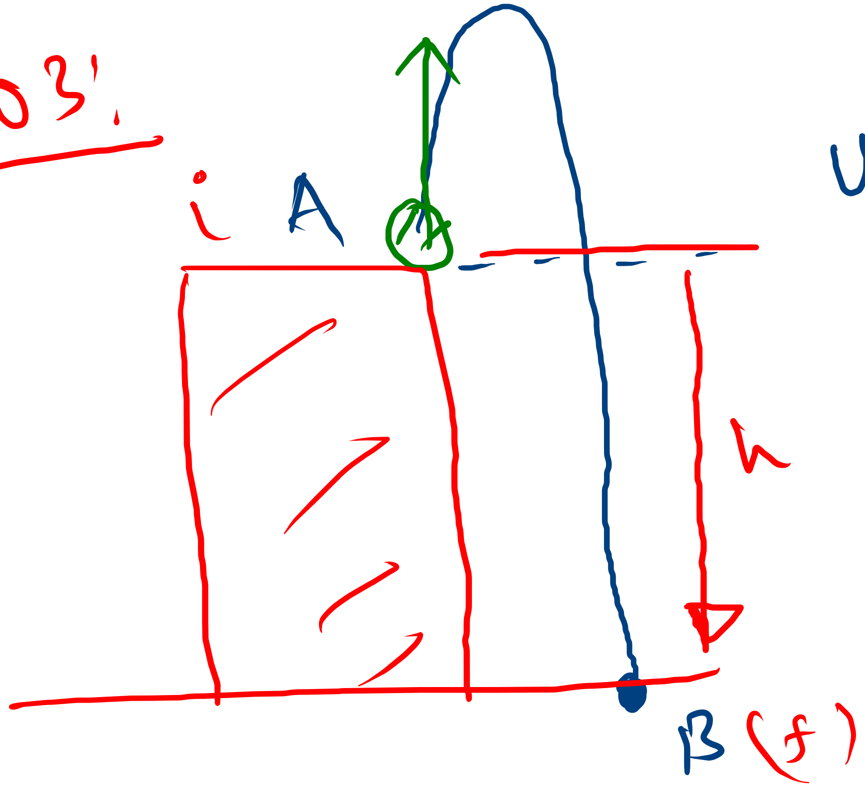
$$+h = +ut - \frac{1}{2}gt^2$$

Case-02:



$$h = ut + \frac{1}{2}gt^2$$

Case-03:



$u\uparrow$ $h\downarrow$ $g\downarrow$

$-$ $+$ $+$

$$\boxed{+h = -ut + \frac{1}{2} g t^2}$$

Problem-09

net result

A gas balloon is going upward with uniform velocity. After 4.5 sec an object gets detached from the balloon and reaches the ground after 7 more seconds. Find the velocity of the balloon and the height from where the object fell off.

ballon,

$AB = h = 4.5u$

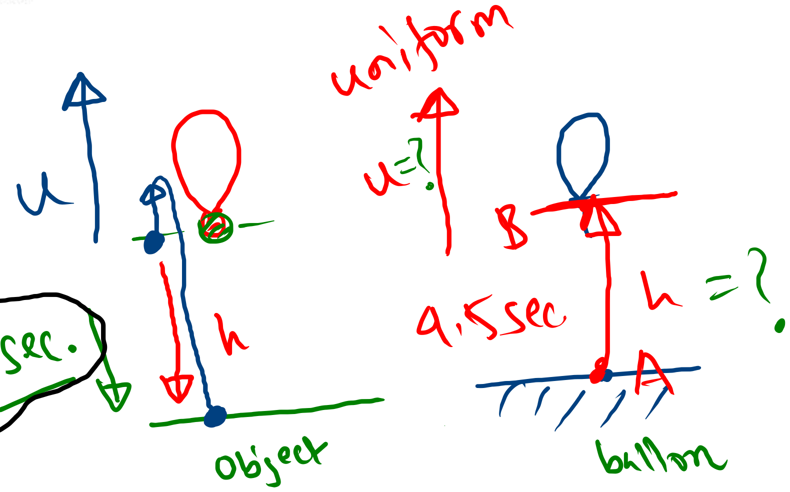
$u_{ballon} = u_{object} = u$

object,

$u \uparrow \quad h \downarrow \quad g \downarrow$

$- \quad + \quad +$

$t = 7$ sec.



$h = -ut + \frac{1}{2}gt^2$

$\therefore 4.5u = -u \times 7 + \frac{1}{2} g \times 7^2$

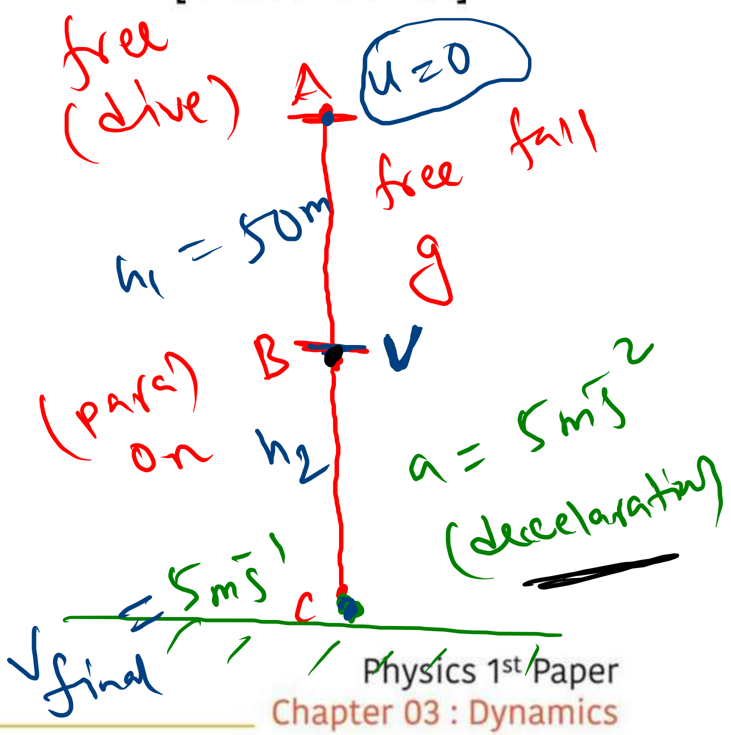
$u = ?$

$h = 4.5u = ?$

Problem-10

A skydiver opens the parachute after falling freely for 50m. As a result the rate of decrease of his speed is $5ms^{-2}$. He reaches the ground at $5ms^{-1}$ velocity.
What is the height where he became free? [BUET '11-'12]

$$h = h_1 + h_2 = ?$$



AB $\rightarrow$ $$v^2 = 0^2 + 2gh_1$$

$$\therefore v = \sqrt{2gh_1} = 14\sqrt{5}\,ms^{-1}$$

BC $\rightarrow$ $$v_{final}^2 = v^2 - 2ah_2$$

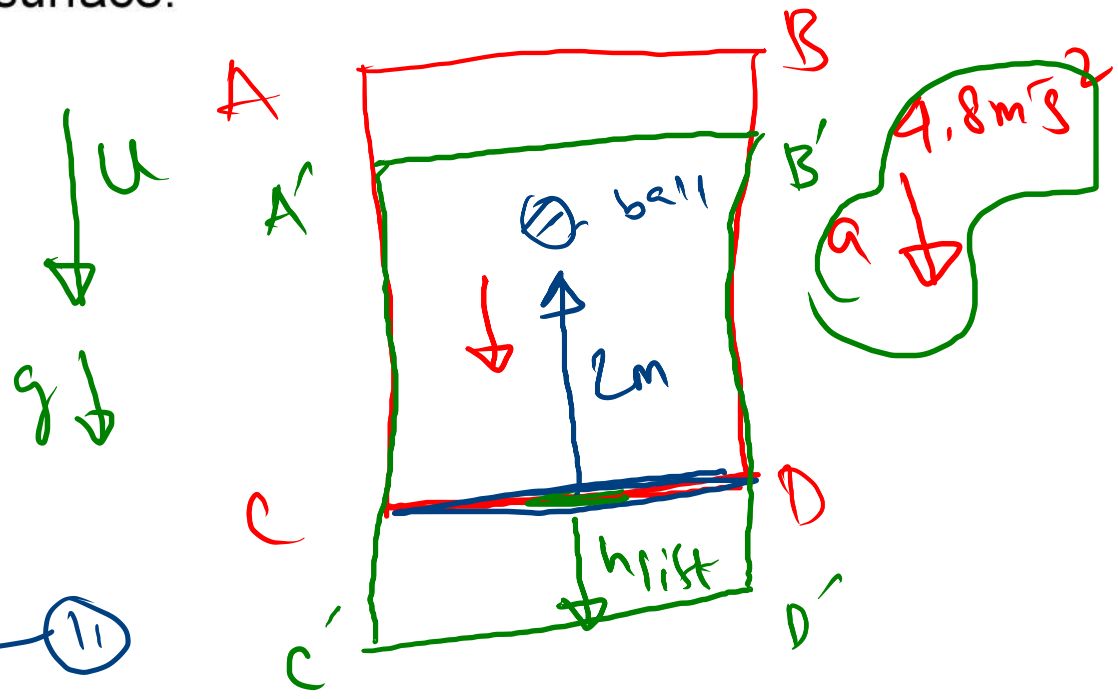
$$\therefore \boxed{h_2 = 95.5m}$$

$$\therefore h = h_1 + h_2 = 50 + 95.5$$
$$= 195.5\,m$$

Problem-11

A lift is going down at $4.8\ ms^{-2}$ net acceleration. A ball is falling freely from 2m height inside the lift. Find the time it will take to reach the lift's surface.

