Engineering Admission Program-2020

PHYSICS

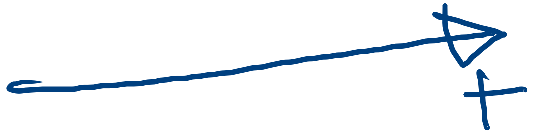
Lecture : P-03

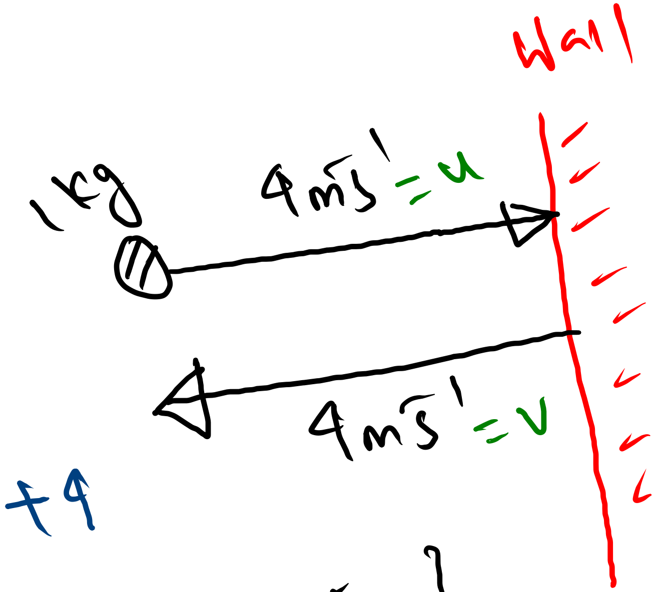
Chapter 04 : Newtonian Mechanics

Impulse of Force

impulse of force, $\vec{J} = m\vec{v} - m\vec{u}$

impulse force, $\vec{F} = \vec{J}/t$





$J = mv - mu$

$= 1kg(-4-4)ms^{-1}$

$J = -8\ kgms^{-1}$

direction

$u = +4$

$v = -4$

$J = ?$

$|\vec{J}| = 8kgms^{-1}$;

$\vec{J}$

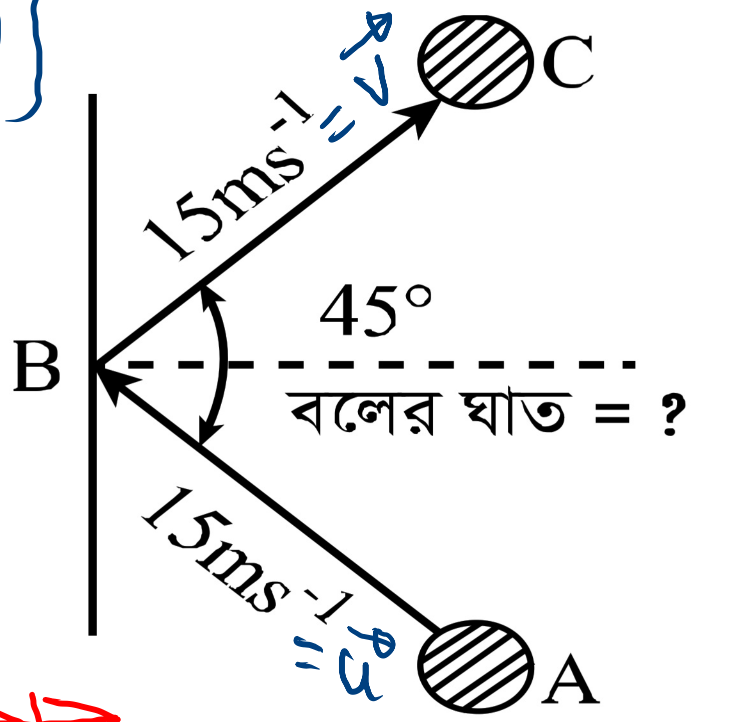
Impulse of Force

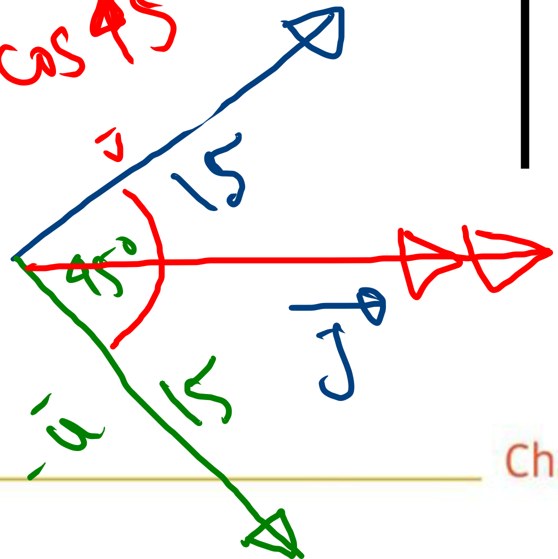
A ball of 1 kg mass, coming from point A hits a wall at point B and goes to point C like shown in diagram . Find the impulse on the ball by the wall.

$$\vec{J} = m(\vec{v} - \vec{u}) = m\{\vec{v} + (-\vec{u})\}$$

$$|\vec{J}| = m \times |\vec{v} + (-\vec{u})|$$

$$J = 1kg \times \sqrt{15^2 + 15^2 + 2 \times 15 \times 15 \cos 45^\circ}\ ms^{-1}$$





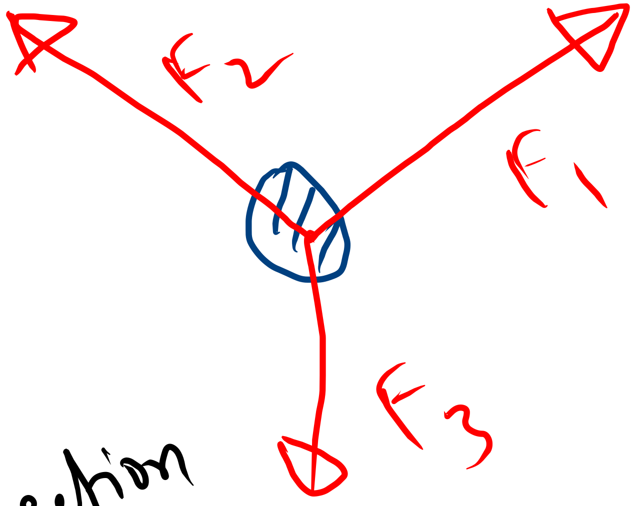
Impulse of force = ?

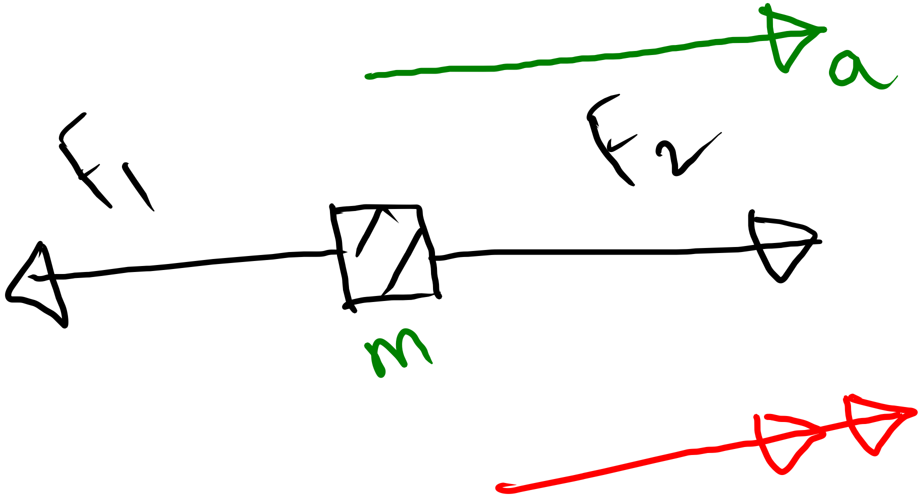
Newton's Formulae

$\vec{F} = m\vec{a}$

$\sum \vec{F} = m\vec{a}$

* $\vec{F}_{resultant}$ যে direction এ $\vec{a}$ সেই direction





$F_2 > F_1$

$F_{resultant} = F_2 - F_1 = ma$

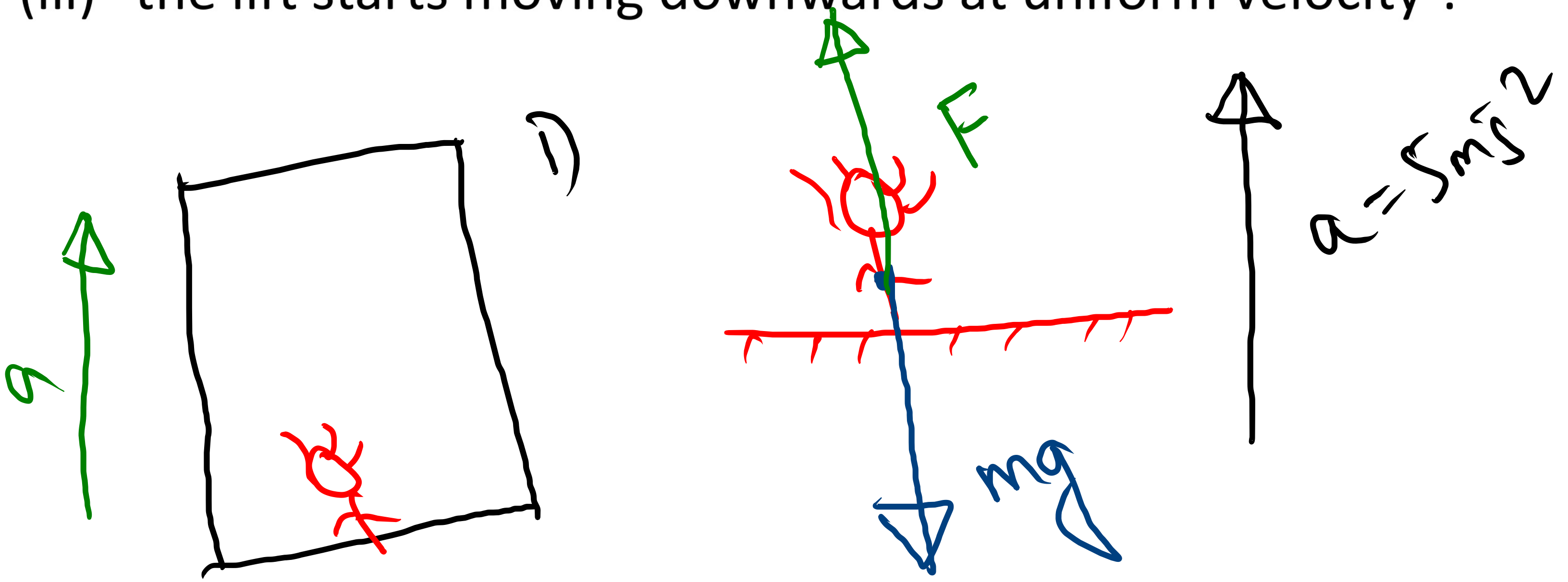
Lift

A man of mass 65 kg stands in a lift. What force does the floor exert on him when

(i) the lift starts moving upwards with an acceleration of 5 ms^{-2}

(ii) the lift starts moving downwards with an acceleration of 5 ms^{-2}

(iii) the lift starts moving downwards at uniform velocity ?

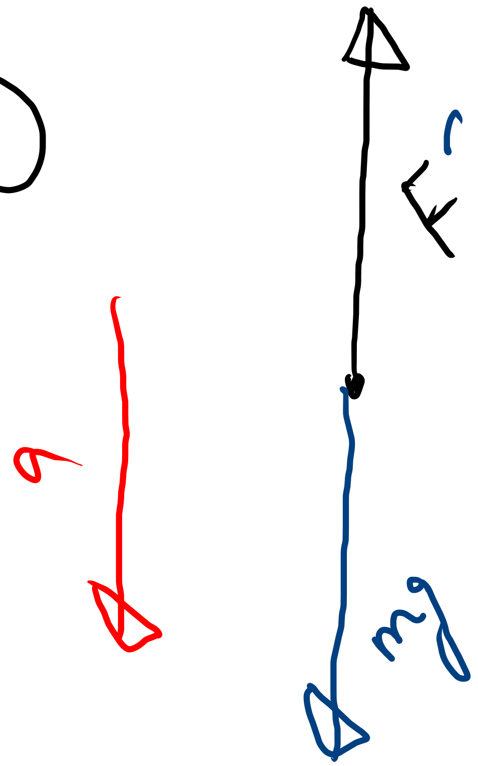


$F > mg$

$F - mg = ma$

$\therefore F = ma + mg = ?$

(ii)



$mg > F'$

$\therefore mg - F' = ma$

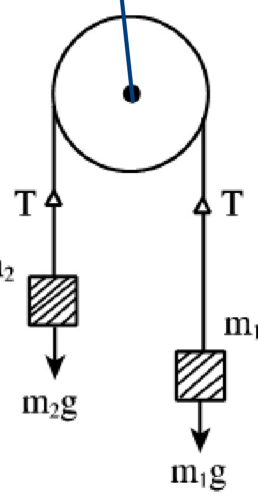
$\therefore F' = mg - ma = ?$

(iii) uniform velocity, $a = 0$, $F = mg$

Pully

Consider two masses m_1 and m_2 connected by a massless string passing over a light smooth pully. Find the acceleration of objects and tension in the string.

[m_1 =10kg and m_2=5 kg]



$T > m_2 g$

$T - m_2 g = m_2 a$

$m_1 g > T$

$m_1 g - T = m_1 a$

$m_1 g - m_2 g = (m_1 + m_2) a$

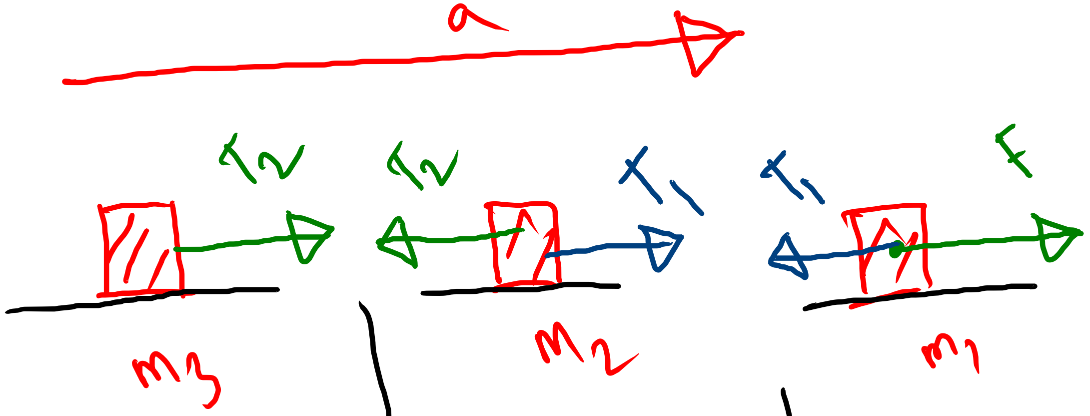
$\therefore a = \frac{m_1 - m_2}{m_1 + m_2} g$; $m_1 > m_2$

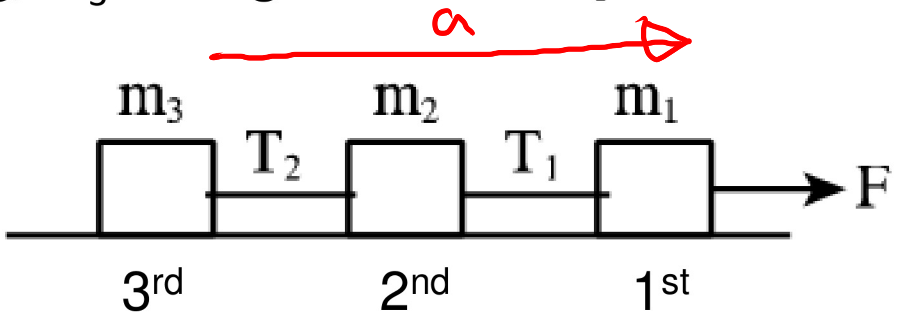
$T = ?$

Masses Connected by Strings, Tension

(friction less)

Three masses m_1,m_2 and m_3 are connected by strings .If m_1 is pulled by a force F, find the common acceleration. [m_1 =10 kg, m_2 =20 kg, m_3 =30 kg and F=600N]





$T_2 = m_3 a$ — (i)

$T_1 > T_2$

$T_1 - T_2 = m_2 a$ — (ii)

$F > T_1$

$F - T_1 = m_1 a$ — (iii)

(i) + (ii) + (iii) ⇒

$F = (m_1 + m_2 + m_3) a$

$\therefore a = ?$

$T_2 = ?$

$T_1 = ?$

Conservation of Momentum

$$\sum \vec{P}_i = \sum \vec{P}_f$$

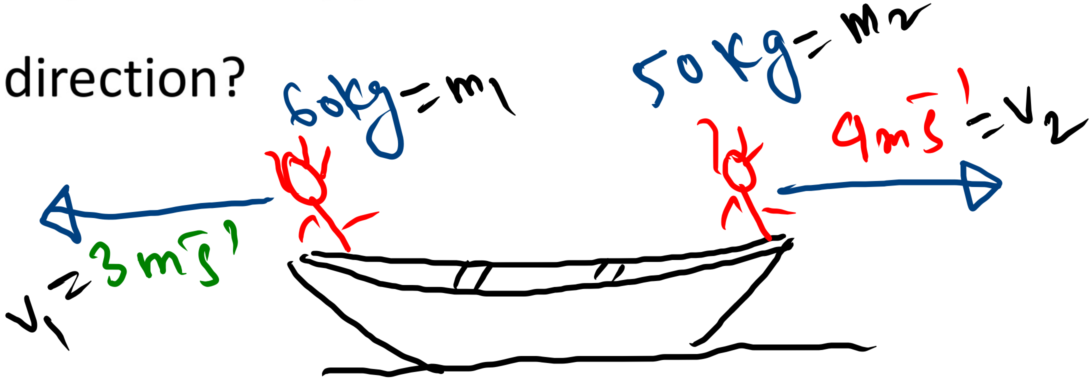
i.e. $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

the velocity just before collision

after collision

Conservation of Momentum

A boy of 60 kg mass jumped at $3\ ms^{-1}$ velocity from a 100 kg boat floating in still water. At the same time, if another boy of 50 kg jumps in the opposite direction at $4\ ms^{-1}$, how fast will the boat move and in which direction?



$m_1v_1 + m_2v_2 + m_3v_3 = 0$

$\therefore v_3 = ?$

$v_1 = +3$
$v_2 = -4$
$v_3 = ?$

$v_3 = v_{boat} = ?$

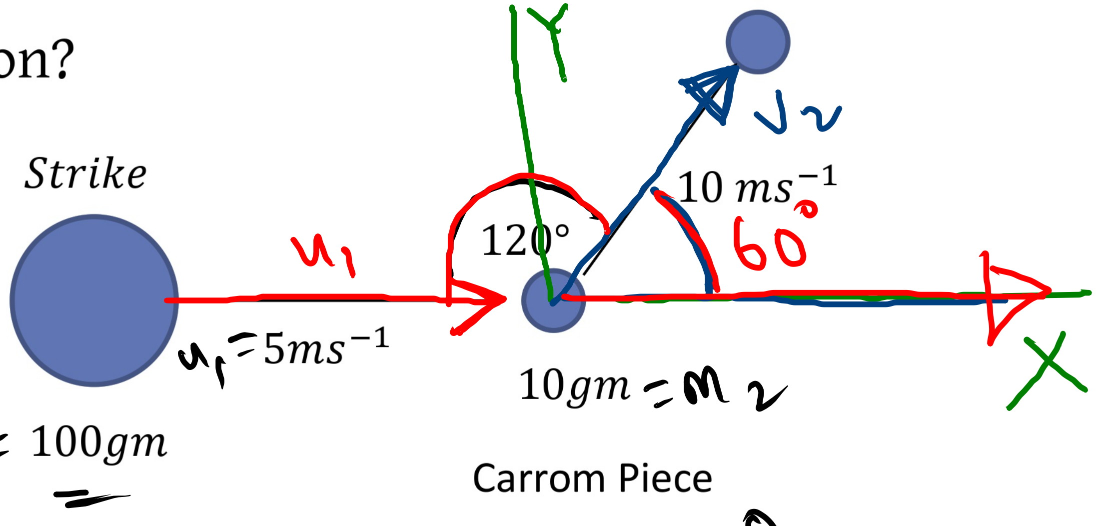
$u_1 = u_2 = u_3 = 0$

$v_3 = +0.2\ ms^{-1}$

(+ve ←)

Conservation of Momentum

A 'Strike' of 100gm mass hits a stationary carrom piece of 10gm mass at a velocity of $5\,ms^{-1}$. As a result, the piece moves at an angle of 120° with the direction of the incoming 'Strike' at a velocity of $10\,ms^{-1}$. After the collision, how fast will the strike move & in which direction?



x axis,

$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$

$\therefore v_{1x} = ?$

$u_{1x} = 5\cos 0° = 5$

$u_{1y} = 5\sin 0° = 0$

$u_{2x} = u_{2y} = 0$

$v_{2x} = 10\cos 60°$

$v_{2y} = 10\sin 60°$

y axis,

$$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$$

$$V_{1y} = ?$$

$$\therefore \quad V_1 = \sqrt{V_{1x}^2 + V_{1y}^2}$$

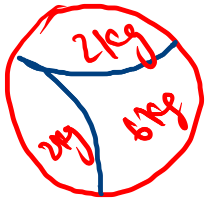
direction → $\theta = \tan^{-1} \frac{V_{1y}}{V_{1x}}$

Conservation of Momentum

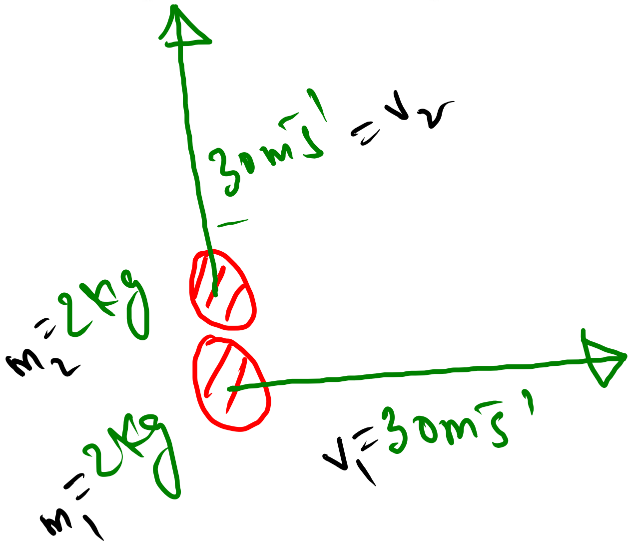
A bomb of mass 10 kg, initially at rest, explodes into three fragments of masses in ratio 1:1:3. The two pieces of equal mass fly off perpendicular to each other, each with a speed of 30m/s. What is the velocity of the heavier fragment?

→ 2kg, 2kg, 6kg

$$u_1 = u_2 = u_3 = 0$$



$$\therefore m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$



$$\therefore \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$$

$$\therefore \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2)$$

$$|\vec{p}_3| = |\vec{p}_1 + \vec{p}_2|$$

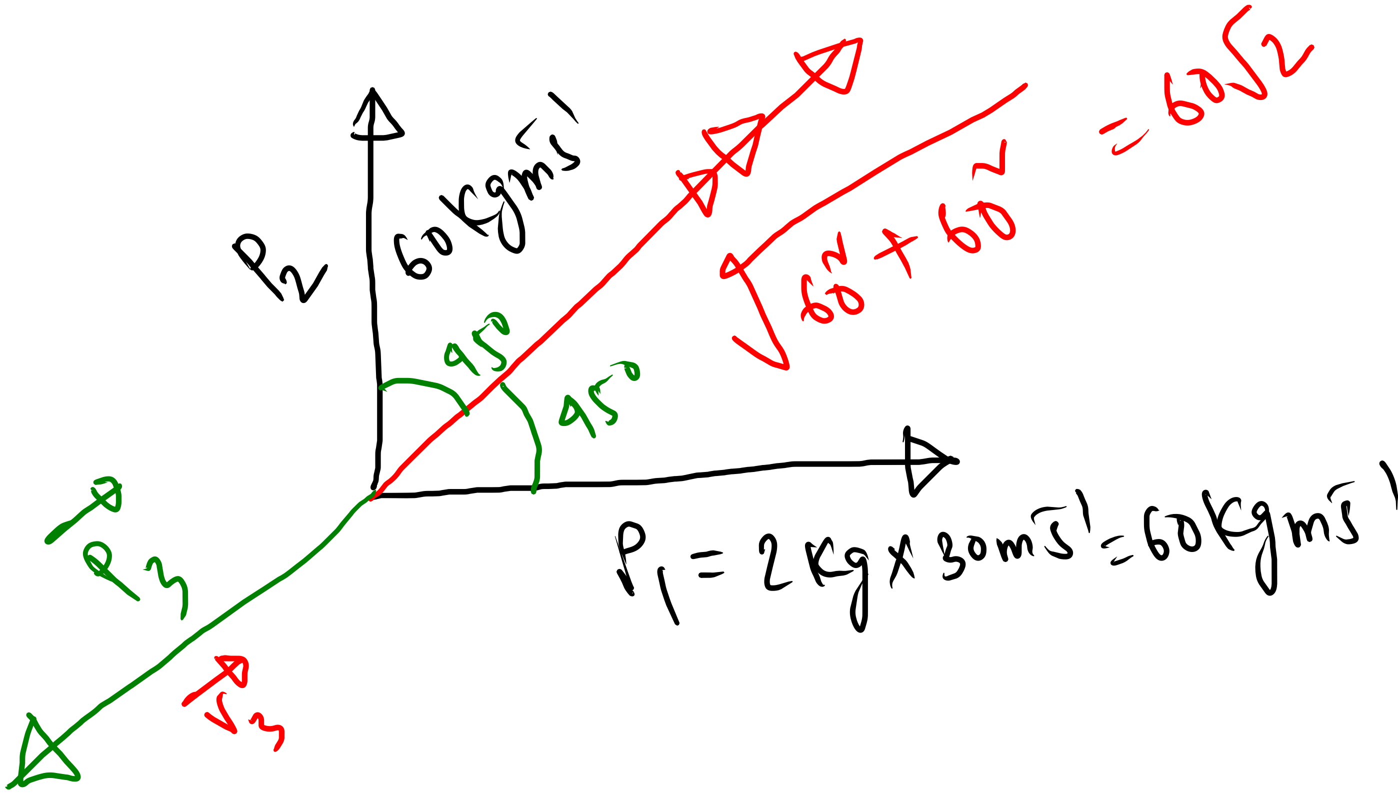
& $\vec{p}_3$ direction হবে resultant opposite দিকে

$$\therefore P_3 = 60\sqrt{2}\ \text{kg m s}^{-1}$$

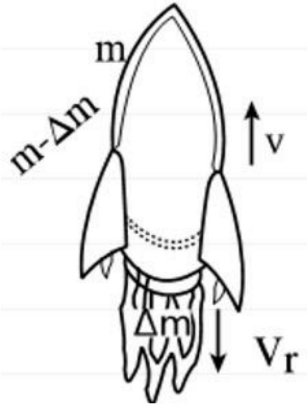
$$\therefore m_3 v_3 = 60\sqrt{2}$$

$$6 v_3 = 60\sqrt{2}$$

$$\therefore v_3 = 10\sqrt{2}\ \text{m s}^{-1}$$



Rocket



Upward Thrust, $F = \frac{dm}{dt} v_r$

v_r = vel. of fuel

$\vec{P}_{rocket} + \vec{P}_{fuel} = 0$

$\therefore \frac{d\vec{P}_{rocket}}{dt} = -\frac{d\vec{P}_{fuel}}{dt}$

$\therefore \vec{F}_{rocket} = -\frac{d}{dt}(m\vec{V}_r) = -\frac{dm}{dt}\vec{V}_r$

$F = \frac{dp}{dt}$

$F_{rocket} = \left(\frac{dm}{dt}\right) V_r$

$\frac{dm}{dt}$ → fuel burning rate

Rocket

The mass of a rocket with 10,000 kg fuel inside is 15,000 kg. The fuel is burnt at a rate of 200 kg/s and the gas is emitted at a speed of 2000 m/s.

(i) What is the upward thrust of rocket?

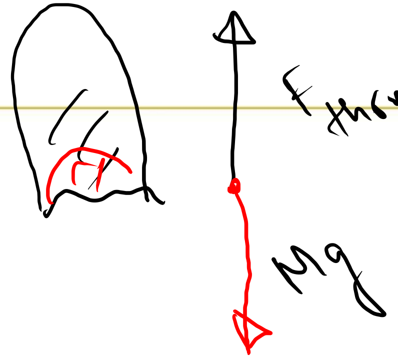
(ii) What is the net force on rocket a) initially and b) just before the fuel is burnt completely? [neglect the variation of g]

$\frac{dm}{dt} = 200$ kg/s

$v_r = 2000 \text{ ms}^{-1}$

i) $F_{thrust} = \left(\frac{dm}{dt}\right) v_r = ?$

(ii)



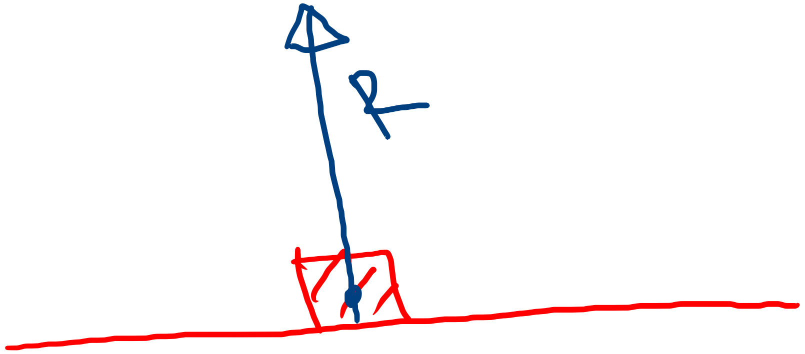
$F_{net} = F_{thrust} - Mg$

a) initially, $M = m_{rocket} + m_{fuel}$

(b) burnt fully, $M = m_{rocket}$

Friction

- Friction force always acts along the plane and opposite to the motion of the body.
- Friction Force is directly proportional to the normal reaction force of the plane.



$\vec{R}$ will always be perpendicular to the plane.

$F_{friction} \propto R$

$\therefore F_{friction} = \mu R$ → μ: friction co-efficient

Maximum force of static friction (limiting static force)

upto the moment of the starting of the motion

$$F_s = \mu_s R$$

μ_s → static friction co-efficient

Problem

A block of mass 30 kg is at rest on a floor. The coefficient of static friction between the block and the floor (μ_s) is 0.5 . Minimum how much force is required to apply horizontally so that the block starts moving?

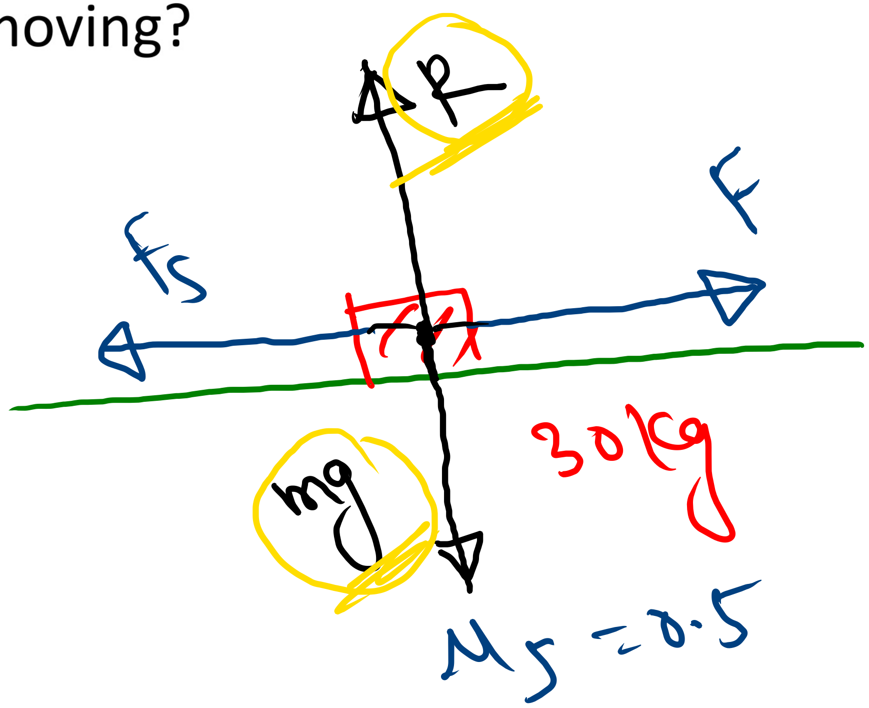
$F > F_s$

$R = mg$

$F_{min} = F_s$

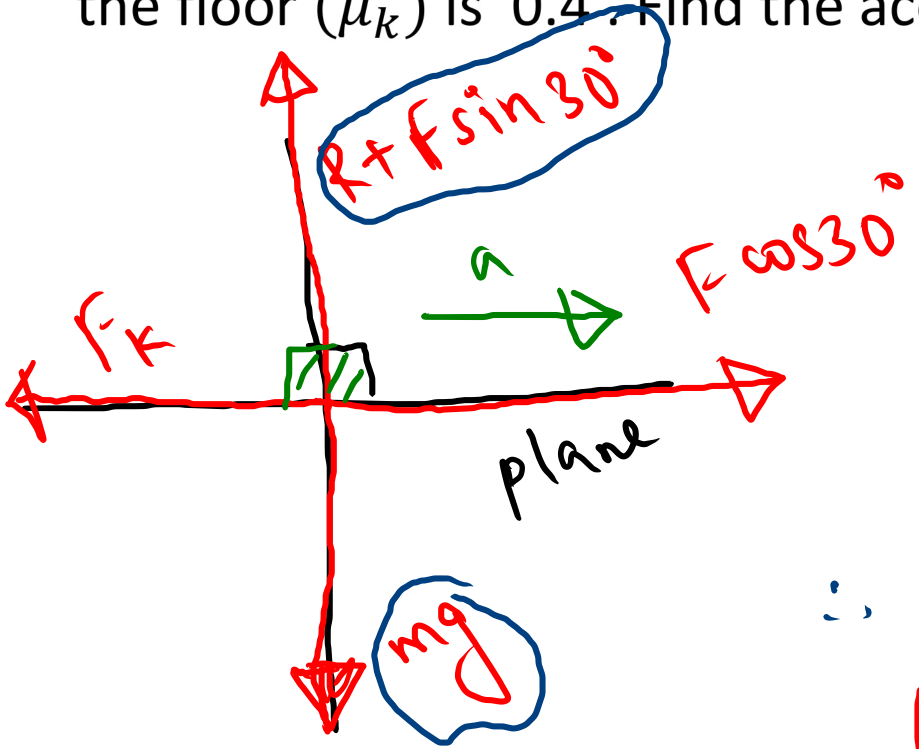
$= \mu_s R$

$F_{min} = \mu_s mg = ?$



Problem

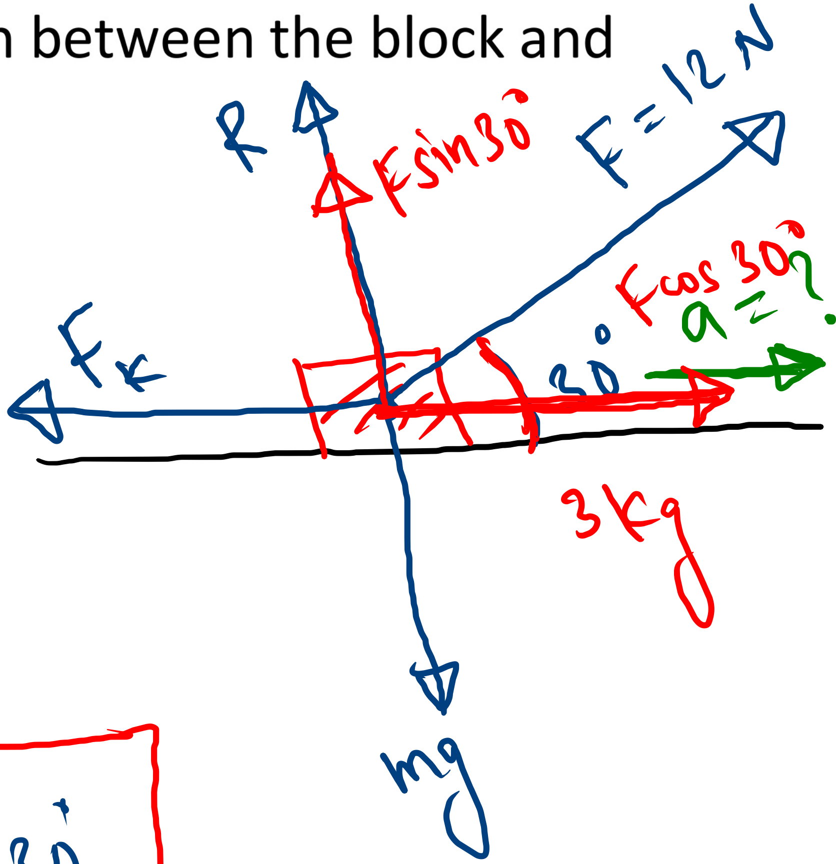
A block of mass 3 kg slides along a floor while a force of magnitude 12 N is applied to it at an upward angle of $30°$ .The coefficient of kinetic friction between the block and the floor (μ_k) is 0.4 . Find the acceleration of the block.



$\therefore a_y = 0$

$\therefore R + F\sin 30° = mg$

$\therefore R = mg - F\sin 30°$



x axis (plane) → $F\cos 30^\circ > f_k$

$\therefore F\cos 30^\circ - f_k = ma$

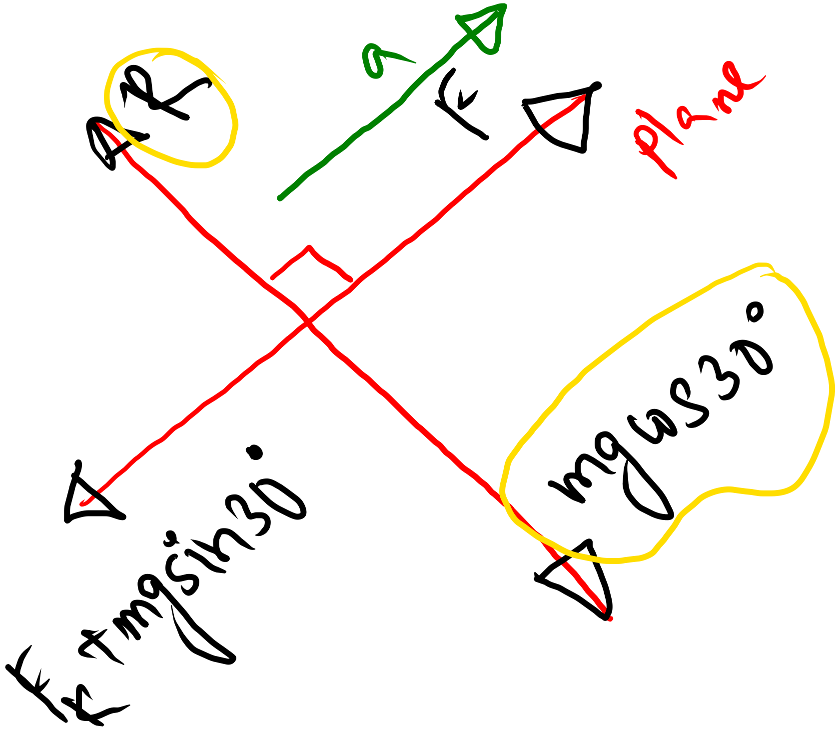
$\therefore F\cos 30^\circ - \mu_k R = ma$

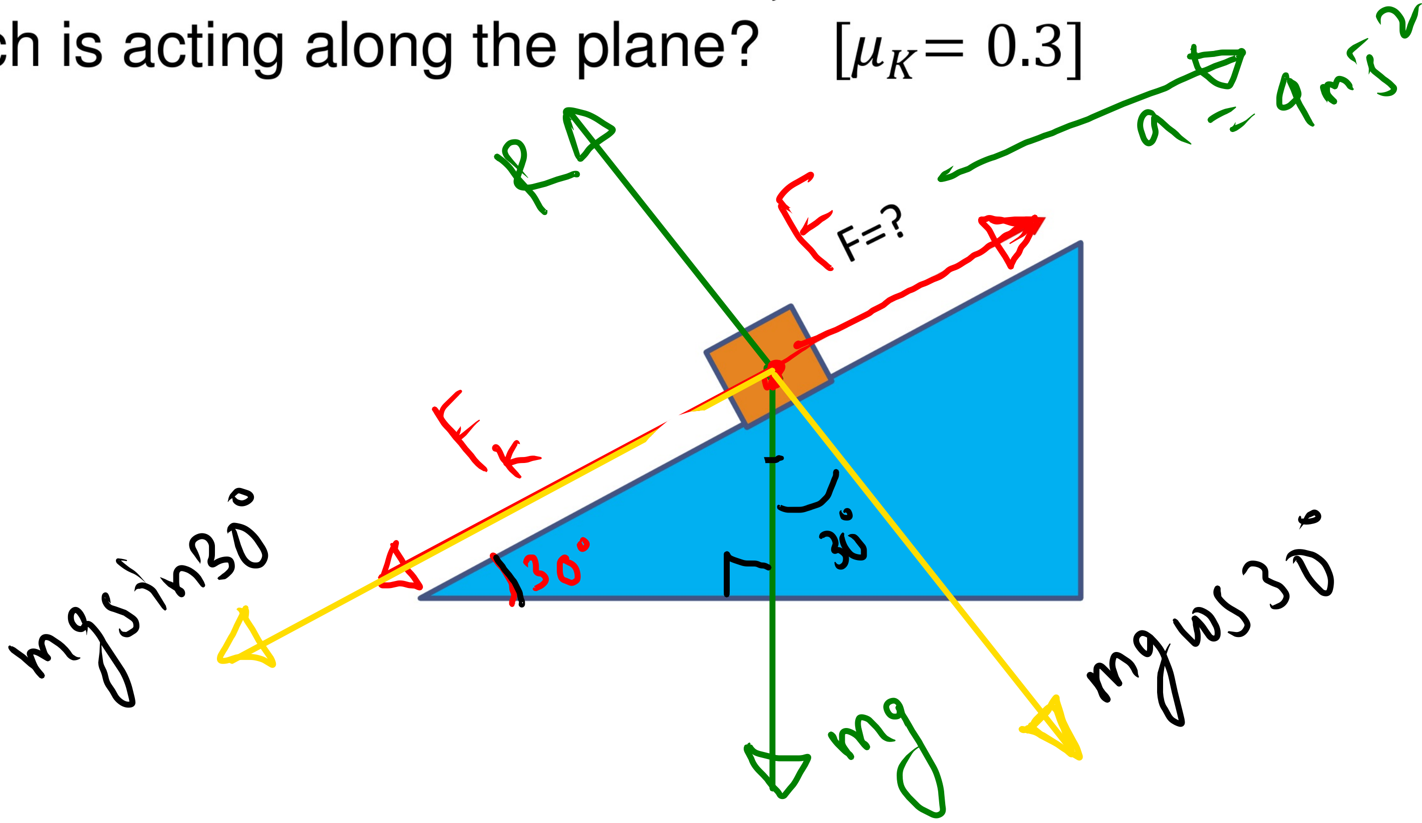
$\therefore F\cos 30^\circ - \mu_k(mg - F\sin 30^\circ) = ma$

$a = ?$

Friction on Inclined Plane

The plane is inclined at an angle of $30°$ with the horizon. If the block moves upward along the plane at an acceleration of $4\ ms^{-2}$, then what is the required magnitude of F which is acting along the plane? $[\mu_K = 0.3]$





$\because a_y = 0$, $R = mg\cos30°$

plan,

$$F - (F_k + mg\sin 30^\circ) = ma$$

$F_k \rightarrow \mu_k F_n \rightarrow \mu_k mg\cos 30^\circ$

$F = ?$

$$F - (\mu_k mg\cos 30^\circ + mg\sin 30^\circ) = ma$$

Practice Problem

Suppose a car of 1000 kg is placed on an inclined road and the coefficient of static friction between the wheel of the car and road is 0.8. Then upto what angle of inclination the car will stay at rest?

Sol^n:

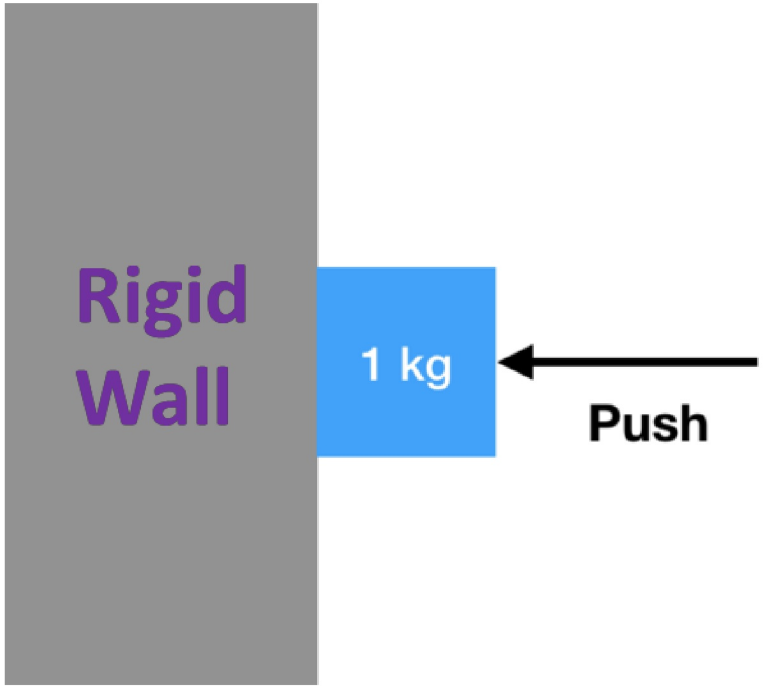
$R = mg\cos\theta$;

$F_s = mg\sin\theta \Rightarrow \mu_s R = mg\sin\theta$

$\therefore \theta = \tan^{-1}(\mu_s)$

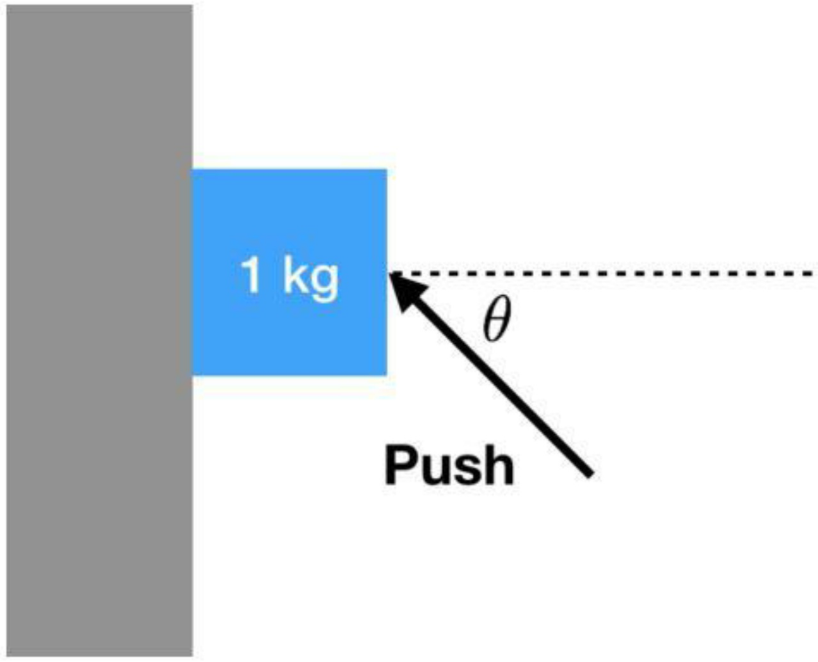
Practice Problem

In the figure, the magnitude of push is 10 N and the block was at rest initially. Find the acceleration of block. [$\mu_s = 0.4$, $\mu_k = 0.1$]



Practice Problem

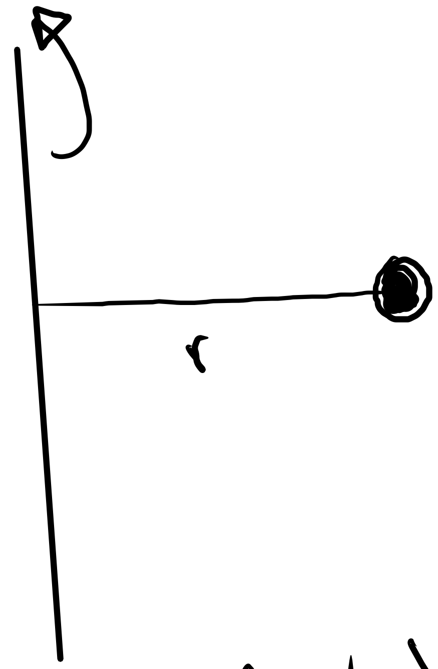
In the figure , the magnitude of push is 10 N and the block was at rest initially. Find the acceleration of block. [$\mu_s = 0.4$, $\mu_k = 0.1$ and $\theta = 45^o$]

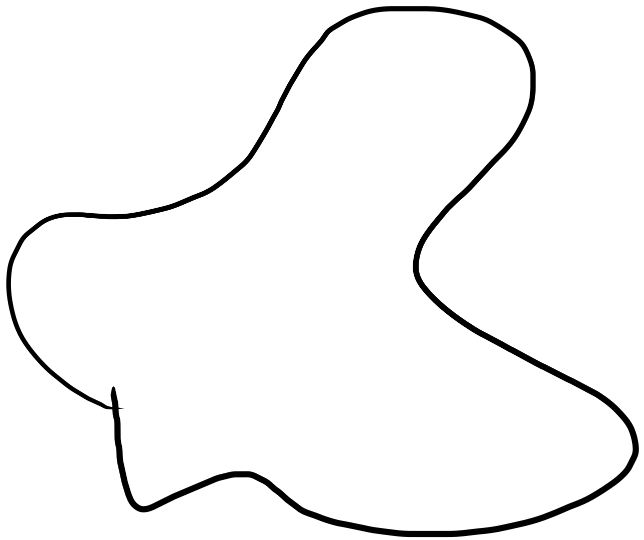


Moment of Inertia

#

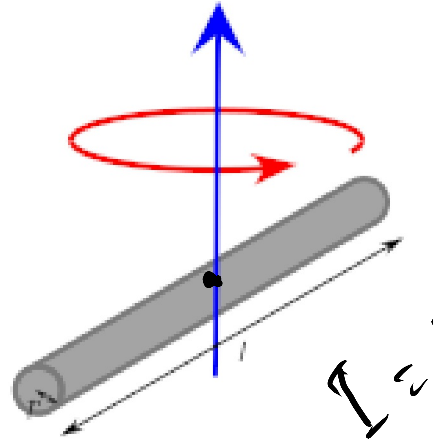
$$I = mr^2$$





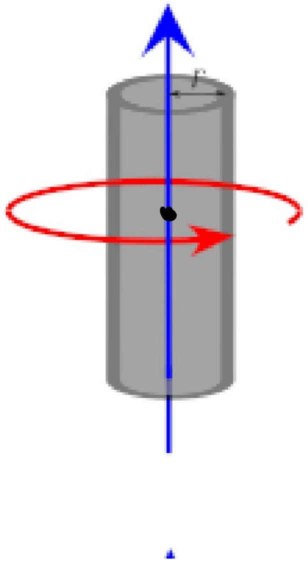
$$I = \int r^2 \, dm$$

Moment of Inertia



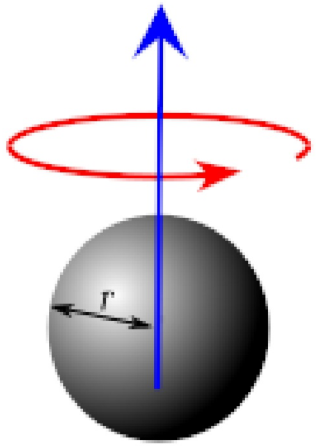
Rod about center

$I = \frac{1}{12} m l^2$



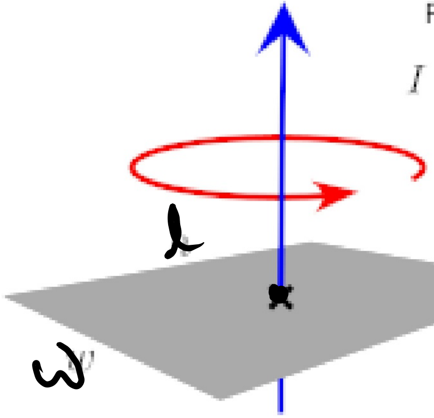
Rod or disc about axis

$I = \frac{1}{2} m r^2$



Sphere

$I = \frac{2}{5} m r^2$



Plane about center

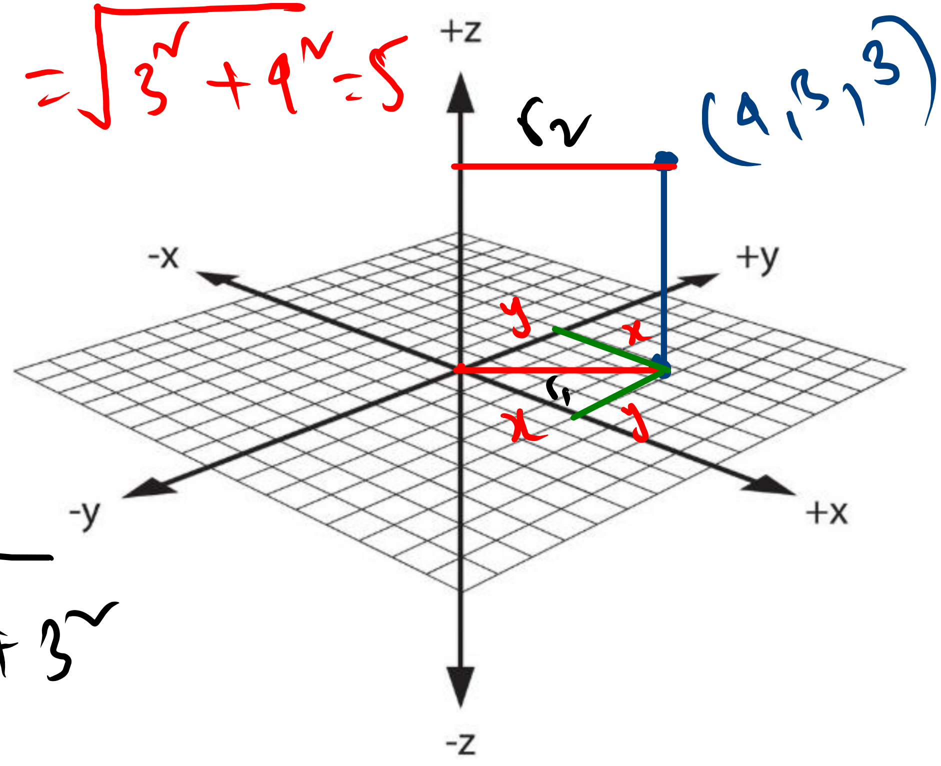
$I = \frac{1}{12} m (h^2 + w^2)$

Moment of Inertia

The co-ordinates of three particles of 4, 5 and 6 unit mass are (4, 3, 0), (4, 3, 3) and (2, -1, 4), respectively. Find their moment of inertia and radius of gyration about z-axis.

(i) $I_{z_1} = m_1 r_1^2$; $r_1 = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5$

$\therefore I_{z_1} = 4 \times 5^2$ unit



$r_2 = r_1 = \sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2}$

$I_{z_2} = ?$

distance w.r.t. z axis $= \sqrt{x^2+y^2}$

(iii) $r_3 = \sqrt{2^2+(-1)^2} = ?$ $I_{z_3} = m_3 r_3^2 = ?$

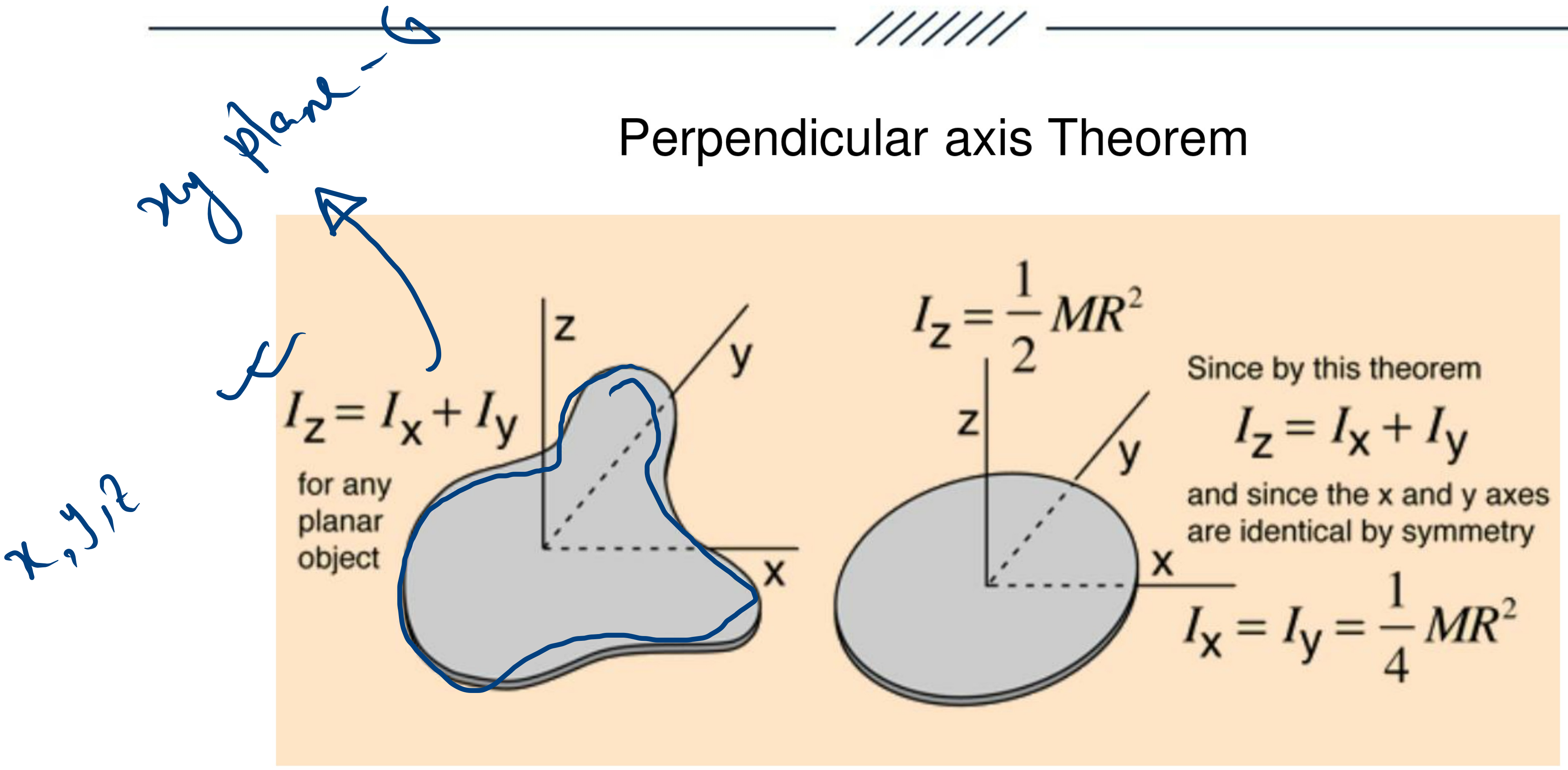
wrt y axis, $r = \sqrt{x^2+z^2}$

① $I_{z_1} = m_1 k_1^2 \Rightarrow k_1 = ?$

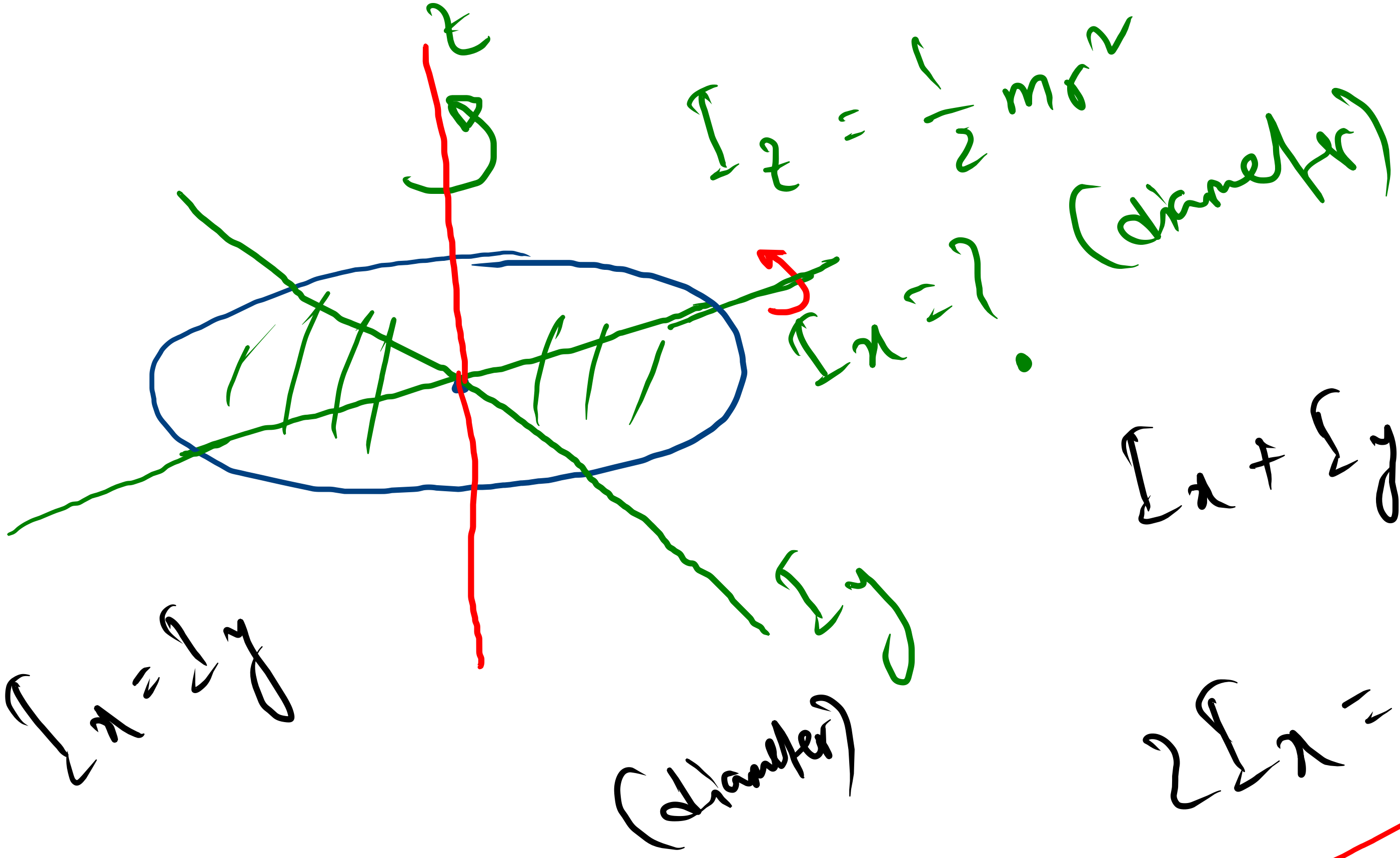
Similarly, $k_2 = ?$ $k_3 = ?$

Moment of Inertia

Perpendicular axis Theorem



yz plane-এ object থাকলে, $I_x = I_y + I_z$



$I_x = I_y$

$I_x + I_y = I_z = \frac{1}{2} m r^2$

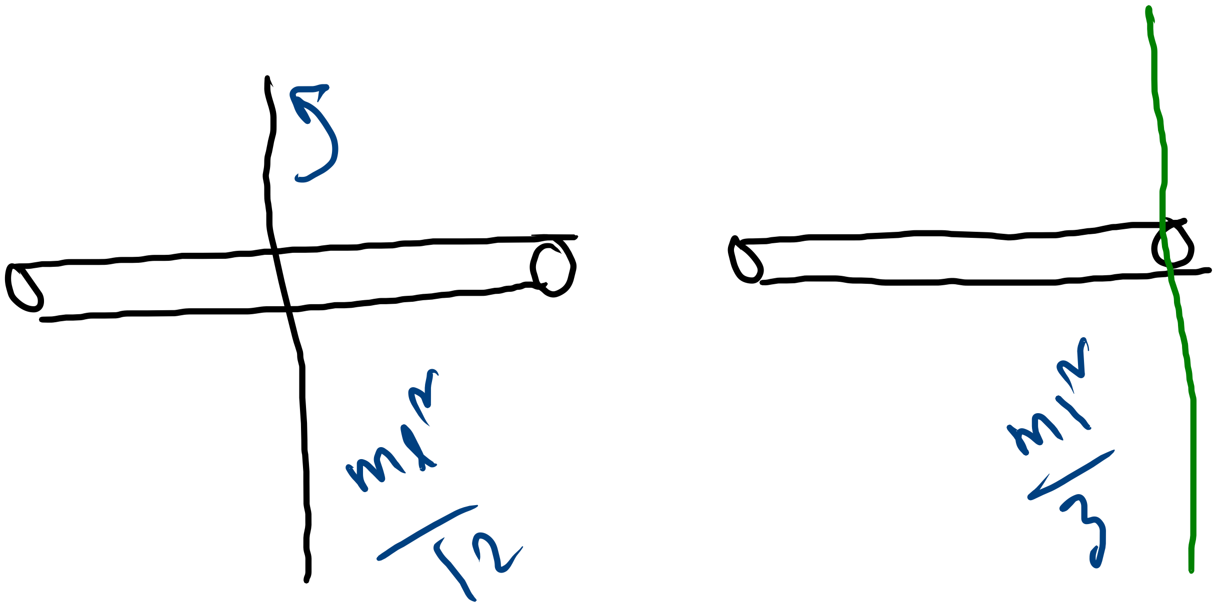
$2 I_x = \frac{1}{2} m r^2$

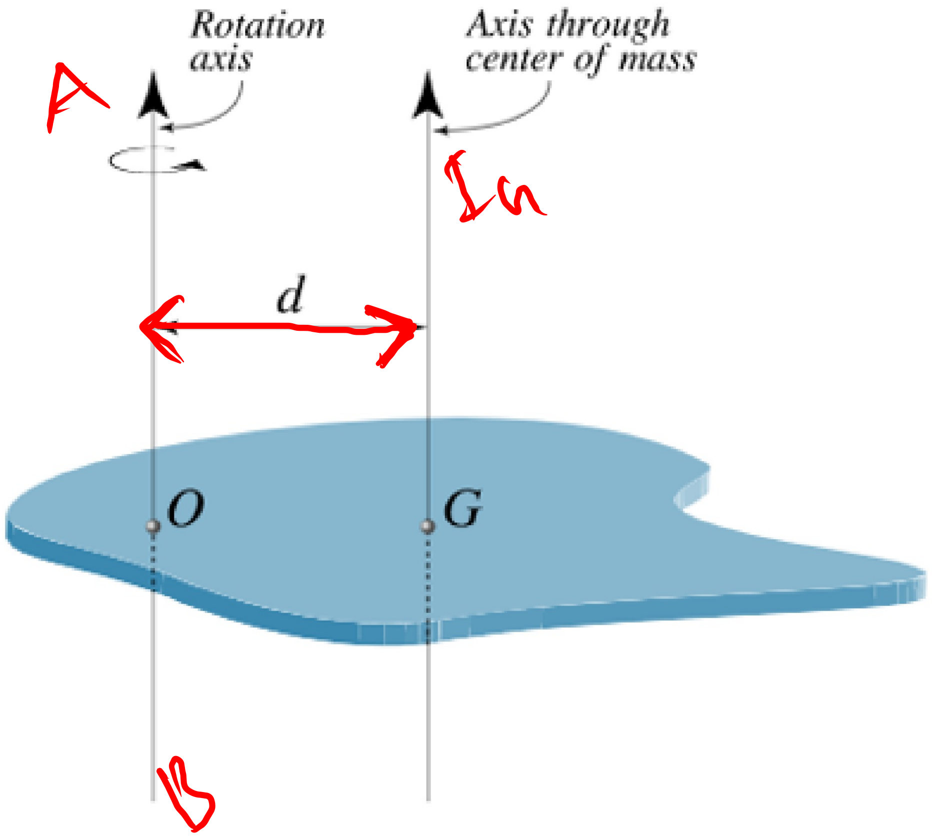
$I_x = \frac{1}{4} m r^2$

Parallel axis

Parallel axis Theorem

$$I_{AB} = I_G + md^2$$





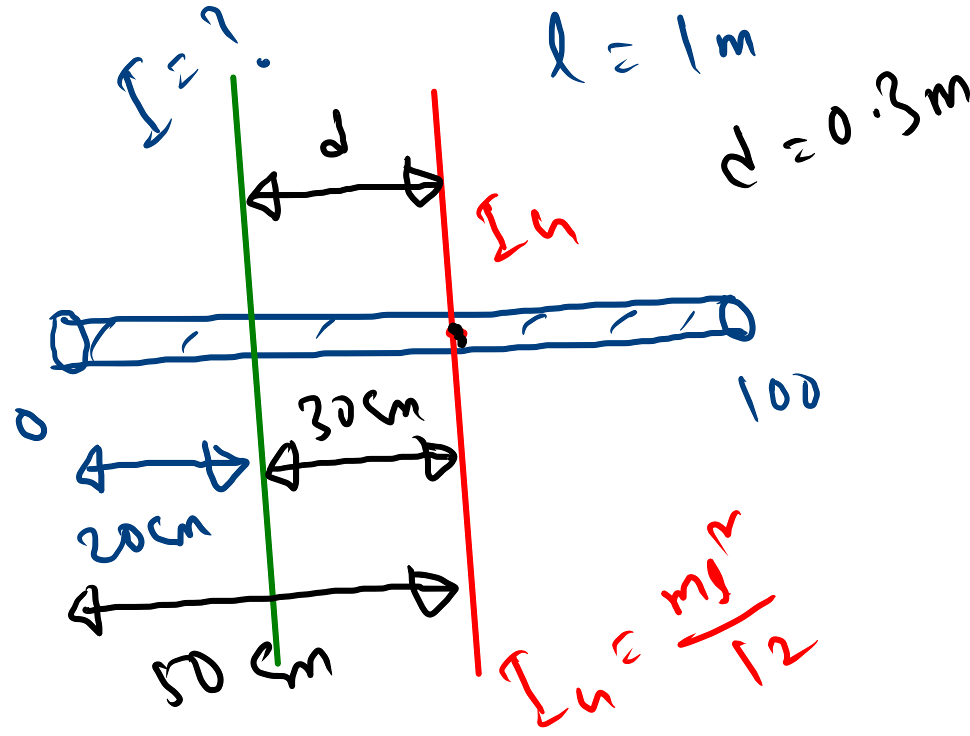
Moment of Inertia

Calculate the rotational inertia of a meter scale, with a mass of 0.5kg, about an axis perpendicular to the scale and located at the 20 cm mark. Consider the scale as a thin rod.

$$I = I_n + md^2$$

$$I = \frac{ml^2}{12} + md^2 = ?$$

$$I = ?$$



Conservation of angular momentum

An object is rotating 5 times per second in a circular path using a string of 20cm. Suddenly the radius of circle is halved ,what will be angular velocity of the object?

$L = I\omega$

$\omega_1 = 5 \text{ rev s}^{-1}$

$r_1 = 20cm$

$r_2 = r_1/2 = 10cm$

$\omega_2 = ?$

$I_1\omega_1 = I_2\omega_2$

$\therefore \frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} = \frac{mr_1^2}{mr_2^2}$

$\therefore \omega_2 = ?$

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

Torque

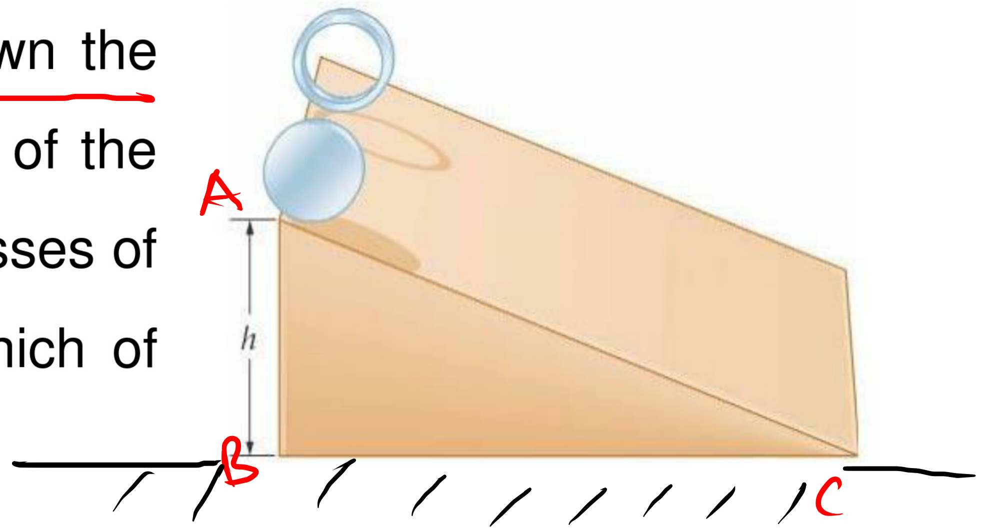
The mass of a wheel is 6kg. Its radius of gyration is 30cm. What is the value of torque which produces 3 rads^{-2} acceleration?

$F = ma$

$\tau = I\alpha = mk^2 . \alpha =$

Rolling at an inclined plane

Two bodies; a ring and a solid sphere roll down the same inclined plane without slipping. The radii of the bodies are respectively 1m and 3m and the masses of the bodies are respectively 10kg and 1kg. Which of the bodies reaches the ground at first?



Sphere,

$$m_1 g h = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 \omega_1^2$$

$$m_1 g h = \frac{1}{2} m_1 v_1^2 + \frac{1}{2}\left(\frac{2}{5} m_1 r_1^2\right)\left(\frac{v_1}{r_1}\right)^2$$

$$\therefore gh = \frac{1}{2} v_1^2 + \frac{1}{5} v_1^2$$

$$\therefore v_1 = \sqrt{\frac{10}{7} gh}$$

ring

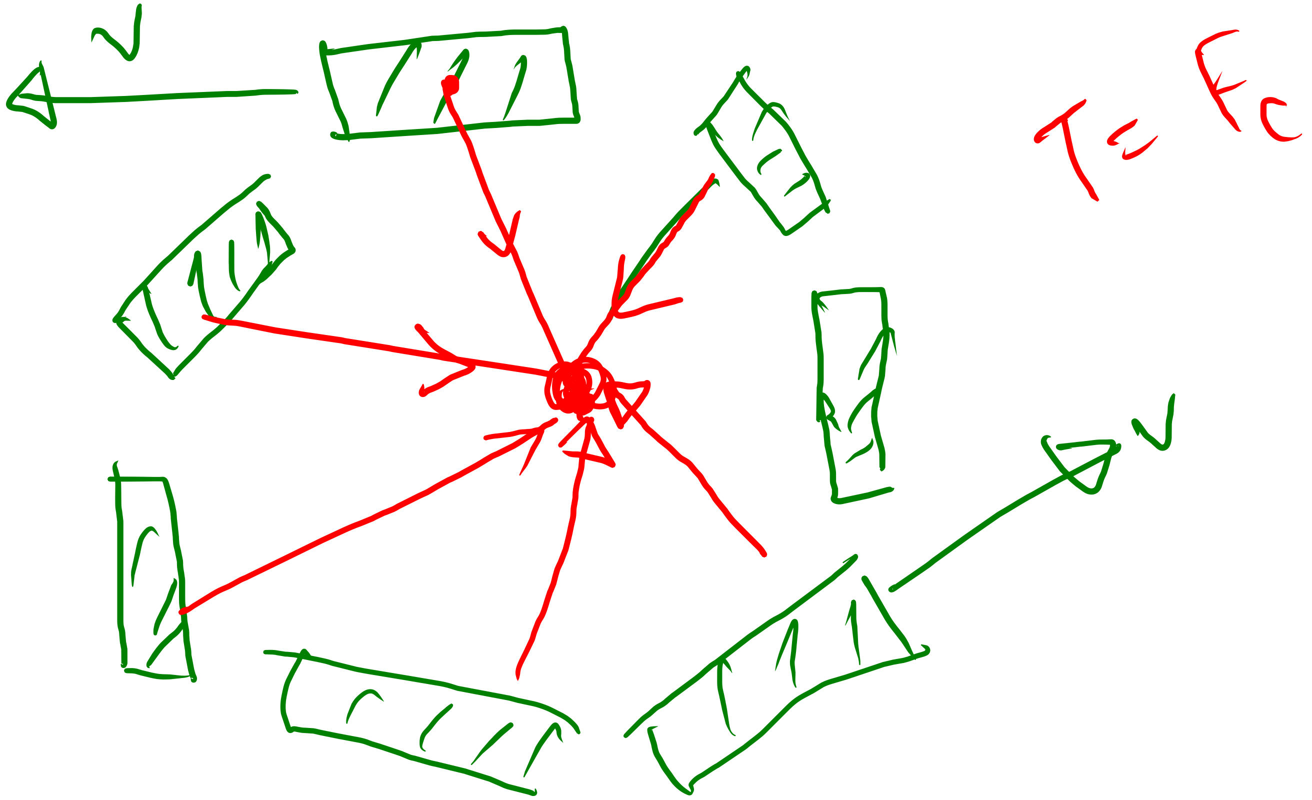
$$m_2 g h = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 \omega_2^2$$

$$= \frac{1}{2} m_2 v_2^2 + \frac{1}{2}\left(\frac{1}{2} m_2 r_2^2\right)\left(\frac{v_2}{r_2}\right)^2$$

$$\therefore \quad v_2 = \sqrt{\frac{8}{6} g h}$$

Since, $v_1 > v_2$ $\therefore$ the sphere will reach the ground first.

Centripetal force → necessary force



Centripetal force

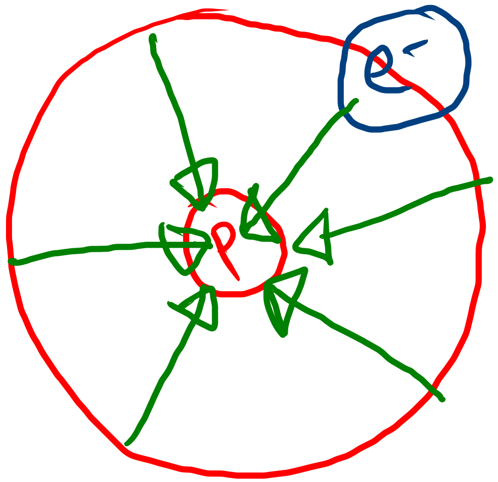
A Hydrogen electron revolves around the nucleus of its atom in a circular path of radius 0.53 Å. What is the centripetal force of electron? Find the velocity of the electron,.

$F_{coulomb} = F_c$

$\therefore \frac{1}{4\pi\epsilon_0} \frac{Q_p Q_e}{r^2} = F_c = ?$

$F_c = \frac{mv^2}{r}$

$v = ?$



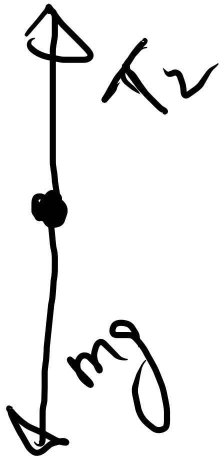
Centripetal force

A object of mass m kg is rotating on a vertical plane.

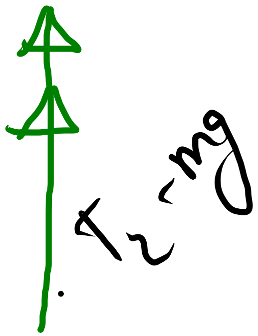
(i) What is difference of tension in the string between the highest point and lowest of point of the circle?

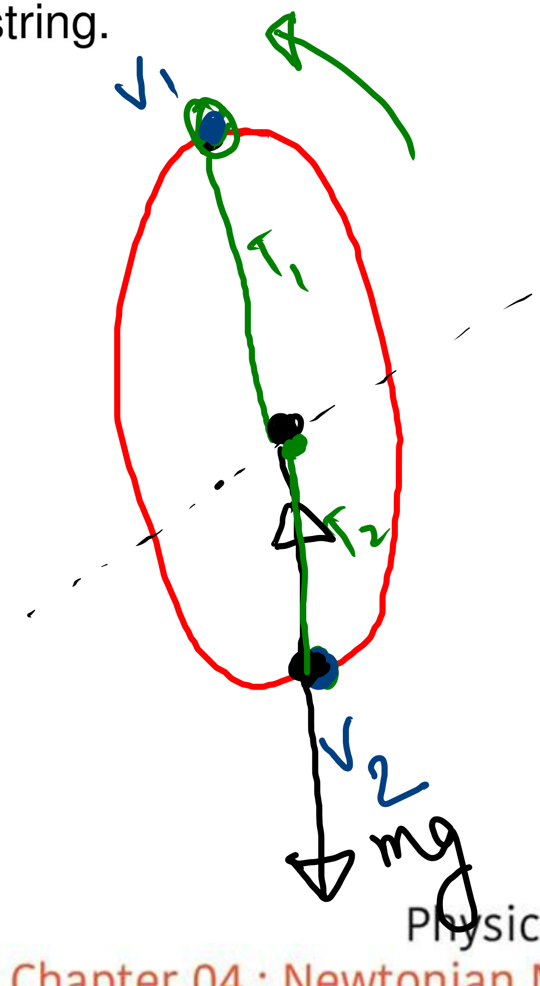
(iv) When the string makes θ angle with the vertical ,find the tension in the string.

lowest,



$T_2 > mg$





$$T_2 - mg = \frac{mv_2^2}{r}$$

$$T_2 = \frac{mv_2^2}{r} + mg$$

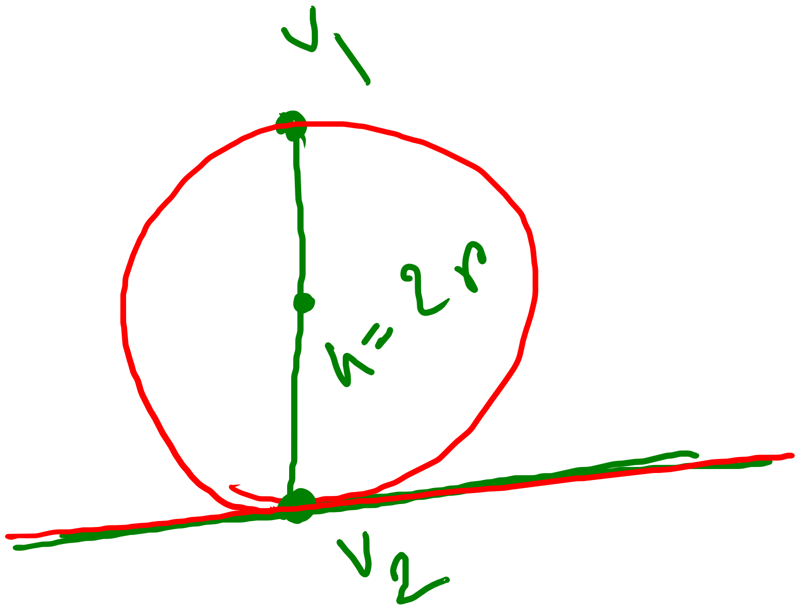
highest:

$$T_1 + mg = \frac{mv_1^2}{r}$$

(ii) $$T_1 = \frac{mv_1^2}{r} - mg$$

$$T_2 - T_1 = \frac{m}{r}\left(v_2^2 - v_1^2\right) + 2mg$$

$$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + mg(2r)$$



$$\therefore v_2^2 - v_1^2 = 4gr$$

$$\therefore T_2 - T_1 = \frac{m}{r}\cdot 4gr + 2mg = 6mg$$

∞

Banking

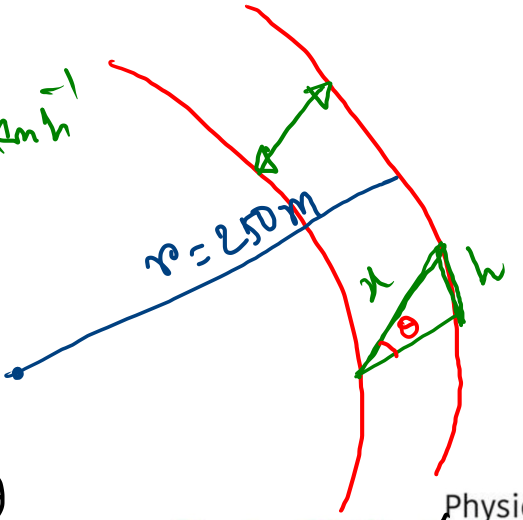
The radius of a curved rail line is 250m and the distance between the two rails is 1m. How much the outside rail is to be raised above the inside rail for required banking in the case of a running train with a speed of 50km per hour?

$\tan\theta = \frac{v^2}{rg}$

$x = 1\,m$

$h = ?$

$v = 50\,kmh^{-1}$



$\sin\theta = \frac{h}{x}$

$\therefore h = x\sin\theta$

$h = x\sin\left\{\tan^{-1}\left(\frac{v^2}{rg}\right)\right\}$ → general

$\theta \to 0^\circ$ $(\theta \approx 1\text{-}9^\circ)$

$\sin\theta = \tan\theta = \frac{h}{x} = \frac{v^2}{rg}$ (Special)

Quiz 01

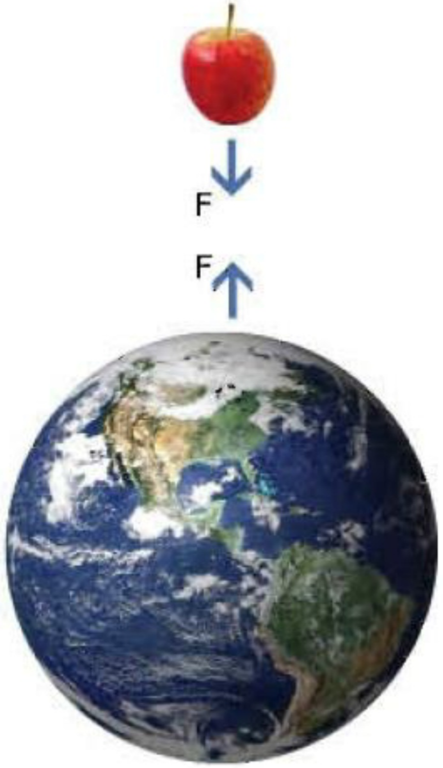
If the earth attracts an apple with a force of F newton and the apple attracts the earth with a force of f newton, which one is correct ?

(a) $F>f$

(b) $F<f$

(c) $F=F$

(d) None



Quiz 02

What is the moment of inertia of a 3m long and 4kg mass thin uniform rod is rotating through an axis?

(a) 4 kgm^2

(b) 12 kgm^2

(c) 4 kgm^{-2}

(d) 12kgm^{-2}

Quiz 03

If a balloon of mass 5kg and radius 0.25m is rolling with an angular velocity of 50 rad/sec, what is its kinetic energy?

(a) 0.078J
(b) 390.63J
(c) 0.78J
(d) 585.94J

Quiz 04

What is the ratio of the dimension of the torque to the dimension of force?

(a) MLT^{-2}

(b) L

(c) $\mathrm{ML}^{2}\mathrm{T}^{-2}$

(d) ML^{-1}

Quiz 05

A particle rotates in a circular path of radius 25cm 2 times a second. What is the acceleration of the particle in ms^{-2}?

(a) 3

(b) π^2

(c) $4\pi^2$

(d) 16

Quiz 06

An 8N force acts on a stationary object of mass 16kg for 4s.

The change of velocity of the object is-

(a) 0.5 m/s

(b) 2 m/s

(c) 4 m/s

(d) 8 m/s

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে।