VARSITY ‘Ka’ ADMISSION PROGRAM-2020

HIGHER MATH

Lecture : M-02

Chapter 02 : Vector

Chapter 08 : Function and Graph of Function

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

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Chapter-08
Function and Graph of Function

Idention of Functions from Mapping

Range = co-domain → surjective

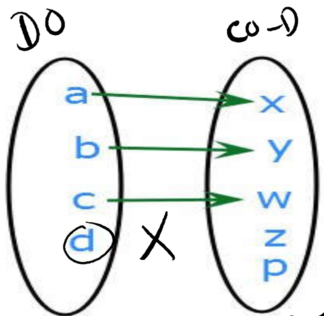
not func

codomain: Range

set of dependent variable

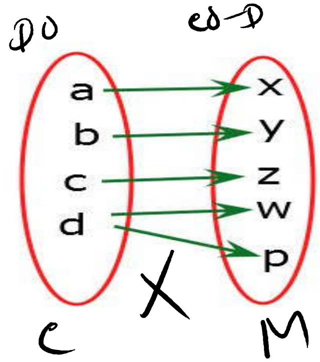
Domain: set of independent variable

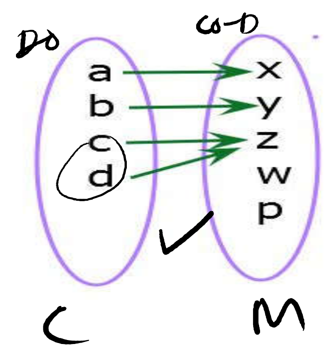
x

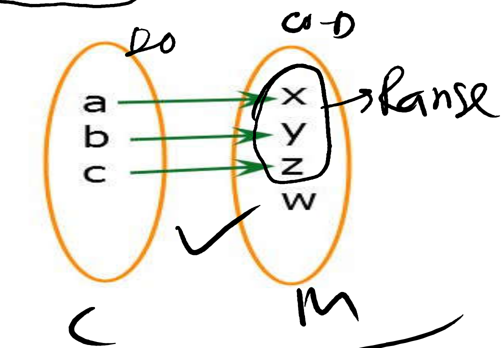


children = c

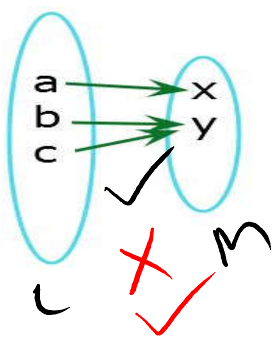
M = mother

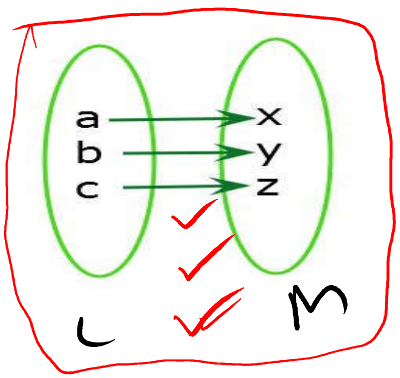






Range is a subset of co-D





one-one/injective: X

On-to/surjective: X

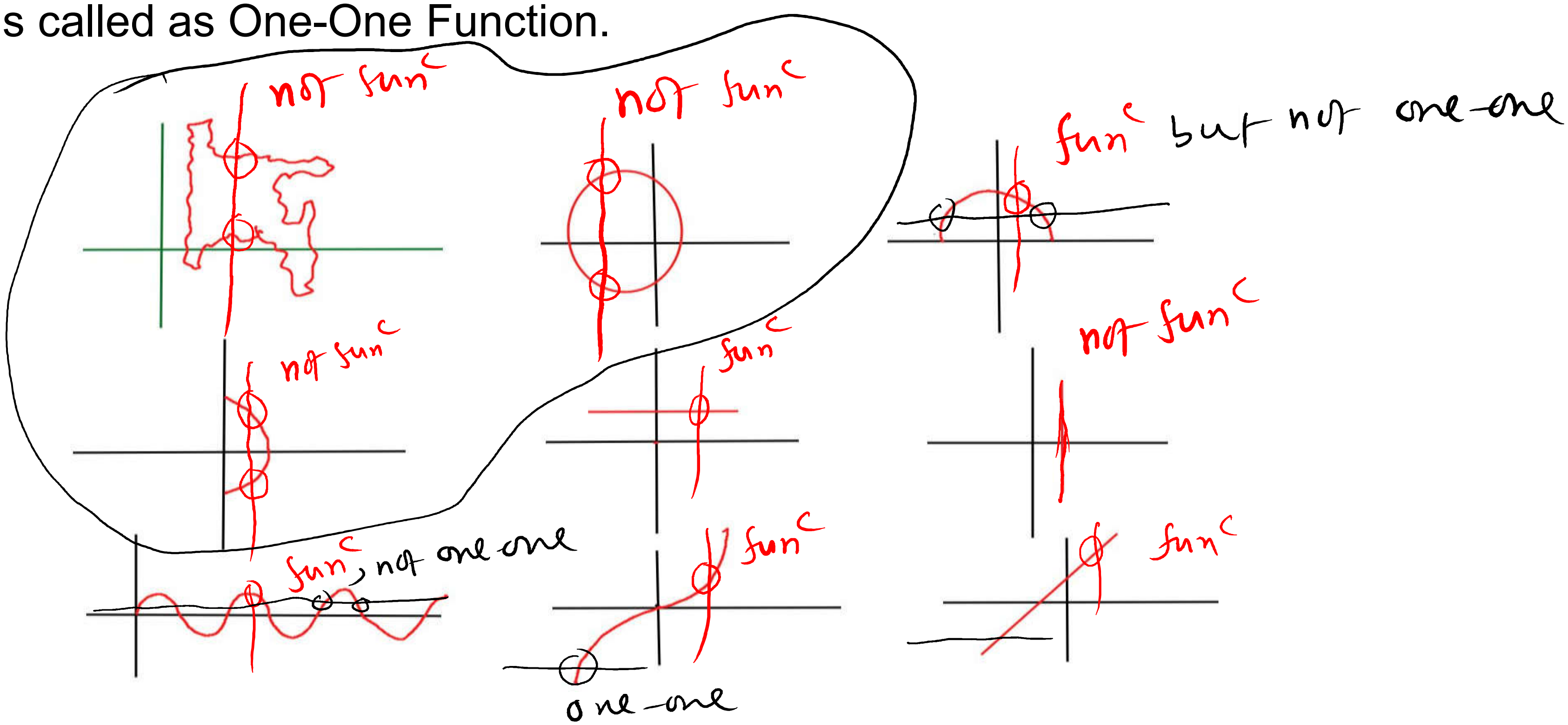
func

Bijective func: one-one + on-to

Identification of Functions and One-One Function from Graph

If y –axis or its' parallel line intersects the graph of a relation at one point only, then the relation is called as Function.

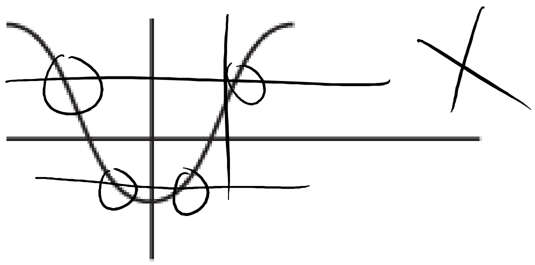
If x –axis or its' parallel line intersects the graph of a function at one point only, then the function is called as One-One Function.



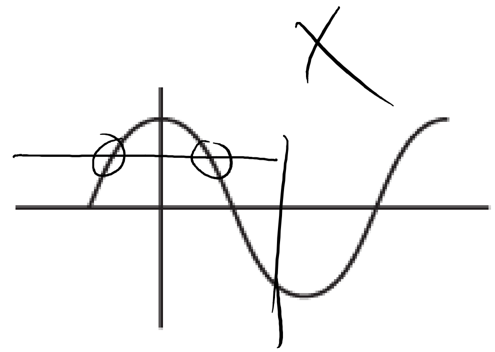
Poll Question 01

Which is One-One Function?

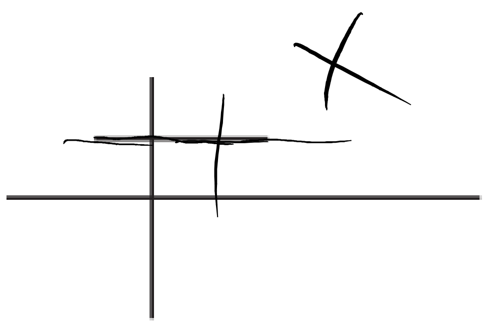
(a)



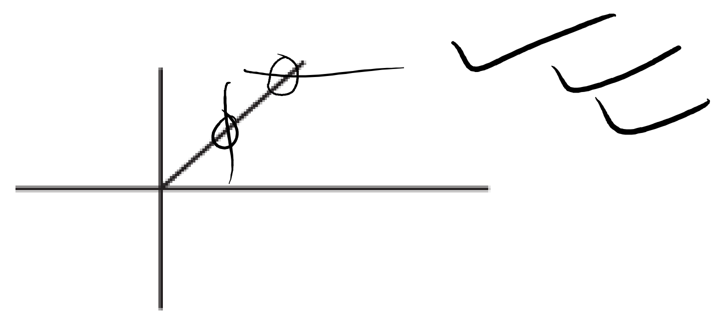
(b)



(c)



(d)



Determination of Domain & Range

For, $y = f(x)$

For which set of real values of x, the values of y or $f(x)$ will be real, is called as Domain of $f(x)$.

Domain: set of independent variable x

For, y = f(x)
For the real values of x which belong to Dom f, the obtained values of y or f(x) is called the Range of f(x).

Range: set of dependent variable y

Determination of Domain and Range of Different Types of Functions

$f(x) = 2x + 1$

$f(x) = ax+b, \quad a, b \in R$

Sol^n:

$D_f : \mathbb{R}$

$R_f : \mathbb{R}$

$D_f : \mathbb{R}$

$R_f : \mathbb{R}$

Determination of Domain and Range of Different Types of Functions

$f: A \longrightarrow B; f(x) =$ Any relation

Domain

codomain

$R \rightarrow R_+, R_-$

$\boxed{0, 1, 2, \frac{2}{3},}$

Example: $f: R_+ \longrightarrow R; f(x) = 2x + 1$

input

$D_f: R_+$

$R_f: R_+$

interval

$(2,3) \rightarrow 2 < x < 3 \rightarrow]2,3[$

$[2,3] \rightarrow 2 \le x \le 3$

$(a,b] \rightarrow a < x \le b$

$[a,b) \rightarrow a \le x < b$

Determination of Domain and Range of Different Types of Functions

$$f(x) = \frac{2x+1}{5x+9}$$

Sol^n:

D_f: $5x+9 \neq 0$

$5x \neq -9$

$x \neq -9/5$

$\therefore D_f : \mathbb{R} - \{-9/5\}$

Let,

$$y = \frac{2x+1}{5x+9}$$

$\Rightarrow 5xy + 9y = 2x+1$

$5xy - 2x = 1 - 9y$

$x(5y-2) = 1-9y$

$$\therefore x = \boxed{\frac{1-9y}{5y-2}}$$

Range:

$5y - 2 \neq 0$

$5y \neq 2$

$y \neq 2/5$

$\therefore$ Range: $\mathbb{R} - \{2/5\}$

$$f(x) = \frac{ax+b}{cx+d}$$

Range: $\mathbb{R} - \left\{\frac{a}{c}\right\}$

$$f(x) = \frac{2x+1}{5x+9}$$

R_f: $\mathbb{R} - \left\{\frac{2}{5}\right\}$

Determination of Domain and Range of Different Types of Functions

$$f(x) = \sqrt{9 - x^2} = \sqrt{3^2 - x^2} = \sqrt{a^2 - x^2} = \sqrt{\text{constant}^2 - \text{variable}^2}$$

Sol^n:

D_f: $9 - x^2 \geq 0$

$3^2 - x^2 \geq 0$

$-x^2 \geq -3^2$

$x^2 \leq 3^2$

$|x| \leq 3$

$+$ $x \leq 3$

$-$ $-x \leq 3$

$\therefore x \geq -3$

$\therefore \boxed{-3 \leq x \leq 3}$

D_f: $-a \leq x \leq a \rightarrow [-a, a]$

$f(x) = \sqrt{a^2 - x^2}$

$[0, a]$

$f(x) = \sqrt{5^2 - x^2}$

$[0, 5]$

$f(x) = \sqrt{6^2 - x^2}$

$[0, 6]$

$f(x) = \sqrt{a^2 - x^2}$

D_f: $-a \leq x \leq a$

R_f: $[0, a]$

$f(x) = \sqrt{10^2 - x^2}$

Range: $[0, 10]$

Poll Question 02

Find the Domain of $f(x) = \frac{1}{\sqrt{36 - 25x^2}}$

(a) $\left(-\frac{6}{5}, \frac{6}{5}\right)$

(b) $\left[-\frac{6}{5}, \frac{6}{5}\right]$

(c) $\left(-\frac{5}{6}, \frac{5}{6}\right)$

(d) $\left[-\frac{5}{6}, \frac{5}{6}\right]$

$$\frac{1}{\sqrt{b - ax^2}}$$

$$\left(-\sqrt{\frac{b}{a}}, \sqrt{\frac{b}{a}}\right)$$

$$\left(-\sqrt{\frac{36}{25}}, \sqrt{\frac{36}{25}}\right)$$

Determination of Domain and Range of Different Types of Functions

$$f(x) = \sqrt{x^2 - 49} = \sqrt{x^2 - 7^2} = \sqrt{\text{variable} - \text{constant}}$$

Range: $[0, \infty)$

Sol^n:

D_f: $x^2 - 49 \geq 0$

$x^2 - 7^2 \geq 0$

$x^2 \geq 7^2$

$|x| \geq 7$

$(+)$ $x \geq 7$

$(-)$ $-x \geq 7$

$\therefore x \leq -7$

$x \geq 7$ or $x \leq -7$

$-\infty$ $\quad$ $+\infty$

-7 $\quad$ 7

$x \leq -7$

$-\infty < x \leq -7$ or $7 \leq x < \infty$

$(-\infty, -7] \cup [7, \infty)$

$\sqrt{ax^2 - b}$

D_f: $\left(-\infty, -\sqrt{\frac{b}{a}}\right] \cup \left[\sqrt{\frac{b}{a}}, \infty\right)$

$(\;) \cup (\;)$

$$\frac{1}{\sqrt{ax^2 - b}}$$

Poll Question 03

Fine the Domain of $f(x) = \frac{1}{\sqrt{25x^2-16}} \neq 0$

$\left(-\infty, -\sqrt{\frac{b}{a}}\right) \cup \left(\sqrt{\frac{b}{a}}, \infty\right)$

$\left(-\infty, -\sqrt{\frac{16}{25}}\right) \cup \left(\sqrt{\frac{16}{25}}, \infty\right)$

(a) $\left(-\infty, -\frac{5}{4}\right) \cup \left(\frac{5}{4}, \infty\right)$

(b) $\left(-\infty, -\frac{4}{5}\right) \cup \left(\frac{4}{5}, \infty\right)$

(c) $\left]-\infty, -\frac{5}{4}\right] \cup \left[\frac{5}{4}, \infty\right[$

(d) $\left]-\infty, -\frac{4}{5}\right] \cup \left[\frac{4}{5}, \infty\right[$

Domain of logarithmic functions:

$\log_a x$

1. x>0
2. a>0 &$a \neq 1$

We also need to know, if, $\log_a x$=p

then, x=a^p

$\log_a \boxed{x} \rightarrow \boxed{x > 0}$

$\boxed{\text{base}}$

$a \neq 0$

$a \neq 1$

$a > 0$

$\log_a \square$

$\square > 0$

Domain of logarithmic functions:

$f(x) = \log(5x - 1)$

Sol^n:

$5x - 1 > 0$

$5x > 1$

$\boxed{x > \frac{1}{5}}$

$\log(2x-3)$

$\downarrow$

$2x - 3 > 0$

$2x > 3$

$x > 3/2$

Poll Question 04

Find the Domain and Range of $f(x) = \frac{x}{|x|} = \frac{0}{|0|} = \boxed{\frac{0}{0}}$ ×

(a) $d_f = \mathbb{R},\ R_f = \mathbb{R}$

(b) $d_f = \mathbb{R}\backslash\{0\}$, $R_f = \{-1, +1\}$

(c) $d_f = \mathbb{R}_+,\ R_f = [-1, +1]$

(d) $d_f = \mathbb{R}_-,\ R_f = \{0\}$

signed data: { }

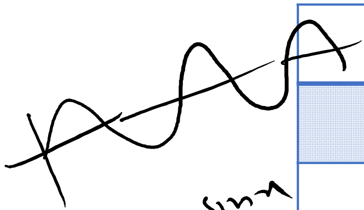
$\frac{x}{|x|} = \frac{2}{|2|} = \frac{2}{2} = 1$

$\frac{x}{|x|} = \frac{-2}{|-2|} = \frac{-2}{2} = -1$

$\frac{x}{|x|} = \frac{3.5}{|3.5|} = \frac{3.5}{3.5} = 1$

$\frac{-3.5}{|-3.5|} = \frac{-3.5}{3.5} = -1$

Domain & Range of Trigonometric Function:



$f(x) = \sin x$

Function	Domain	Range
sin x	**R**	[-1,1]
cos x	**R**	[-1,1]
tan x	**R** – {(2n+1)π/2, n∈**Z**}	**R**
cot x	**R** – {nπ, n∈**Z**}	**R**
sec x	**R** – {(2n+1)π/2, n∈**Z**}	(-∞,-1] ∪ [1, ∞)
cosec x	**R** – {nπ, n∈**Z**}	(-∞,-1] ∪ [1, ∞)

[-1,1)

Domain & Range of Trigonometric Function:

Find the domain & range of f(x) = 2 + 3sinx.

$D_f: \mathbb{R}$

Range: $[\min^m, \max^m]$

$[-1, 5]$

$\min^m: 2+3\times(-1) = 2-3 = -1$

$\max^m: 2+3\times 1 = 5$

Shortcut (Inverse Function)

$$f(x) = \frac{ax+b}{cx+d}$$

$$f^{-1}(x) = \frac{-dx+b}{cx-a}$$

$$f(x) = \frac{5x+6}{2x-3}$$

$$f^{-1}(x) = \frac{-3x+6}{2x+5}$$

$$y = 5x+6$$

$$y-6 = 5x$$

$$x = \frac{y-6}{5}$$

$$= \frac{x-6}{5}$$

$$f(x) = \frac{2x+6}{3x}$$

$$= \frac{2x+6}{3x+0}$$

$$f^{-1}(x) = \frac{0 \times x+6}{3x-2}$$

$$= \frac{6}{3x-2}$$

$$y = \frac{2x+6}{3x}$$

$$3xy = 2x+6$$

$$3xy-2x = 6$$

$$x(3y-2) = 6 \quad \therefore x = \frac{6}{3y-2}$$

Poll Question 05

If $f: R \to R; f(x) = 2x + 1$then what will be the value of $f^{-1}(x)$?

(a) $\frac{x+1}{2}$

(b) $\frac{x-2}{1}$

(c) $\frac{x-1}{2}$

(d) None

$y = 2x + 1$

$y - 1 = 2x$

$\therefore x = \frac{y-1}{2}$

$= \frac{x-1}{2}$

Composite Function:

Composite Function:

If $f(x) = \sqrt{x-1}, g(x) = x^2 + 2$, then find $(gof)(2) = ?$

$(gof)(2)$

$= g(f(2))$

$= g(1)$

$= 1^2 + 2$

$= 3$

$f(x) = \sqrt{x-1}$

$f(2) = \sqrt{2-1} = 1$

Problem related to the function value:

If $f: R \to R;\ f(x) = \begin{cases} 3x+1, & x > 3 \\ x-2, & -2 \le x \le 3 \\ 2x+3, & x < -2 \end{cases}$, then find $f(2), f(4), f(-1), f(-3)$.

$\therefore f(2) = x-2$

$f(4) = 3x+1$

$f(-1) = x-2$

$f(-3) = 2x+3$

Some special functions:

- **Even function:** $f(x) = \cos x$

  $f(-x) = \cos(-x) = \cos x = f(x)$

  $f(x) = x^2$

  $f(-x) = (-x)^2 = x^2$

- **Odd function:** $f(x) = \sin x$

  $f(-x) = \sin(-x) = -\sin x = -f(x)$

- **Identity Function:** $f(x) = x$

  output = input

- **Constant function:** $f(x) = 10$

  fixed

Poll Question 06

Which one is even function?

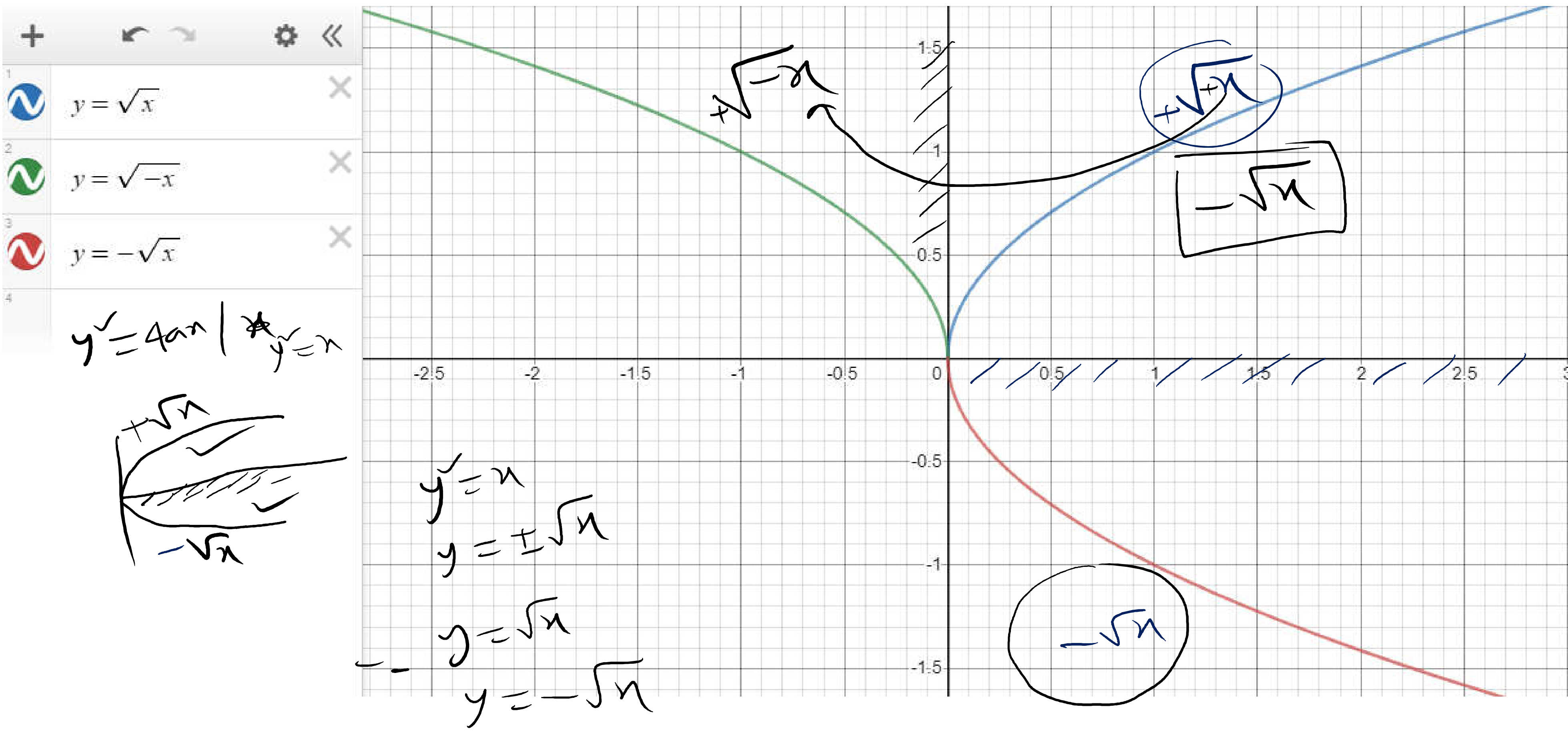
(a) $f(x) = tanx$ $\longrightarrow$ $f(-x) = \tan(-x) = -\tan x = -f(x)$

(b) $f(x) = x^2 + 2x$ $\longrightarrow$ $(-x)^2 + 2(-x) = x^2 - 2x$

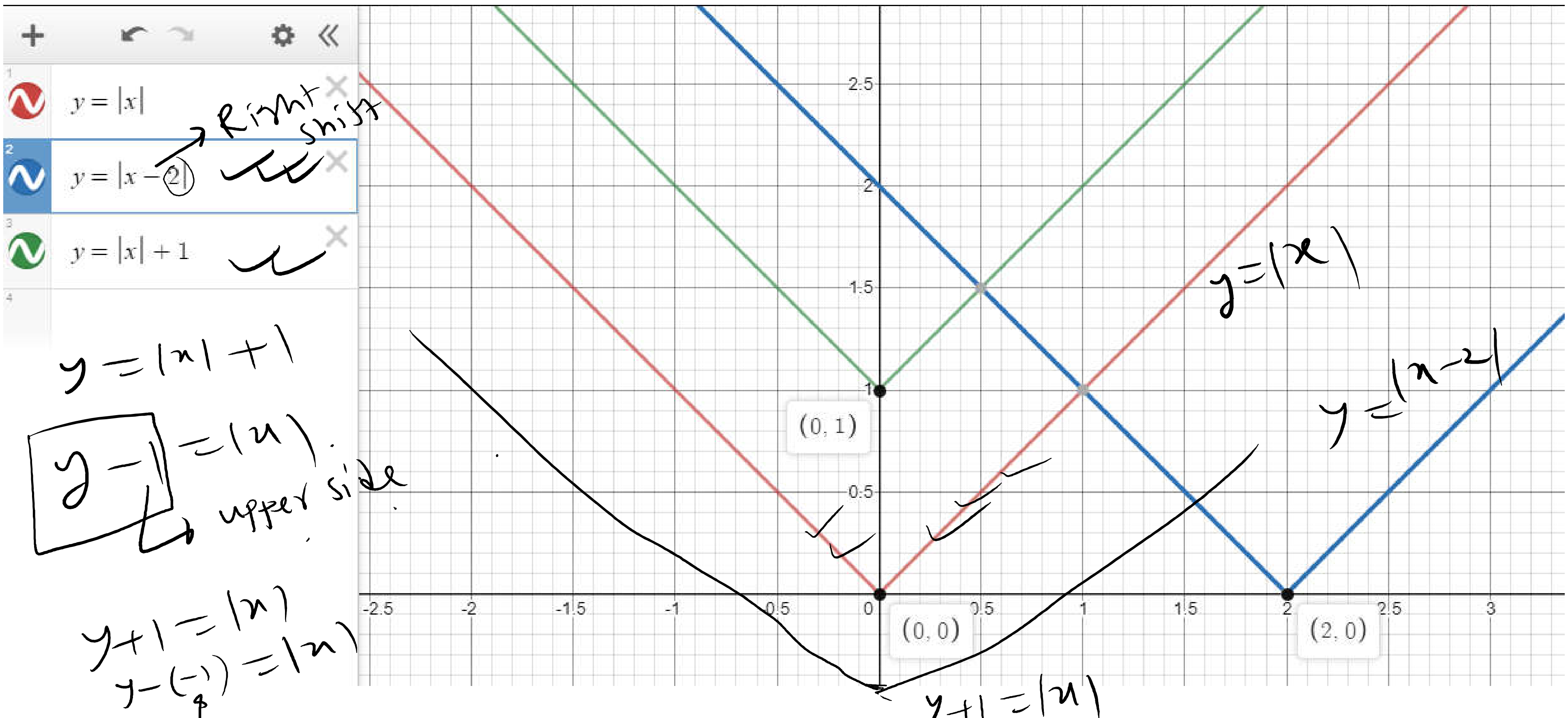
(c) $f(x) = sinx + 2$ $\longrightarrow$ $\sin(-x) + 2 = -\sin x + 2$

(d) None ✓

Graph of Functions (Symmetry)



Graph of Functions (Shifting):



Chapter-02 : Vector

Determination of magnitude and internal angle

Concept:

(i) For a vector $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$, $|\vec{A}| = \sqrt{x^2 + y^2 + z^2}$

(ii) If the angle between two vectors $\vec{A}$ and $\vec{B}$ is θ,

then $\vec{A}.\vec{B} = AB\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}.\vec{B}}{AB}$;

Determination of magnitude and internal angle

If $\vec{P} = 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$ and $\vec{Q} = 4\hat{\imath} - 2\hat{\jmath} - \hat{k}$ then what's the angle between $\vec{P}$ and $\vec{Q}$?

$$\bar{P} \cdot \bar{Q} = PQ \cos\theta$$

$$\therefore \cos\theta = \frac{\bar{P} \cdot \bar{Q}}{PQ}$$

$$= \frac{4 \times 4 + (-2) \times (-2) + 4 \times (-1)}{\sqrt{4^2 + (-2)^2 + 4^2}\ \sqrt{4^2 + (-2)^2 + (-1)^2}}$$

$$=$$

Poll Question 07

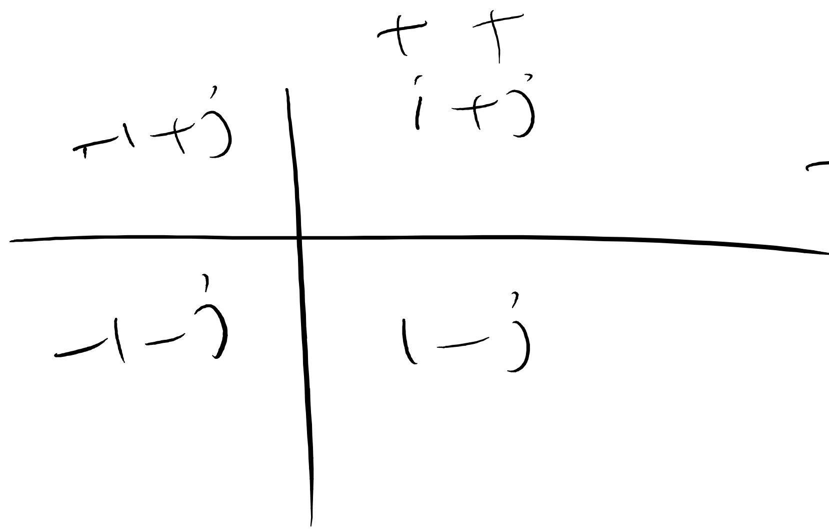
Find the angle that the vector $\overline{A} = \hat{i} + \hat{j}$ makes with the x-axis.

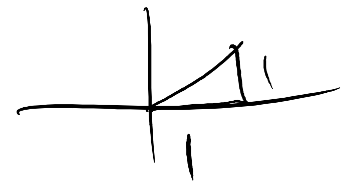
(a) 45° ✓

(b) 210°

(c) Both a & b

(d) None





$\tan\theta = \frac{1}{1}$

$= \tan 45°$

$\theta = 45°$

Related to unit vector

Find the perpendicular unit vector on the plane formed by
$\overline{A} = \hat{i} + \hat{j} + \hat{k}$ *and* $\overline{B} = \hat{i} - 3\hat{j} + \widehat{2k}$.

$$\overline{A} \times \overline{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \hat{i}(2+3) - \hat{j}(2-1) + \hat{k}(-3-1)$$

$$= 5\hat{i} - \hat{j} - 4\hat{k}$$

$$\eta = \pm \frac{(\overline{A} \times \overline{B})}{|\overline{A} \times \overline{B}|}$$

$$= \pm \frac{(5\hat{i} - \hat{j} - 4\hat{k})}{\sqrt{5^2 + (-1)^2 + (-4)^2}}$$

Related to perpendicular or parallel vector

Concept:

(i) Condition on two perpendicular vectors, $\vec{A}.\vec{B} = 0$

(ii) Condition on two parallel vectors, $|\vec{A} \times \vec{B}| = 0$

Shortcut for MCQ : $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$; $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ & if $\vec{A}||\vec{B}$ then $\boxed{\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}}$

Related to perpendicular or parallel vector

For which value of a, $\vec{A} = 8\hat{i} + \hat{j} - a\hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} + 5\hat{k}$ will be perpendicular on each other?

$\bar{A} \cdot \bar{B} = 0$

$8 \times 4 + 1 \times (-2) + (-a) \times 5 = 0$

$a = (\quad)$

Poll Question 08

For which value of m, $4\hat{i} + 3\hat{j} + 5\hat{k}$ & $8\hat{i} + 6\hat{j} + \frac{m}{5}\hat{k}$ will be parallel?

(a) $\frac{10}{25}$

(b) $\frac{5}{3}$

(c) 50

(d) None

$$\frac{3}{6} = \frac{5}{m/5}$$

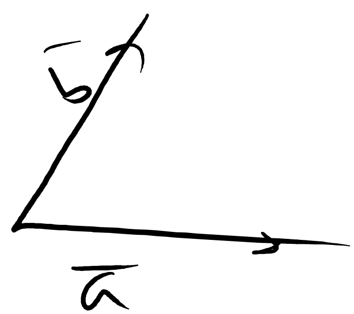
$$\frac{1}{2} = \frac{25}{m}$$

$$\therefore m = 50$$

Related to projection and component

Related to projection and component

If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ & $\vec{b} = 4\hat{i} + 8\hat{j} - \hat{k}$ then find the component of $\vec{b}$ on $\vec{a}$ & projection of $\vec{b}$ along $\vec{a}$. [BUET'08-09, 09-10, 10-11,12-13,13-14; KUET' 05-06,09-10; DU'16−17]



projection:

$b\cos\theta = \boxed{\dfrac{\bar{a}\cdot\bar{b}}{a}}$ → scaller Quantity

component: $\left(\dfrac{\bar{a}\cdot\bar{b}}{a}\right)\hat{a}$

Related to area

Concept:

$\vec{A}$and $\vec{B}$ are two vectors,

- If they indicate two side of a triangle then area, $\Delta = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$
- If they indicate two diagonal of a parallelogram then area, $\Delta = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$
- If they indicate two side of a parallelogram then area,$\Delta = \left|\vec{A} \times \vec{B}\right|$

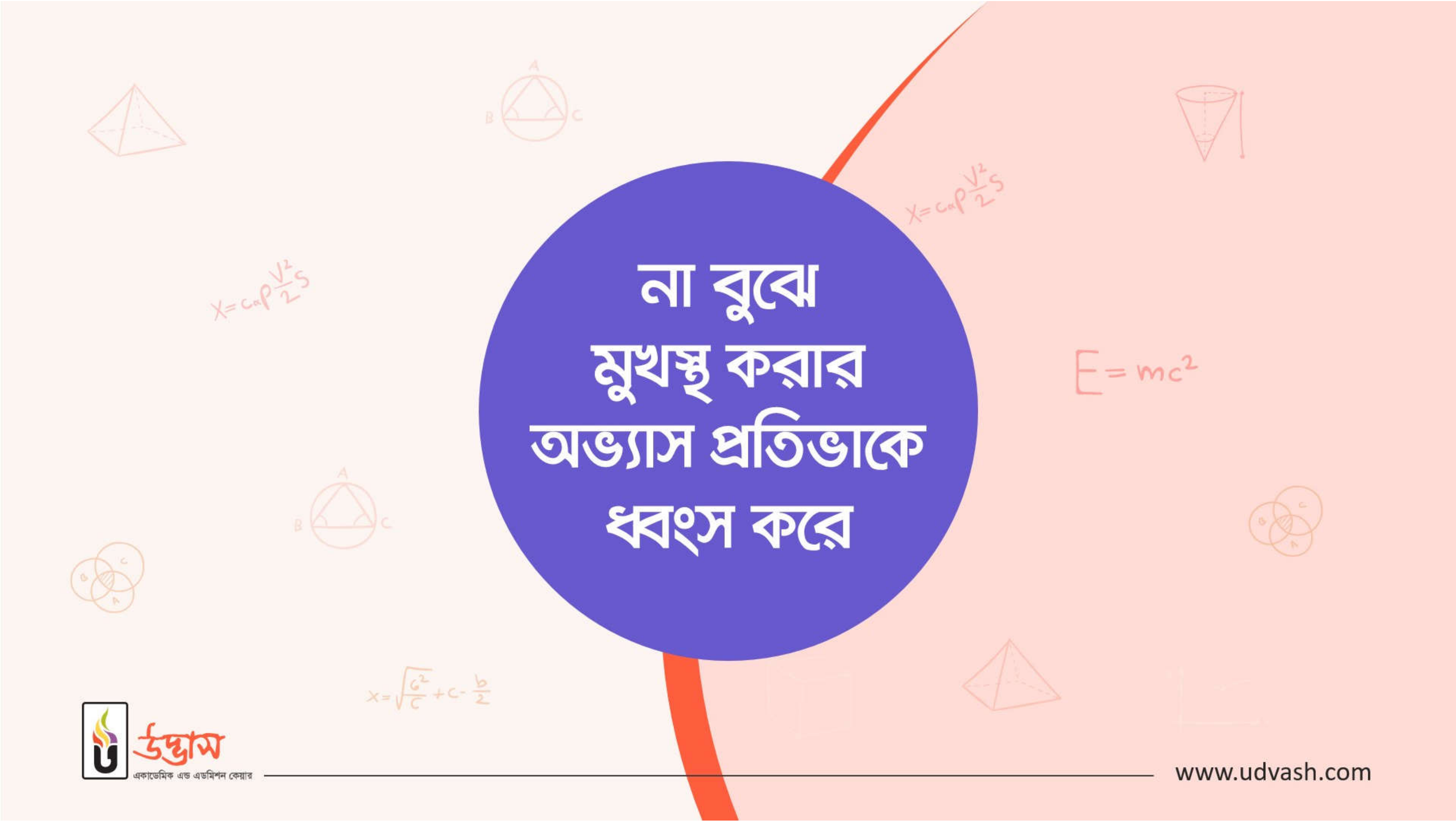
Related to area

If $\vec{P} = 4\hat{i} - 4\hat{j} + \hat{k}$ & $\vec{Q} = 2\hat{i} - 2\hat{j} - \hat{k}$ is expressed as two adjacent sides of a parallelogram, then find it's area. [CUET'15-16, DU'17-18]

$$\bar{P} \times \bar{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 1 \\ 2 & -2 & -1 \end{vmatrix}$$

$$= 6\hat{i} + 3\hat{j}$$

$$|\bar{P} \times \bar{Q}| = \sqrt{6^2 + 3^2}$$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