$$h_{lift} = ut + \frac{1}{2} a t^2 \quad \text{— (i)}$$



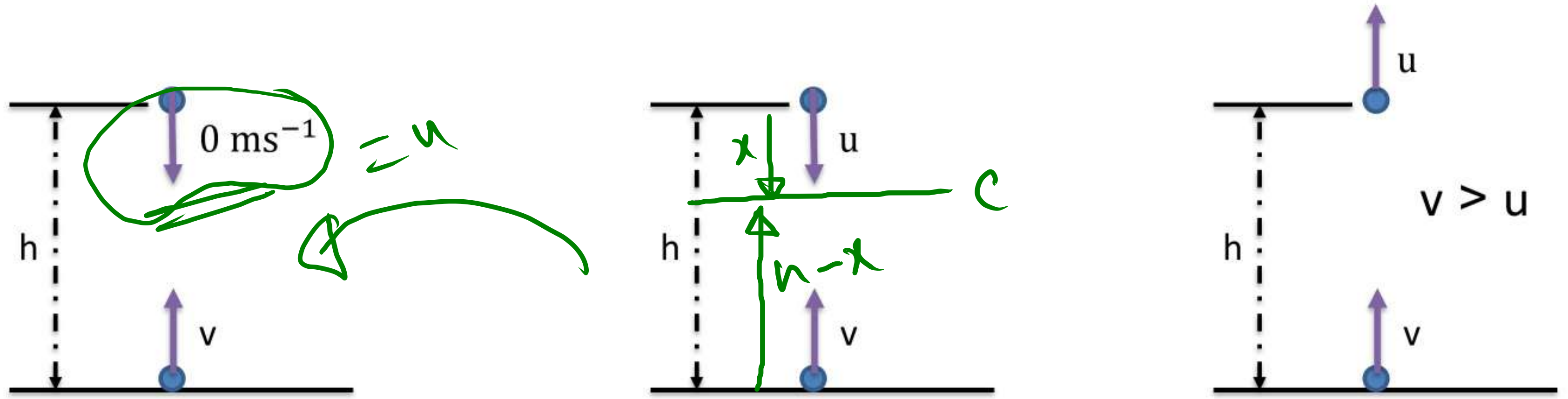
ball, $u_{ball} = u_{lift} = u$

$$2 + h_{lift} = ut + \frac{1}{2} g t^2 \quad \text{— (ii)}$$

$$\text{(ii)} - \text{(i)} \Rightarrow \boxed{2 = \frac{1}{2}(g - a)t^2}$$

$$t = ?$$

When the bodies will meet?



$t = \frac{h}{0+v}$

$t = \frac{h}{v}$

১ম, $x = ut + \frac{1}{2}gt^2$

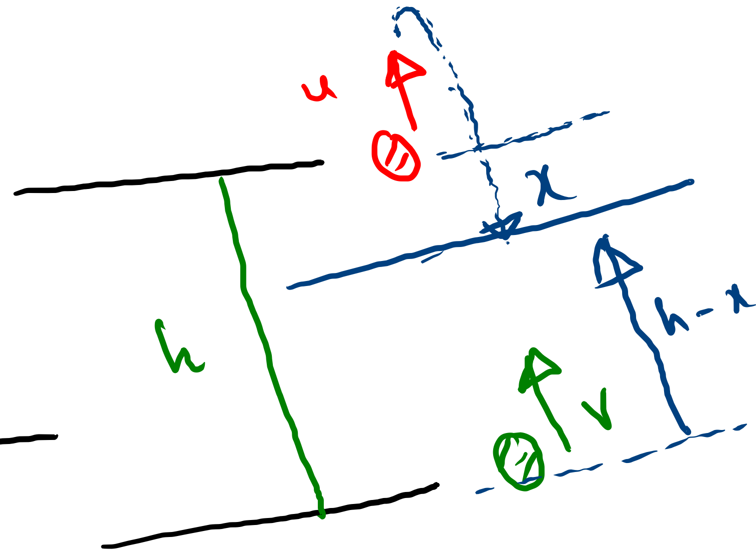
২য়, $h - x = vt - \frac{1}{2}gt^2$

$h = (u+v)t$

$$\boxed{t = \frac{h}{u+v}}$$

3rd Case ; object 1, $x = -ut + \frac{1}{2}gt^2$

object 2, $h - x = vt - \frac{1}{2}gt^2$



$$h = (v-u)t$$

$$\therefore t = \frac{h}{v-u}$$

Problem-12

A body is left downward from 400m height & at the same time another body is thrown vertically upward $50ms^{-1}$ velocity. When will they meet? [KUET '11-'12]

$$t = \frac{h}{u+v}$$

$$t = \frac{h}{0+v}$$

$$t = \frac{h}{v}$$

Practice Problem

An object is thrown upward at $30\ ms^{-1}$ from the top point of a tower and after 4s another object is released from the same height with no initial velocity. If they reach the ground at the same time, What is the height of the tower? [g = 10 ms^{-2}]

Let, time taken by second one is t sec
For the second object,

$h = \frac{1}{2}gt^2$

For the first one,

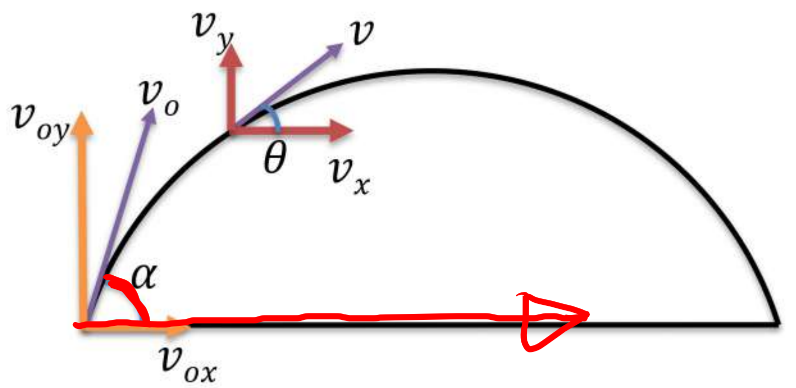
$h = -30(t+4) + \frac{1}{2}g\,(t+4)^2$

$\Rightarrow 5t^2 = -30t - 120 + 5(t^2 + 8t + 16)$

$\Rightarrow t = 4s$

Height of the tower, $h = \frac{1}{2} \times 10 \times 4^2 = 80\ m$

Projectile



Initial velocity v_0

Angle of Projection α

Horizontal component of initial velocity v_{x0}

Vertical component of initial velocity v_{y0}

$$\begin{cases} v_{ox} = v_0 \cos\alpha \\ v_{oy} = v_0 \sin\alpha \end{cases}$$

g downward

$a_x = 0$;

$v_x = v_{ox} + a_x t$

$$\boxed{v_x = v_0 \cos\alpha}$$

$v_0 \uparrow \quad g \downarrow$

$v_y = +(v_0 \sin\alpha) - gt$

Now, $v_{min} = ?$

$\cancel{v_{min} = 0}$

at, max. height

$v_y = 0$

$v_x = v_0 \cos\alpha$

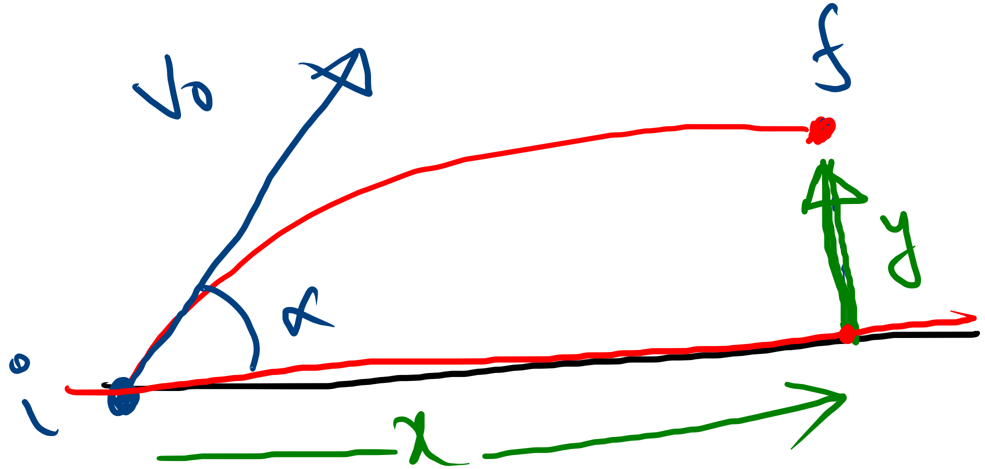
$\therefore v_{min} = v_0 \cos\alpha$

$v_y \rightarrow (+) \uparrow$

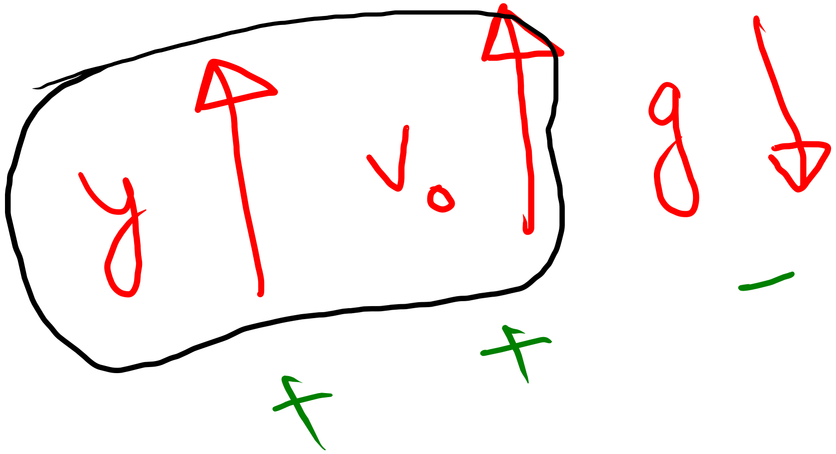
$v_y \rightarrow (-) \downarrow$

$\therefore v = \sqrt{v_x^2 + v_y^2}$

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$



$$\begin{cases} x = (v_0 \cos\alpha)\, t \\ +y = +(v_0 \sin\alpha)\, t - \frac{1}{2} g t^2 \end{cases}$$



Projectile

Equation of the parabolic path :

$$y = x \tan\alpha - \frac{g\,x^2}{2\,v_0^{\,2}\cos^2\alpha}$$

Relation between position and Range:

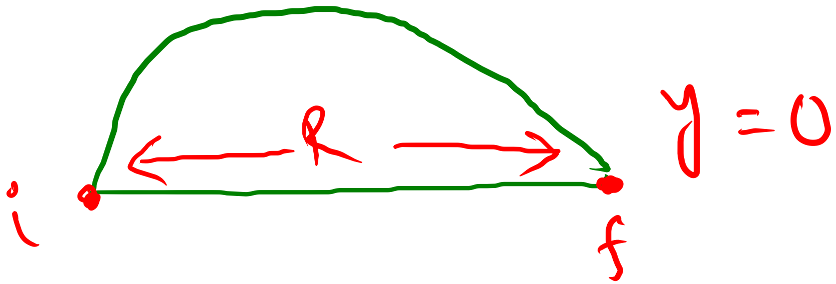
$$y = x \tan\alpha \left(1 - \frac{x}{R}\right)$$

Range

Maximum Height, $H = \dfrac{v_0^{\,2}\sin^2\alpha}{2\,g}$

Time Period, $T = \dfrac{2\,v_0\sin\alpha}{g}$

Horizontal Range, $R = \dfrac{v_0^{\,2}\sin 2\alpha}{g}$



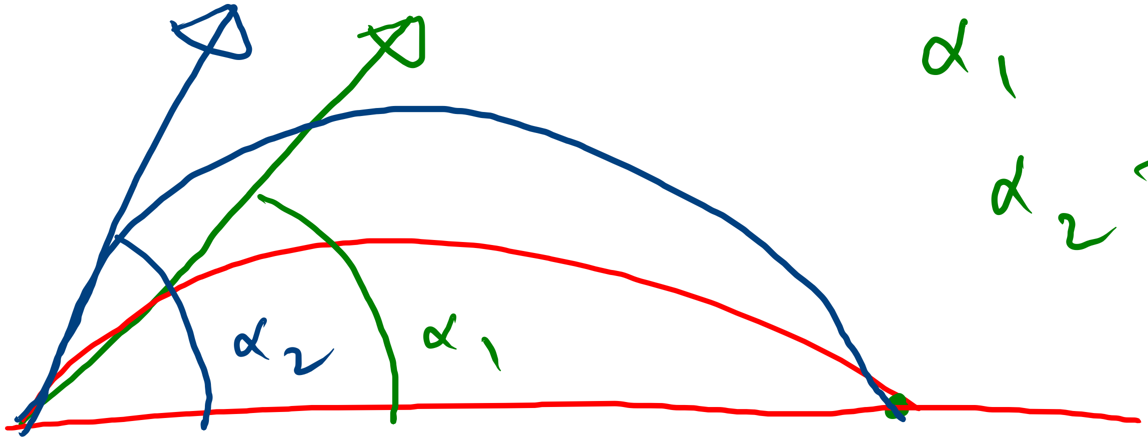
$v_0 = 10\,ms^{-1}$

$R = 30\,ms^{-1}$

$\alpha = ?$

$$\boxed{R = \frac{v_0^2 \sin 2\alpha}{g}}$$

$\alpha =$



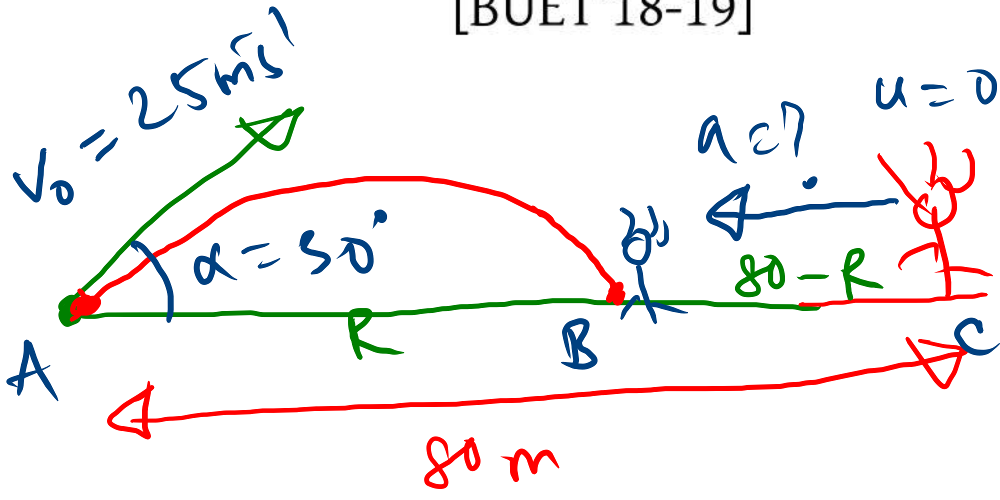
α_1

$\alpha_2 = 90^\circ - \alpha_1$

Problem-13

A ball is projected at an angle of 30° with a velocity of $25ms^{-1}$. A fielder standing 80m away immediately ran to catch the ball. What is the required acceleration if he wants to catch the ball just before it hits the ground? [BUET'18-19]

$$AB = R = \frac{v_0^2 \sin 2\alpha}{g} = 55.23m$$



$$T = \frac{2v_0 \sin\alpha}{g} = 2.55 \text{ sec}$$

fielder,

$$(80 - R) = \frac{1}{2} aT^2$$

$$\therefore a = ? \quad 7.62ms^{-2}$$

Practice Problem

The maximum height of a projectile is equal to its horizontal range. Find the angle of projection.

$$R = H$$

$$\Rightarrow \frac{{v_0}^2 \sin 2\theta_0}{g} = \frac{{v_0}^2 \sin^2 \theta_0}{2\,g}$$

$$\Rightarrow 2 \sin \theta_0 \cos \theta_0 = \frac{1}{2} \sin^2 \theta_0$$

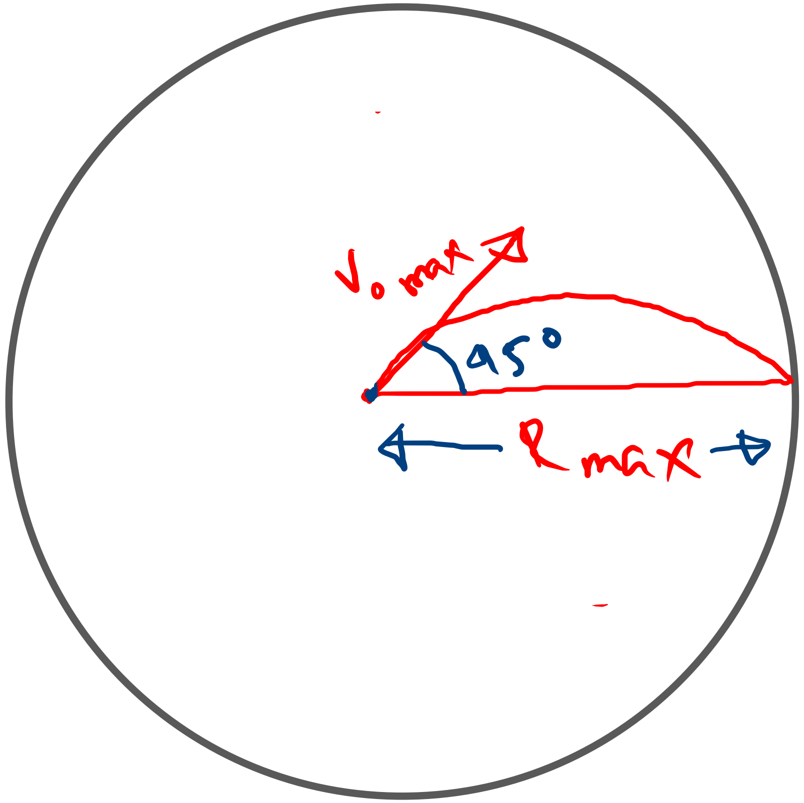
$$\Rightarrow \tan \theta_0 = 4$$

$$\Rightarrow \theta_0 = 75.69°$$

Problem-14

An object bursts and spreads in everywhere. If the maximum speed of a piece be $100\ ms^{-1}$, then what is the circumference of the area will the broken pieces cover in the surface?

$v_{0_{max}}$



$$R_{max} = \frac{v_{0_{max}}^2}{g}$$

$$\boxed{\alpha = 45^\circ}$$

$\therefore$ Circumference $= 2\pi R_{max}$

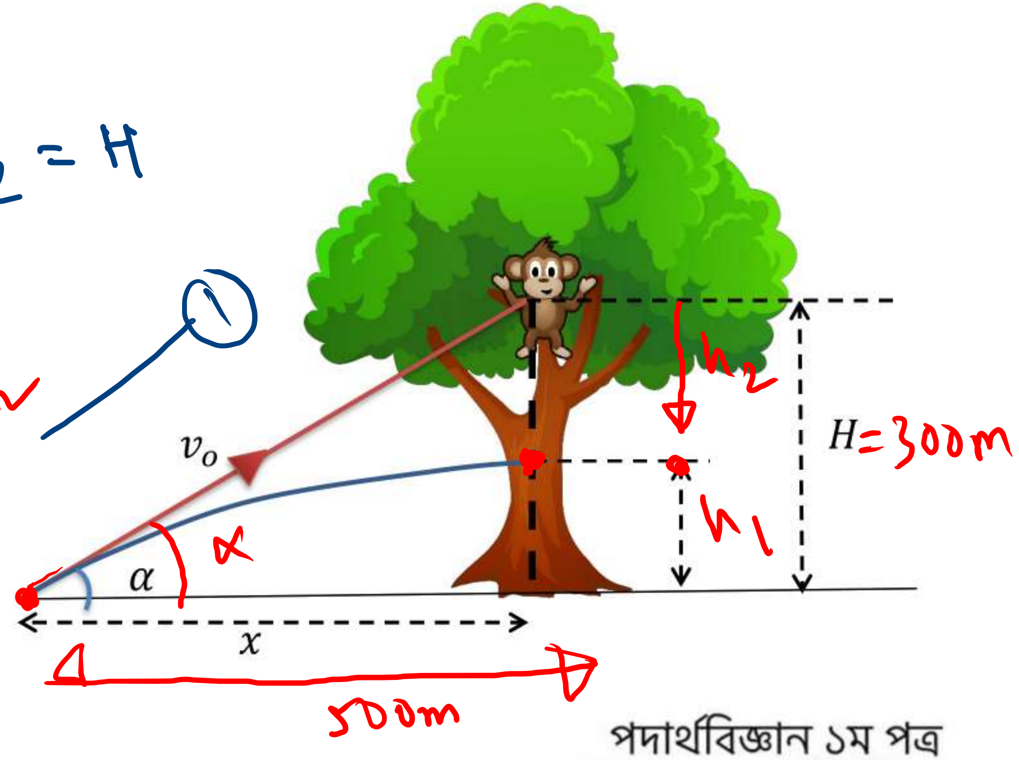
Problem-15

A bullet was fired directly at a monkey in a tree 300m high from any point on the ground. If the monkey falls at that moment, determine if the monkey will be shot. [The tree was 500m away from the point of casting] [BUET '12-'13]

Given,

$$\tan\alpha = \frac{H}{x} = \frac{300}{500}$$

The monkey will be shot if $h_1 + h_2 = H$



proj,

$$h_1 = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \quad (i)$$

$$x = (v_0 \cos\alpha)t \quad (iii)$$

$$h_2 = \frac{1}{2}gt^2 \quad (ii)$$

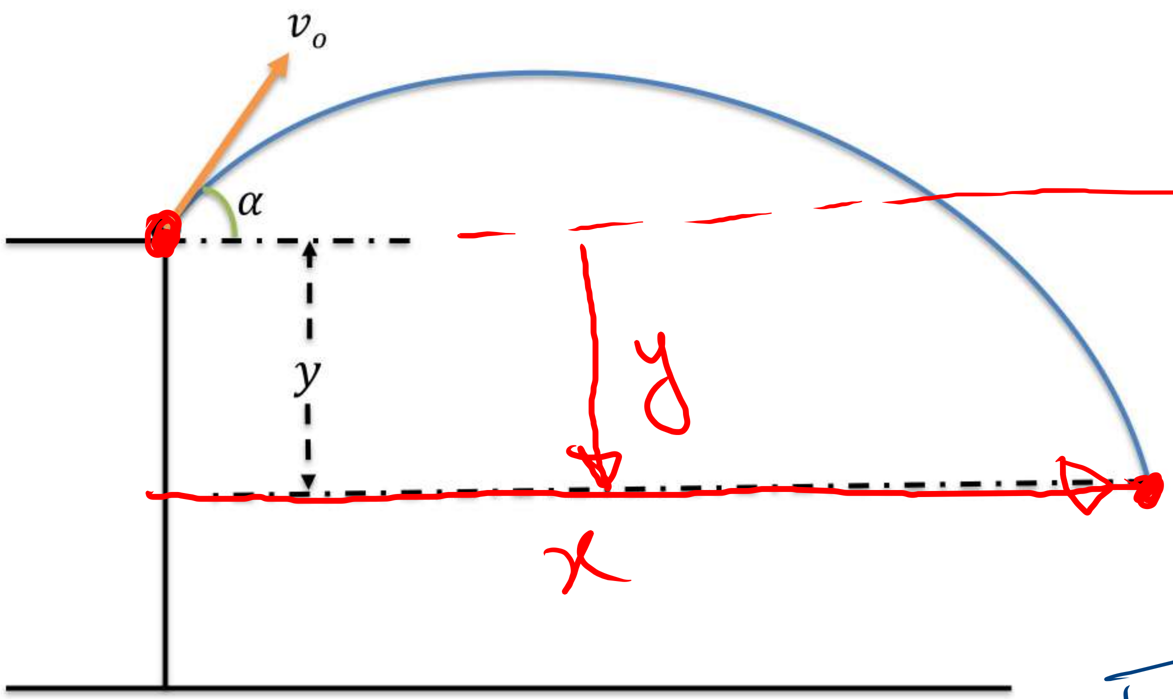
(i) + (ii) $\Rightarrow$, $h_1 + h_2 = (v_0 \sin\alpha)\, t$

(iv) $\div$ (iii) $\rightarrow$, $\dfrac{h_1 + h_2}{x} = \tan\alpha = \dfrac{H}{x}$

$\therefore h_1 + h_2 = H$

$\therefore$ The monkey will be shot.

Projectile Thrown from a Height



$x = (v_0 \cos\alpha)\, t$

$y \downarrow$ + $v_0 \uparrow$ − $g \downarrow$ +

$$\boxed{+y = -(v_0 \sin\alpha)\, t + \frac{1}{2} g t^2}$$

Problem-16

A stone is projected from a 80 m height with $38ms^{-1}$ velocity at an angle of $30°$ with horizontal. Where will it hit the ground? What will be the velocity?

$x = (v_0 \cos\alpha) t$

$y = 80m$

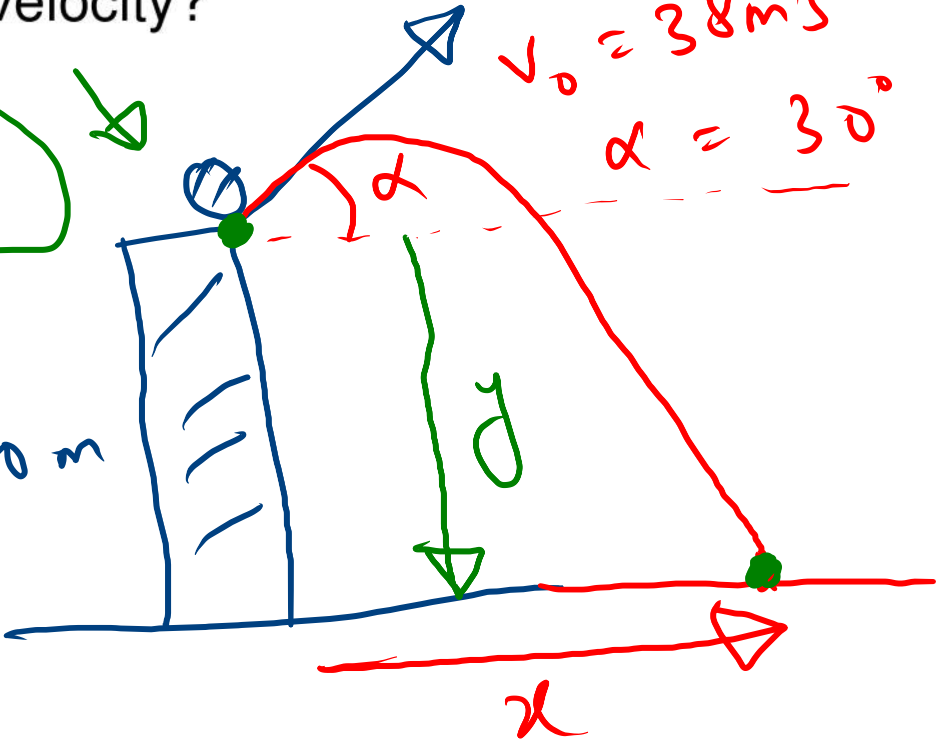
$y = -(v_0 \sin\alpha) t + \frac{1}{2} g t^2$

$80 = -(38 \sin 30°) t + 4.9 t^2$

$v_0 \uparrow$

$g \downarrow$

$t = ?$



velocity:

$$v_x = v_0 \cos\alpha$$

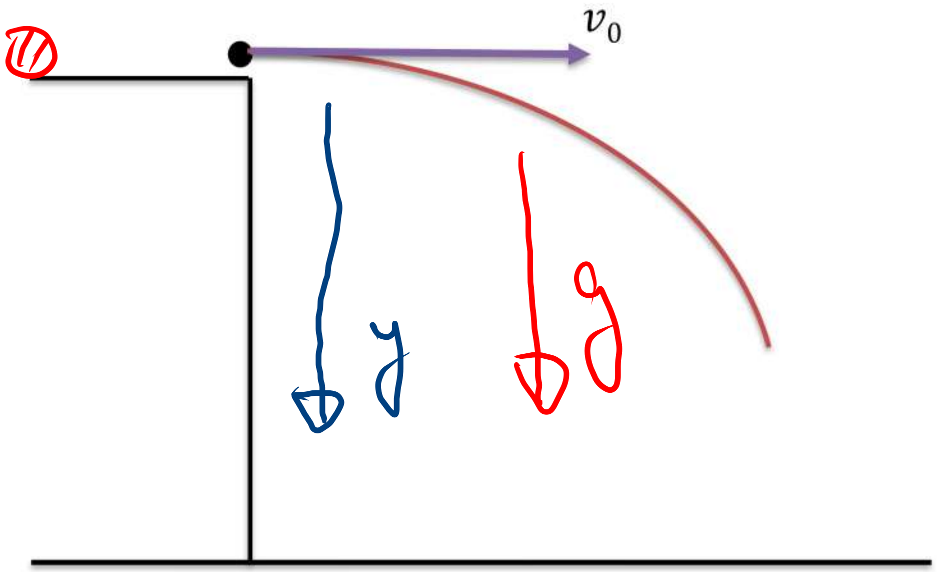
$$v_y = v_0 \sin\alpha - gt$$

$$v = \sqrt{v_x^2 + v_y^2} = ?$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = ?$$

Horizontally Projected Body

$v_0 \longrightarrow x$ axis (along) ; $\boxed{\alpha = 0^\circ}$



$x = (v_0 \cos 0^\circ)\, t = v_0 t$

$\boxed{x = v_0 t}$

$y = \underbrace{(v_0 \sin 0^\circ)\, t}_{\to 0} \quad \frac{1}{2} g t^2$

$\boxed{y = \frac{1}{2} g t^2}$

Problem-17

What will be the required velocity to jump from the 1st roof to the 2nd roof if you run horizontally?

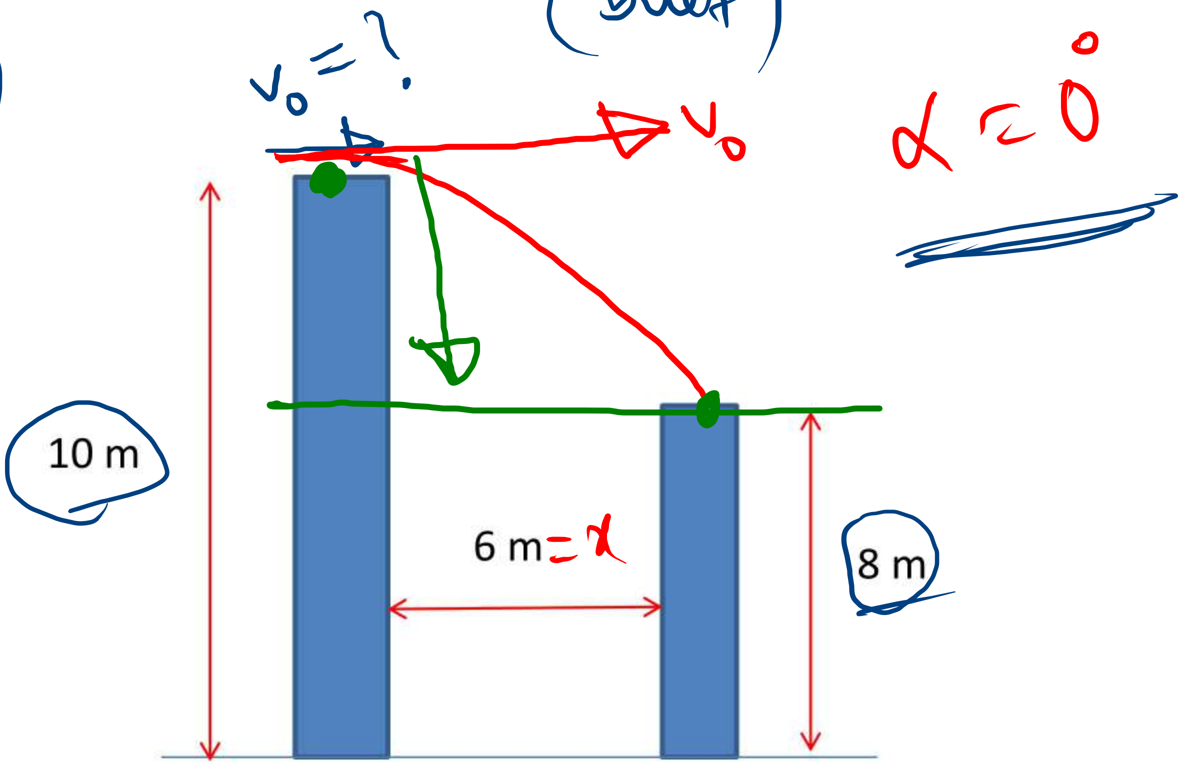
$$x = v_0 t$$

$$v_0 = ?$$

$$y = (10 - 8) = 2\,m$$

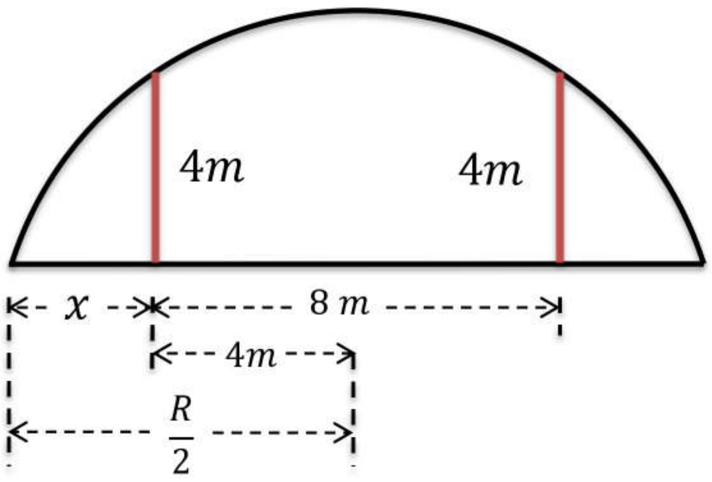
$$y = \frac{1}{2} g t^2$$

$$t = ?$$



Practice Problem

A projectile, projected at $30°$ with horizontal, just manages to cross two 4m high walls both separated by 8m from one another in its path. What is its range?



$$y = x\tan\theta_0\left(1 - \frac{x}{R}\right)$$

$$\Rightarrow 4 = \left(\frac{R}{2} - 4\right)\tan 30°\left(1 - \frac{\frac{R}{2}-4}{R}\right)$$

$$\Rightarrow 4\sqrt{3} = \left(\frac{R-8}{2}\right)\left(\frac{R+8}{2R}\right)$$

$$\Rightarrow 4\sqrt{3} = \left(\frac{R^2-64}{4R}\right)$$

$$\Rightarrow 16\sqrt{3}R = R^2 - 64$$

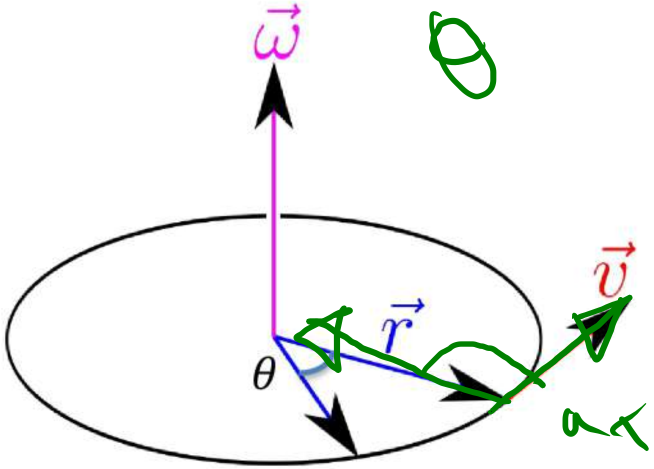
$$\Rightarrow R = 29.84\ m$$

Angular Motion

ω, ω_0

α

θ



Angular velocity	$\omega = \frac{d\,\theta}{dt}$
Angular acceleration	$\alpha = \frac{d\,\omega}{dt}$
Linear velocity	$v = \omega\, r$
Linear/Tangential acceleration	$a_T = \alpha\, r$
Centripetal acceleration	$a_c = \frac{v^2}{r} = \omega^2 r$
Resultant acceleration	$a = \sqrt{a_T{}^2 + a_c{}^2}$

For any object moving in uniform angular acceleration,

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0{}^2 + 2\alpha\theta$$

$$\theta = \frac{\omega + \omega_0}{2} t$$

$a = \sqrt{a_T^2 + a_c^2}$

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Problem-18

Length of the minute's hand of a clock is 10 cm. What is the linear velocity at its corner?

$V = \omega r$

$\omega = \frac{2\pi}{T} = ?$

$v = ?$

$r = 10$ cm

$T = 60$ min

$= 3600$ s

Problem-19

A fan revolving at 1500 rpm is switched off. It comes to rest in 4 min. How many complete cycles will it complete before coming to rest? [KUET '19-'20]

$t = 4\,min = 4 \times 60\,s$

$\omega_0 = \dfrac{1500\,rev}{1\,min}$

$= \dfrac{1500 \times 2\pi\,rad}{60\,s}$

$= 50\pi\,rad\,s^{-1}$

$\omega_0 = $

$\omega = 0$

$\theta = \left(\dfrac{\omega + \omega_0}{2}\right) t$

$\theta = 6000\pi\,rad$

no. of cycles $= \dfrac{\theta}{2\pi}$

$= 3000\,rev$

Problem-20

Velocity at a certain moment of a particle in angular motion with 0.5 m radius is $1.6\, ms^{-1}$.
If its angular acceleration is $16\, rads^{-2}$.
Find the magnitude and the direction of the resultant acceleration.

$r = 0.5 m$

$v = 1.6 ms^{-1}$

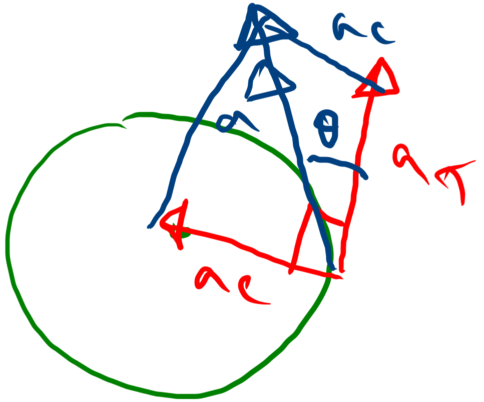
$\alpha = 16 rads^{-2}$

$a_T = \alpha r = ?$

$a_c = \frac{v^2}{r} = ?$

$a = \sqrt{a_T^2 + a_c^2}$

$\tan\theta = \frac{a_c}{a_T}$



Useful Link

https://phet.colorado.edu/sims/html/projectile-motion/latest/projectile-motion_en.html

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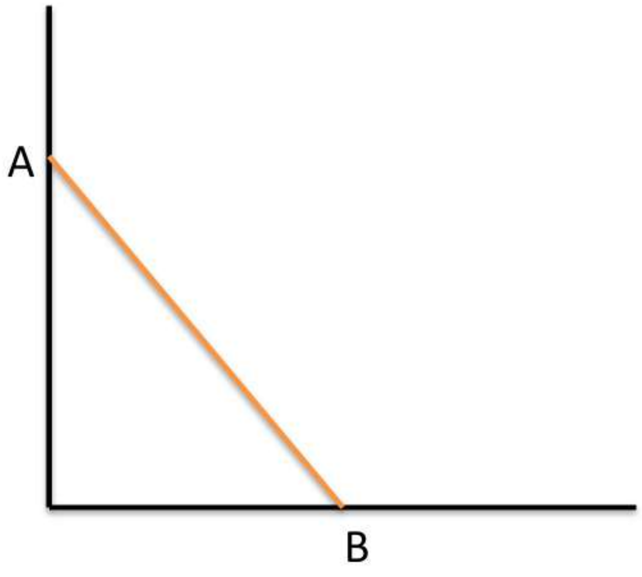
More Problems

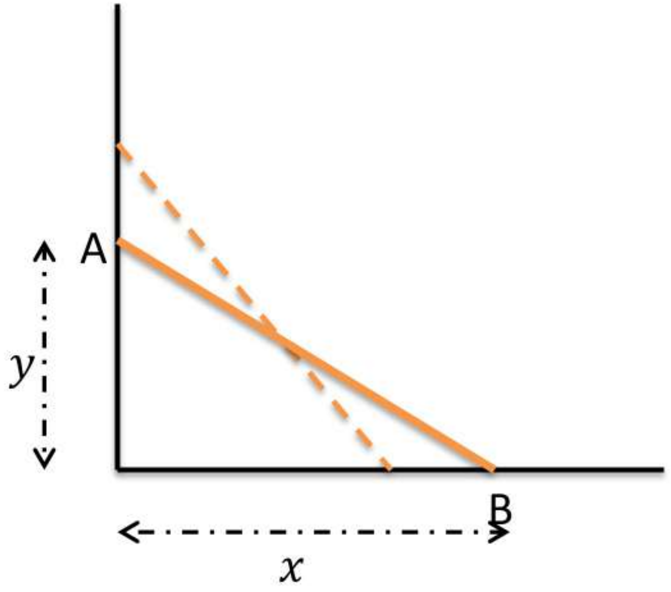
Problem-21

AB, 5m long ladder was kept as shown in the figure. Suddenly it starts to slide down. When point A is 3m above the ground, velocity of point B is $2ms^{-1}$. What will be the velocity of A?



Solution

AB, 5m long ladder was kept as shown in the figure. Suddenly it starts to slide down. When point A is 3m above the ground, velocity of point B is $2ms^{-1}$.
What will be the velocity of A?



$$x^2 + y^2 = 25$$

$$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\Rightarrow x\, vB + y\,(-vA) = 0$$

$$\Rightarrow vA = \frac{x}{y}\, vB$$

$$\Rightarrow vA = \frac{\sqrt{25-9}}{3} \times 2$$

$$\Rightarrow vA = \frac{8}{3}\, ms^{-1}$$

Problem-21

Equation of motion of a body, $v = \sqrt{s}\, t$
Find displacement for 9s to 15s.

Solution

Equation of motion of a body, $v = \sqrt{s}\, t$

Find displacement for 9s to 15s.

$$v = \sqrt{s}\, t$$

$$\Rightarrow \frac{ds}{dt} = \sqrt{s}\, t$$

$$\Rightarrow \frac{1}{\sqrt{s}} ds = t\, dt$$

$$\Rightarrow \int_0^s \frac{1}{\sqrt{s}} ds = \int_9^{15} t\, dt$$

$$\Rightarrow 2\sqrt{s} = \frac{1}{2}[15^2 - 9^2]$$

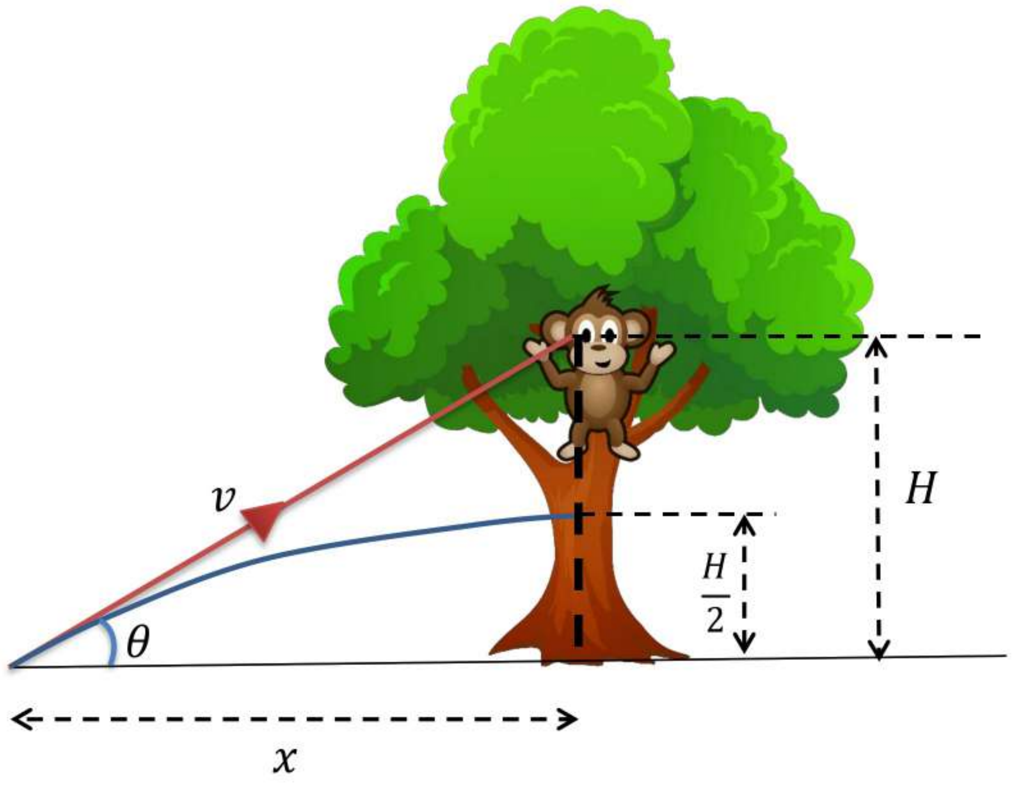
$$\Rightarrow s = 36\, m$$

Problem-22

A bullet is fired from ground aiming at a monkey sitting on a tree. If the bullet hits the tree at half of the height of the monkey, show that, direction of the bullet is horizontal while hitting the tree.

Solution

A bullet is fired from ground aiming at a monkey sitting on a tree. If the bullet hits the tree at half of the height of the monkey, show that, direction of the bullet is horizontal while hitting the tree.



$$y = x \tan\theta_0 \left(1 - \frac{x}{R}\right)$$

$$\Rightarrow \frac{H}{2} = x\frac{H}{x}\left(1 - \frac{x}{R}\right)$$

$$\Rightarrow \frac{1}{2} = 1 - \frac{x}{R}$$

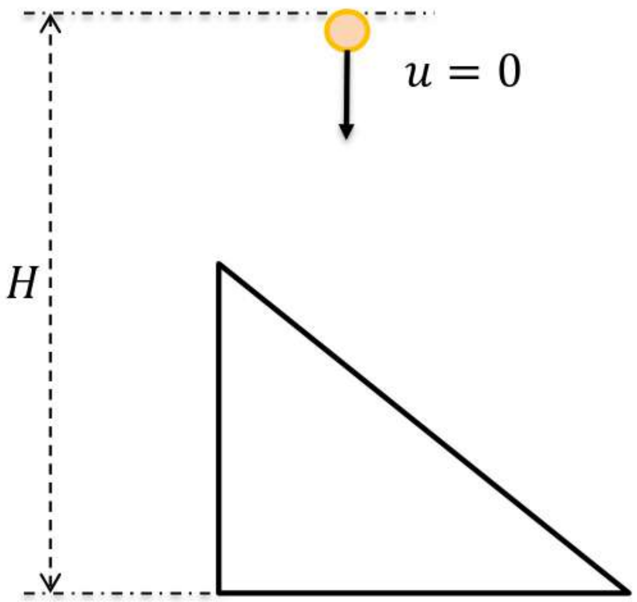
$$\Rightarrow \frac{x}{R} = \frac{1}{2}$$

$$\Rightarrow x = \frac{R}{2}$$

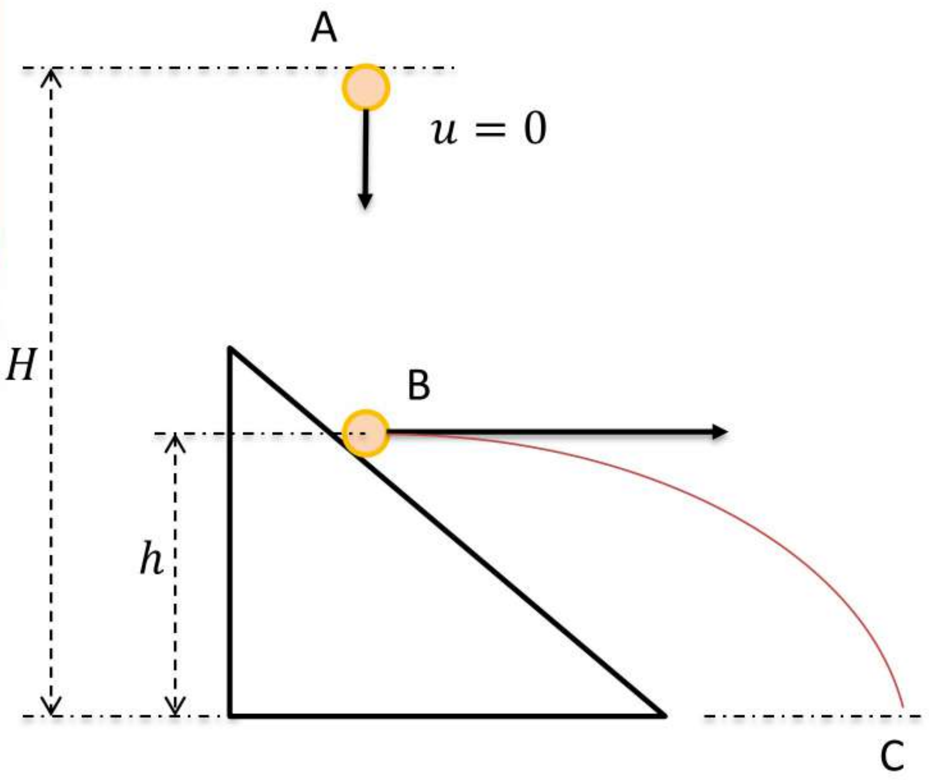
Problem

The ball hits the inclined surface at a height 'h' from ground and then the direction of it's velocity becomes horizontal.

Find the ratio of $\frac{h}{H}$ for which time to touch the ground becomes maximum.



Solution



Time taken to reach B from A,

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

Time for reaching C from B,

$$t2 = \sqrt{\frac{2h}{g}}$$

Total time,

$$t = \sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}}$$

For max time,

$$\frac{dt}{dh} = 0$$

$$\Rightarrow \sqrt{\frac{2}{g}}\left[\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right] = 0$$

$$\Rightarrow \frac{1}{\sqrt{h}} = \frac{1}{\sqrt{H-h}}$$

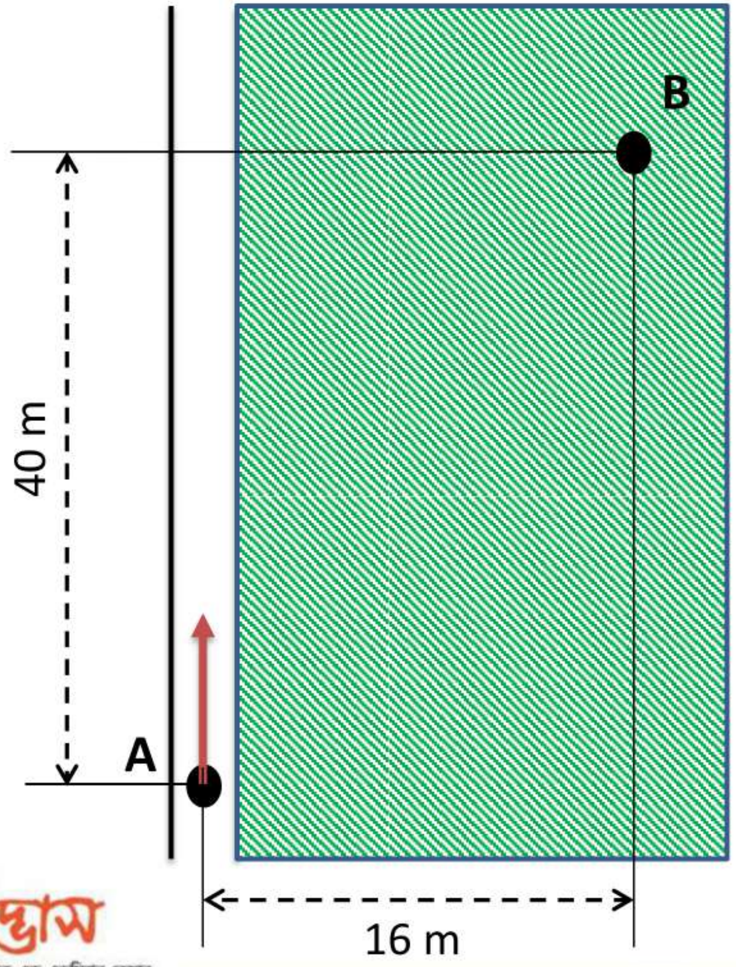
$$\Rightarrow h = H = h$$

$$\Rightarrow 2h = H$$

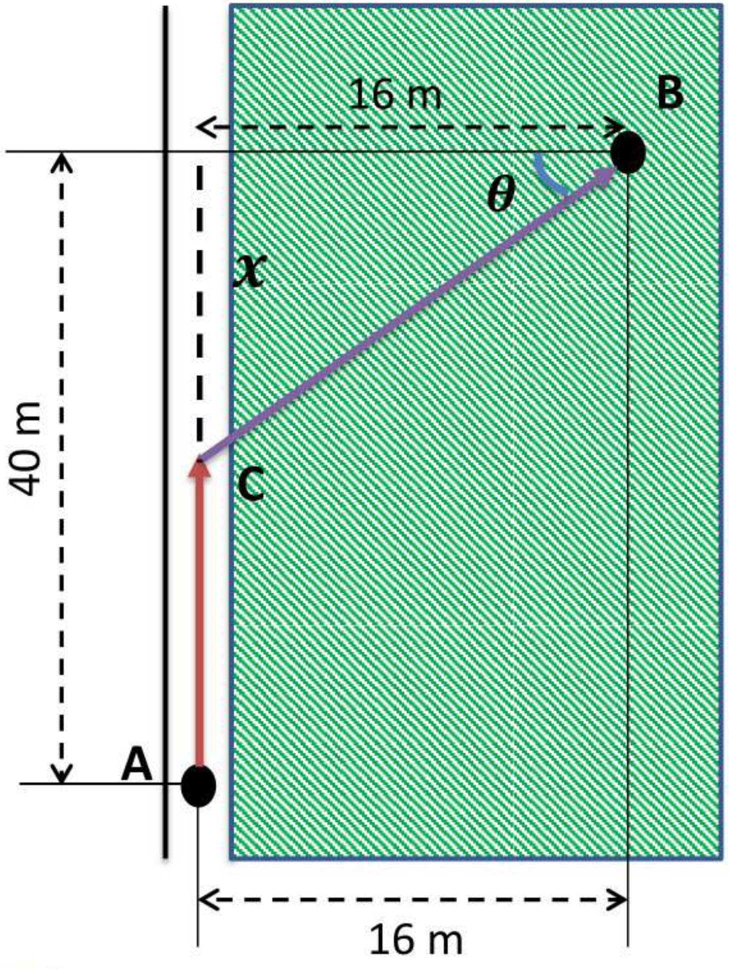
$$\Rightarrow \frac{h}{H} = \frac{1}{2}$$

Problem

A person starts from point A to go to B in the direction shown in the figure. He takes a turn in his path and enter the grassland and move towards B . Velocity of the person at road is 2.5 times of his velocity in grassland. At which point he should take the turn so that it takes min time for him to reach destination?



Solution



Follow the path that **LIGHT** would follow!
Yeah, Fermat's principle of minimum time.

If the man takes the turn at C,

$$\sin\theta = \frac{v_{grass}}{v_{Road}}$$

$$\Rightarrow \frac{x}{\sqrt{x^2 + 16^2}} = \frac{1}{2.5}$$

$$\Rightarrow \frac{x^2}{x^2 + 256} = \frac{1}{2.5^2}$$

$$\Rightarrow x = 6.98\ m$$

After going $(40 - 6.98) = 33.02\ m$ from A, he should take the turn.

Poll Question 01

A projectile thrown from ground returns to ground. When its velocity is minimum?

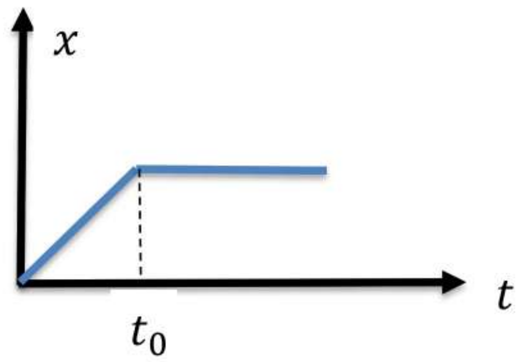
(a) $t = \frac{u^2 \sin^2 \alpha}{2g}$

(b) $t = \frac{2u \sin \alpha}{g}$

(c) $t = \frac{u \sin \alpha}{2g}$

(d) $t = \frac{u \sin \alpha}{g}$

Poll Question 02



What does the graph imply?

(a) The object is continuously moving along X-axis

(b) The object remains stationary

(c) At first the velocity increases, then moves in constant velocity

(d) Uniform velocity along X-asis upto t_0 and then comes at rest

Poll Question 03

A person moves straight with velocity V_1 for some time and then moves in the same direction with V_2 for the same time. What is his average velocity?

(a) $\frac{2v_1v_2}{v_1+v_2}$

(b) $\frac{(v_1+v_2)}{2v_1v_2}$

(c) $\frac{v_1+v_2}{2}$

(d) $\left(\frac{1}{v_1}+\frac{1}{v_2}\right)^{-1}$

d_1, v_1 d_2, v_2

t_1 t_1

$$V_{avg} = \frac{d_1+d_2}{t_1+t_1}$$

$$= \frac{v_1t_1+v_2t_1}{2t_1}$$

$$\therefore \boxed{V_{avg} = \frac{v_1+v_2}{2}}$$

Poll Question 04

Two projectiles are fired horizontally with different velocities. Which one will hit the ground first?

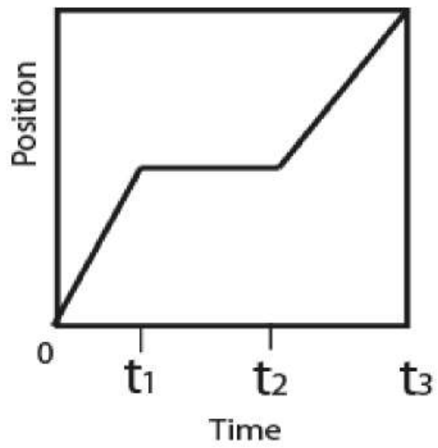
(a) The one with higher velocity
(b) The one with lower vwlocity
(c) Both of them reach together
(d) Depends on their masses

Poll Question 05

Horizontal range of a projectile thrown at 45° is-

(a) Equal to its max height
(b) Equal to twice its max height
(c) Equal to thrice its max height
(d) Equal to four times its max height

Poll Question 06



When was the force applied?

(a) 0 to t_1

(b) t_2 to t_3

(c) t_1 and t_2

(d) a & b both

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে।