VARSITY ‘Ka’ ADMISSION PROGRAM-2020

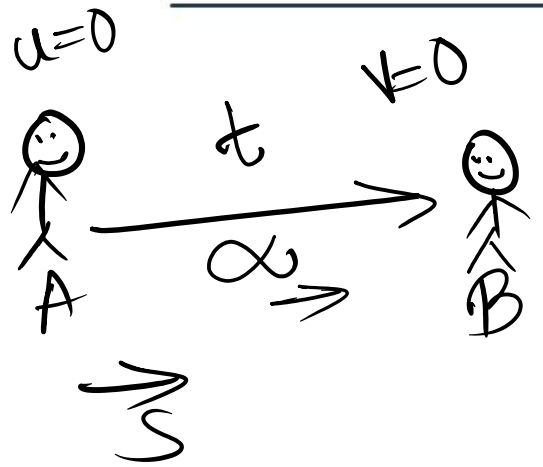
PHYSICS

Lecture : P-02

Chapter 3 : Dynamics

উদ্ভাস
একাডেমিক এন্ড এডমিশন কেয়ার

Quantities of motion:



$\Rightarrow S$

$S = vt$

$S = \left(\frac{u+v}{2}\right)t$

$S = ut + \frac{1}{2}at^2$

t_{th} second, $S_{th} = u + \frac{1}{2}a(2t-1)$

$v = u + at$

$v^2 = u^2 + 2as$

Maximum height, $H = \frac{v_0^2}{2g}$

Horizontal range, R

If you throw it with an angle

Time required to go to maximum height, $t = \frac{v_0}{g}$

$T = \frac{v_0}{g} + \frac{v_0}{g} = \frac{2v_0}{g}$

Poll Question 01

Distance crossed at t^{th} second:

(a) $s = ut + \frac{1}{2}gt^2$

(b) $s = vt$

(c) $s = u + \frac{1}{2}a(2t - 1)$

(d) $s = u + \frac{1}{2}a(2t + 1)$

Poll Question 02

The time required for a vertically thrown object to reach its maximum height:

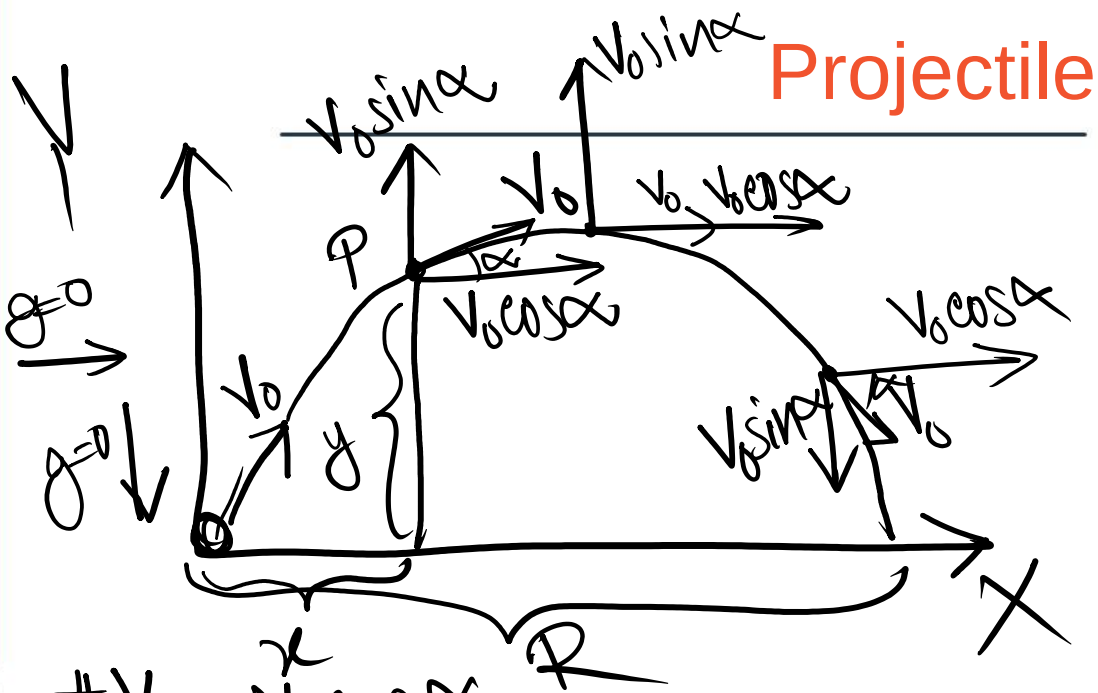
(a) $t = \frac{u}{g}$

(b) $t = \frac{2u}{g}$

(c) $t = \frac{u}{2g}$

(d) $t = u - \frac{g}{2g}$

Projectile and its identity:



$\# V_x \Rightarrow V_0\cos\alpha$

$\# V_y = V_0\sin\alpha - gt$

$\# x = (V_0\cos\alpha)\,t$

$\# y = t(V_0\sin\alpha) - \frac{1}{2}gt^2$

$\# y = x\tan\alpha - \frac{gx^2}{2V_0^2\cos^2\alpha}$

$y = bx - cx^2$ → Parabolla

Maximum Hight,

$H = \frac{(V_0\sin\alpha)^2}{2g}$

$\# R = \frac{V_0^2\sin 2\alpha}{g}$

$\# t = \frac{V_0\sin\alpha}{g}$

$\# T = \frac{2V_0\sin\alpha}{g}$

Poll Question 03

Applicable to the direction of the projectile –

(a) circular

(b) linear

(c) ellipse

(d) parabolic

Poll Question 04

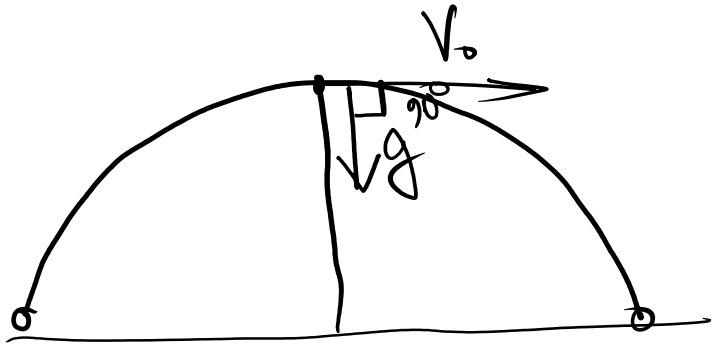
When a projectile reaches the highest point of its course, its direction of velocity and acceleration is-

(a) parallel to each other

(b) opposite to each other

(c) interacting at an angle of $45°$

(d) perpendicular to each other



Mathematical problems related to projectile:

When an object is thrown at an angle of $\mathbf{60^0}$ with a velocity of $40ms^{-1}$, then determine the-

(a) Maximum height

(b) Time to rise to maximum height

(c) Horizontal scale of range

(d) Time to hit the ground (T)

(e) Velocity after 2 s

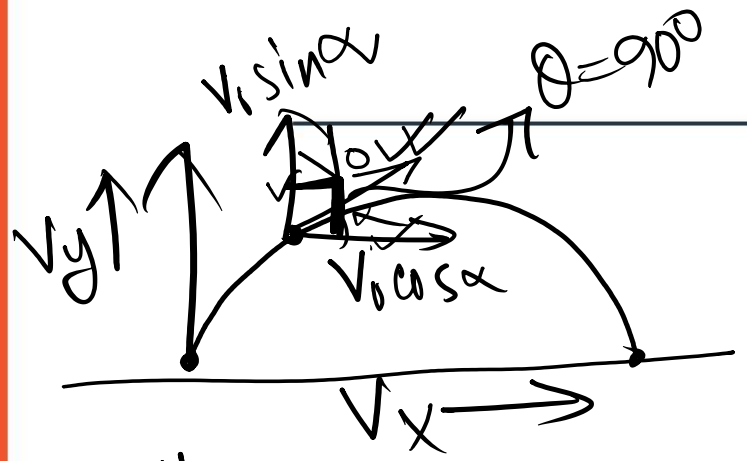
(f) Find the angle formed with the horizontal after 2 s.

(a) $H = \frac{v_0^2 \sin^2\alpha}{2g} = \frac{(40)^2 (\sin 60)^2}{2 \times 9.8} = 61.22m$

(b) $t = \frac{v_0 \sin\alpha}{g} = \frac{40 \sin 60^0}{9.8} = 3.53$

(c) $R = \frac{v_0^2 \sin 2\alpha}{g} = \frac{(40)^2 \sin 120^0}{9.8} = 141.39$

(d) $T = \frac{2v_0 \sin\alpha}{g} = 7.075$



Here, $V_x = V_0 \cos\alpha = 40 \cos 60^\circ = 40 \times \frac{1}{2} = 20$

$V_y = V_0 \sin\alpha - gt = 40 \sin 60^\circ - (9.8 \times 2)$

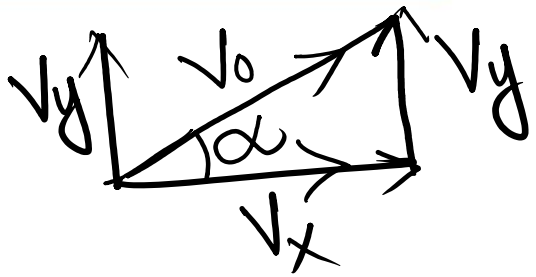
$= 15.04$

$V_0 = \sqrt{V_x^2 + V_y^2 + 2 V_x V_y \cos\theta}$ → $\cos 90^\circ = 0$

$= \sqrt{V_x^2 + V_y^2}$

$= \sqrt{(20)^2 + (15.04)^2}$

$= 25.02 \text{ ms}^{-1}$



$\tan\alpha = \frac{V_y}{V_x}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{15.04}{20}\right)$

$\therefore \alpha = 36.94^\circ$, [Am]

Mathematical problems related to projectile:

Determine how at which angle the horizontal range of a projectile will be equal to its maximum height.

$[R = H]$ #

$\alpha = ?$

$$R = \frac{v_0^2 \sin 2\alpha}{g} = H = \frac{v_0^2 \cdot 2\sin\alpha\cos\alpha}{g} \quad \ldots (1)$$

$$H = \frac{v_0^2 \sin^2\alpha}{2g} \quad \ldots (2)$$

$$(1) \div (2) \Rightarrow \frac{R}{H} = \frac{\frac{v_0^2 \cdot 2\sin\alpha\cos\alpha}{g}}{\frac{v_0^2 \sin^2\alpha}{2g}} = \frac{4\cos\alpha}{\sin\alpha}$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{4H}{R}$$

$$\Rightarrow \tan\alpha = \frac{4R}{R}$$

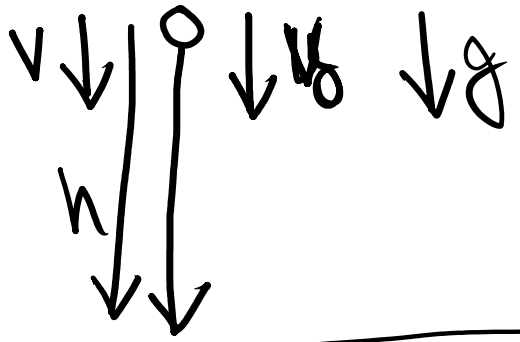
$$\Rightarrow \tan\alpha = 4$$

$$\Rightarrow \alpha = \tan^{-1}(4) = ?$$

[Ans]

Theory of motion and projectile in different cases:

Case°1

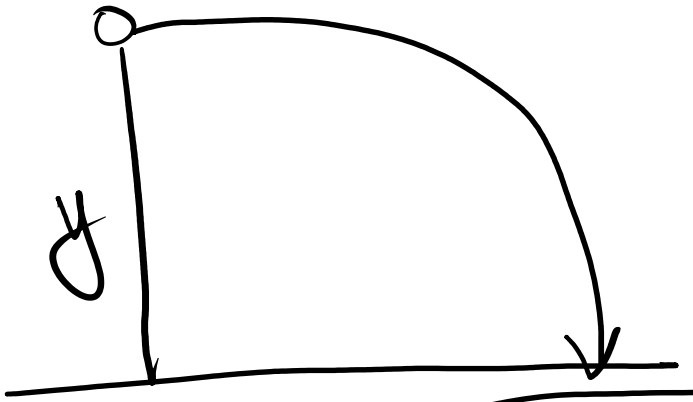


h → Initial point → Terminal Point

$$h = ut + \frac{1}{2}gt^2$$

$$v = u + gt$$

Projectile°



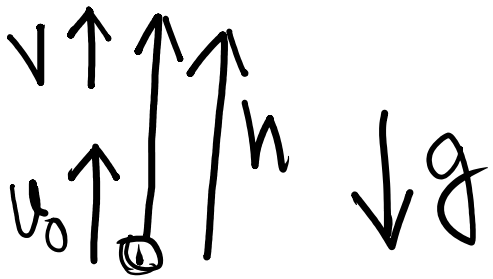
$$y = (V_0 \sin\alpha)t + \frac{1}{2}gt^2$$

$$V_y = V_0 \sin\alpha + gt$$

Theory of motion and projectile in different cases:

Case-2

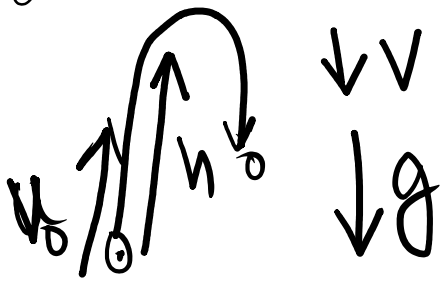
(i) Hasn't reached → H



$$h = v_0 t - \frac{1}{2} g t^2$$

$$v = v_0 - gt$$

(ii) It has reached and crossed → H



$$h = v_0 t - \frac{1}{2} g t^2$$

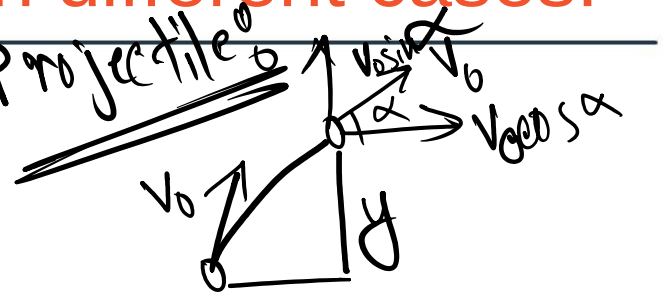
$$v = v_0 - gt$$

t → really huge quantity

$$gt > v_0$$

$$v = (-)ve$$

Projectile

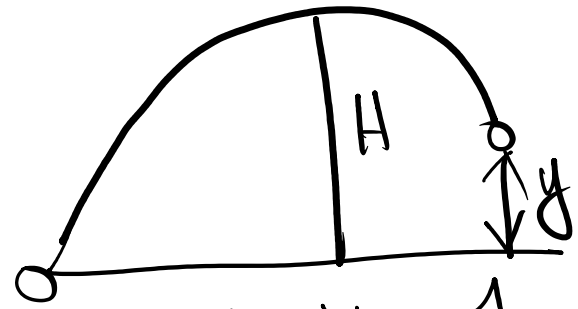


(i)

$$y = (v_0 \sin\alpha)t - \frac{1}{2} g t^2$$

$$v_y = v_0 \sin\alpha - gt$$

(ii)

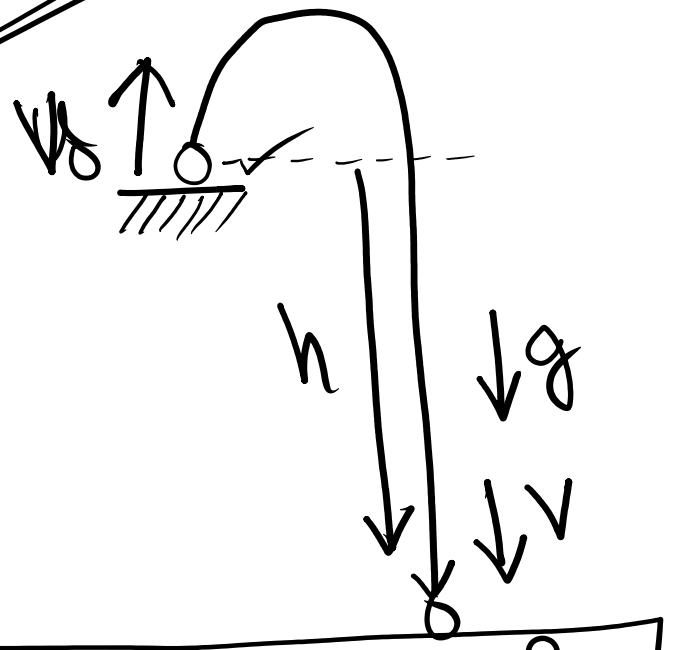


$$y = (v_0 \sin\alpha)t - \frac{1}{2} g t^2$$

$$v_y = (v_0 \sin\alpha) - gt$$

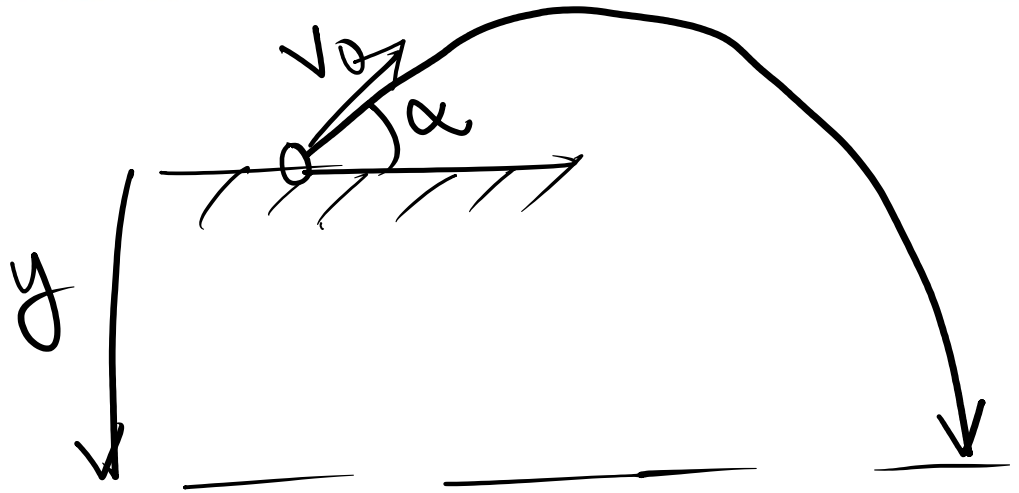
Theory of motion and projectile in different cases:

Case:-03



$$h = -v_0 t + \frac{1}{2} g t^2$$

$$v = -v_0 + gt$$



$$y = -v_0 \sin\alpha + \frac{1}{2} g t^2$$

$$v_y = -u \sin\alpha + gt$$

Poll Question 05

Horizontal range is maximum when an object is thrown at which angle with the horizontal?

(a) 90°

(b) 100°

(c) 45°

(d) 0°

$R = \frac{v_0^2 \sin 2\alpha}{g} \rightarrow \max$

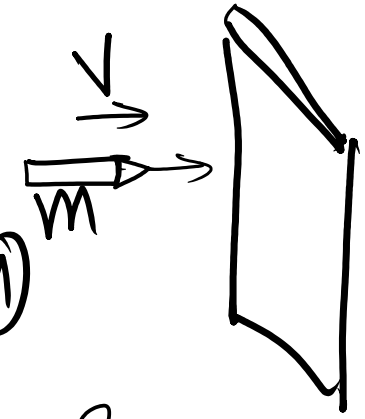
$\sin 2\alpha = \sin 90^\circ = 1$

$\Rightarrow 2\alpha = 90^\circ$

$\therefore \alpha = 45^\circ$

Mathematical problems related to different speeds:

A bullet pierces a board, if its velocity is increased by four-times, how many similar boards can it pierce?



$E = \frac{1}{2}mv^2$ ---- (1)

$E' = \frac{1}{2}m(4v)^2 = 16 \times \frac{1}{2}mv^2$

$= 16E$

∴ Number of Boards = 16

Shortcut:

'increased velocity → n times

∴ number of boards = n^2

$= 4^2$

$= 16$

Poll Question 06

How much multiplied velocity of a bullet is to be able to pierce 9 planks of the same size?

(a) 3 times

(b) 4 times

(c) 5 times

(d) 9 times

Mathematical problems related to different speeds:

$d_1 = v_1 t_1$
$d_2 = v_2 t_2$

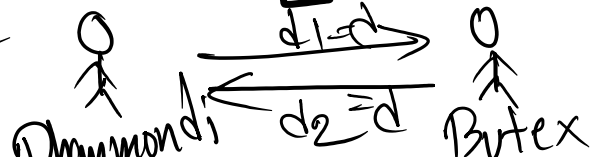
$S = vt$

Zawad comes from Dhanmondi at $4ms^{-1}$ speed to Butex and $5ms^{-1}$ from Butex to Dhanmondi. What is his average speed? What would be his average speed if he traveled $2ms^{-1}$ half the time and $10ms^{-1}$ the other half of his journey from Dhanmondi to Butex?

Ans. $\bar{v} = \frac{4+5}{2} = 4.5$

Case:-01

$d_1 = d_2 = d$



$\therefore$ Average speed, $\bar{v} = \frac{\text{Total distance}}{\text{Total time}}$

$\bar{v} = \frac{8 \times 5}{4+5} = \frac{40}{9}$, Ans

$\bar{v} = \frac{d_1 + d_2}{t_1 + t_2}$

$= \frac{d+d}{\frac{d}{v_1} + \frac{d}{v_2}}$

$= \frac{2d}{d\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}$

$\bar{v} = \frac{2v_1 v_2}{v_1 + v_2}$

Case:-02 $[t_1 = t_2 = t]$

$\bar{v} = \frac{d_1 + d_2}{t_1 + t_2} = \frac{v_1 t + v_2 t}{t + t}$

$= \frac{t(v_1 + v_2)}{2t} = \frac{v_1 + v_2}{2}$

$\therefore \bar{v} = \frac{v_1 + v_2}{2}$, Ans

$= \frac{2+10}{2}$

$= 6ms^{-1}$

Poll Question 07

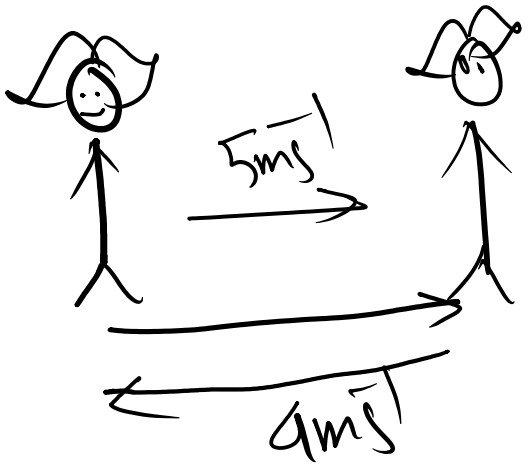
A girl goes to school at 5m / s and comes back at $4ms^{-1}$. What is the average speed of the girl?

(a) $4.5\ ms^{-1}$

(b) $4.44ms^{-1}$

(c) $1\ ms^{-1}$

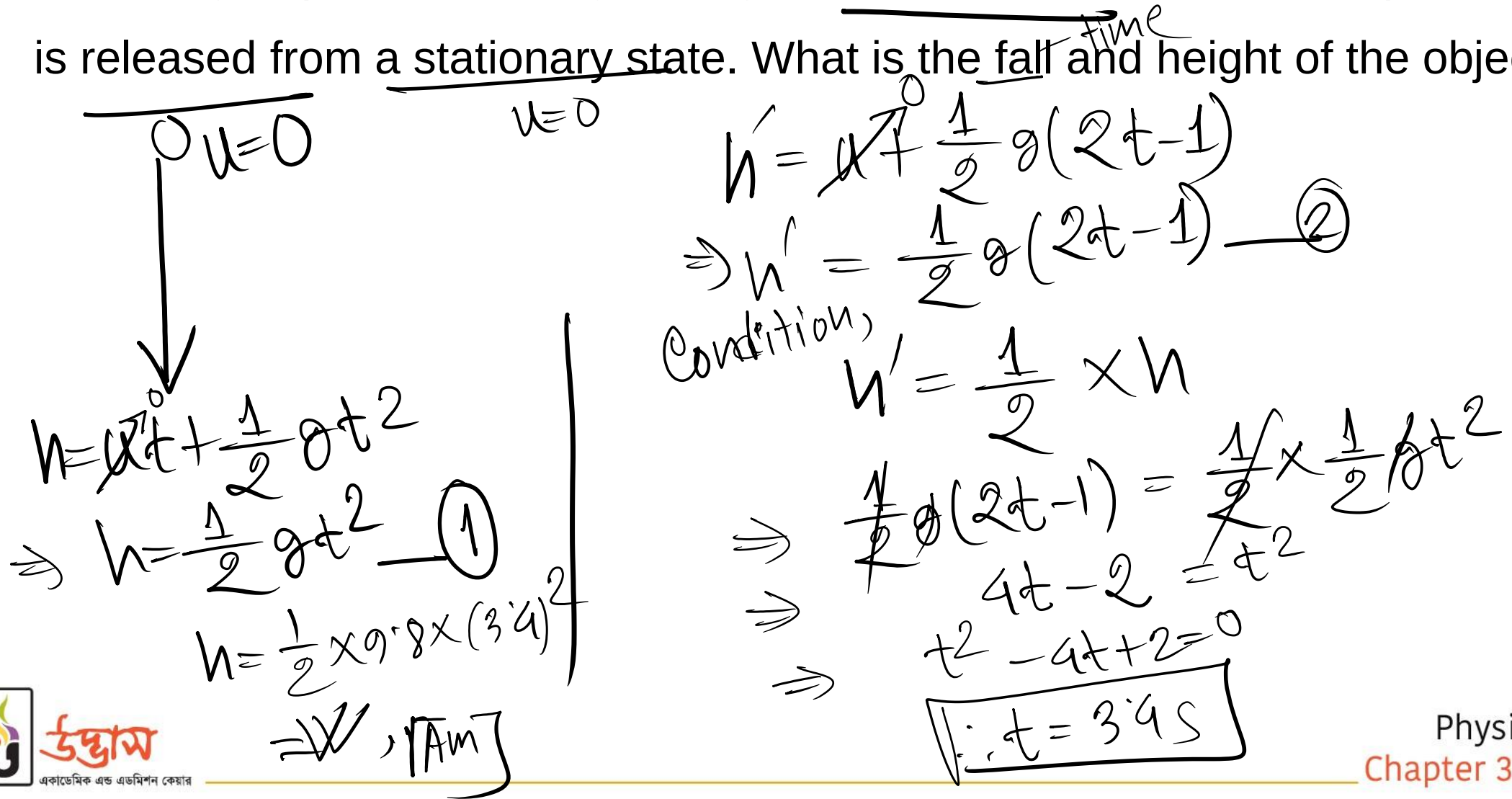
(d) $9\ ms^{-1}$



$$\bar{V} = \frac{2V_1V_2}{V_1+V_2} = 4.44ms^{-1}$$

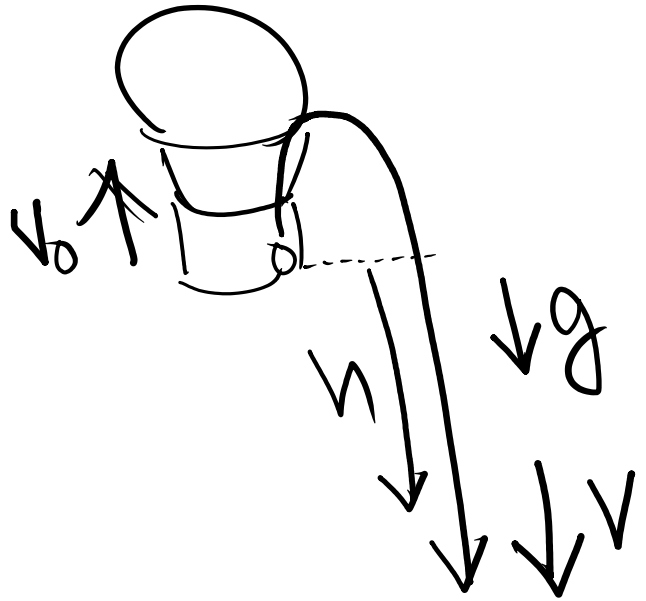
Mathematical problems related to different speeds:

If an object passes halfway through the last second of the whole path and is released from a stationary state. What is the fall time and height of the object?



Mathematical problems related to different speeds:

An object is released from an upward balloon when it was 1764 m above the ground and the object reaches the ground at 20s. What was the velocity of the balloon when the object was dropped?



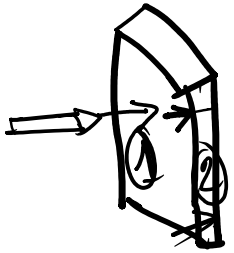
$$h = -v_0 t + \frac{1}{2} g t^2$$

$$\Rightarrow 1764 = -v_0 (20) + \frac{1}{2} \times 9.8 \times (20)^2$$

$$\Rightarrow v_0 = 9.8\ ms^{-1}$$ [Ans]

Mathematical problems related to different speeds:

#When a bullet is fired at a target, it hits a wooden plank. How much longer will the bullet go and stop if its velocity is halved after entering the board 9cm?



1st Case:

$$v_1^2 = v_0^2 + 2as_1$$

$$\Rightarrow a = \frac{v_1^2 - v_0^2}{2s_1} = \frac{\frac{v_0^2}{4} - v_0^2}{2s} = -\frac{3v_0^2}{8s_1}$$

2nd Case:

$$v^2 = v_1^2 + 2as$$

$$\Rightarrow 0 = \frac{v_0^2}{4} + 2\left(-\frac{3v_0^2}{8s_1}\right)s$$

$$\Rightarrow \left(\frac{3v_0^2}{4s_1}\right)s = \frac{v_0^2}{4}$$

$$\Rightarrow s = \frac{s_1}{3} = \frac{9}{3} = 3\text{cm}$$

Shortcut:

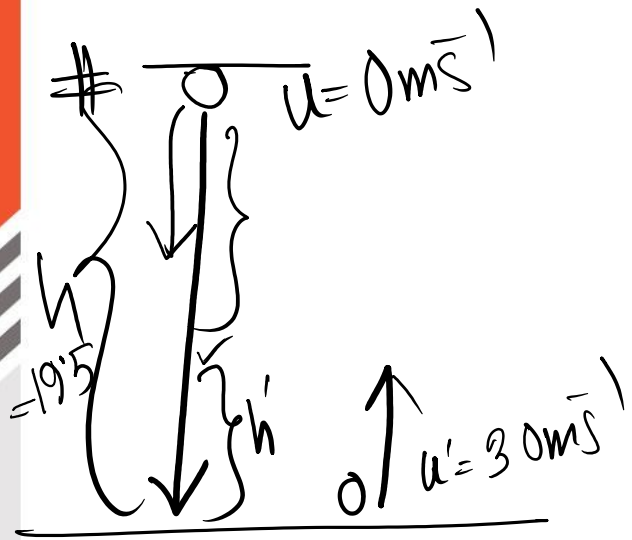
$$\frac{v}{u} = \sqrt{\frac{y-x}{y}} \Rightarrow \frac{u/2}{u} = \sqrt{\frac{y-9}{y}}$$

$\therefore y = 12$ cm

$\therefore$ Remaining $= (12-9) = 3$ cm. Ans

Mathematical problems related to different speeds:

When a ball is thrown from a height of 19.5m , when another ball is thrown at the same time at a speed of 30 ms^{-1}, when and where will they meet?



1st Ball:

$(19.5 - h') = ut + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$ — (1)

2nd Ball:

$h' = 30t - \frac{1}{2}gt^2$ — (2)

(1) & (2) ⇒

$19.5 - 30t + \frac{1}{2}gt = \frac{1}{2}gt^2$

$\Rightarrow 30t = 19.5$

$\therefore t = 0.65$

(1) ⇒

$19.5 - h' = \frac{1}{2} \times 9.8 \times (0.65)^2$

$\Rightarrow h' = 17.43m$ (From ground)

Mathematical problems related to the equation of motion:

An object is moving at a linear speed according to the formula,

$s = \left(\frac{1}{2}t^3 + 2t\right)m$ Determine its velocity after 2s.

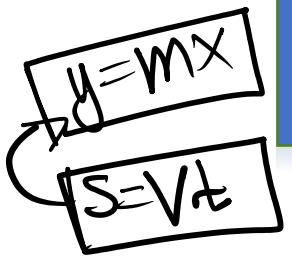
$$S = \frac{1}{2}t^2 + 2t$$

$$\therefore V = \frac{ds}{dt} = \frac{d}{dt}\left(\frac{1}{2}t^3 + 2t\right) = \frac{d}{dt}\left(\frac{1}{2}t^3\right) + \frac{d}{dt}(2t)$$

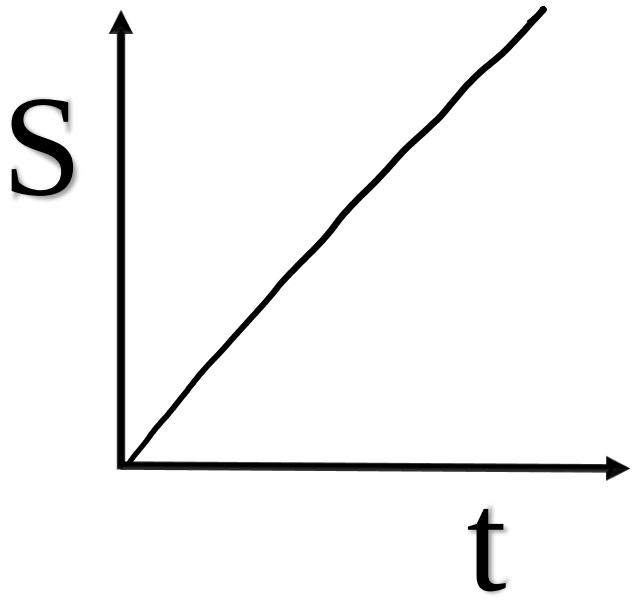
$$= \frac{3}{2}t^2 + 2$$

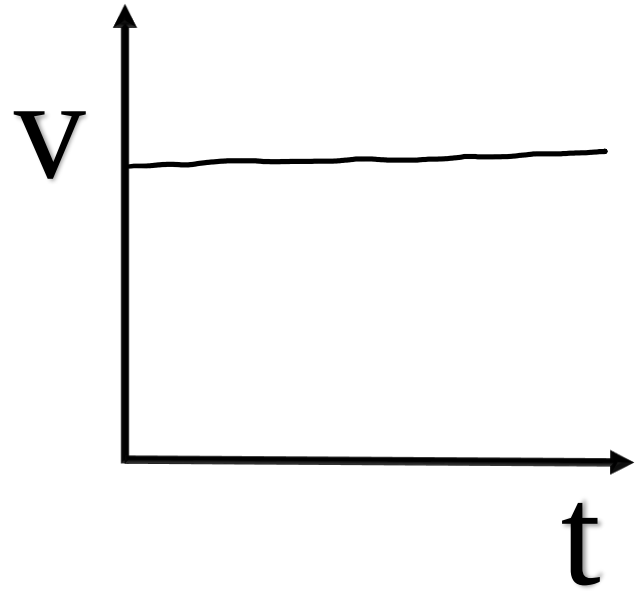
$\therefore$ Velocity after 2S, $V_2 = \frac{3}{2}(2)^2 + 2 = 8\,ms^{-1}$, [Ans]

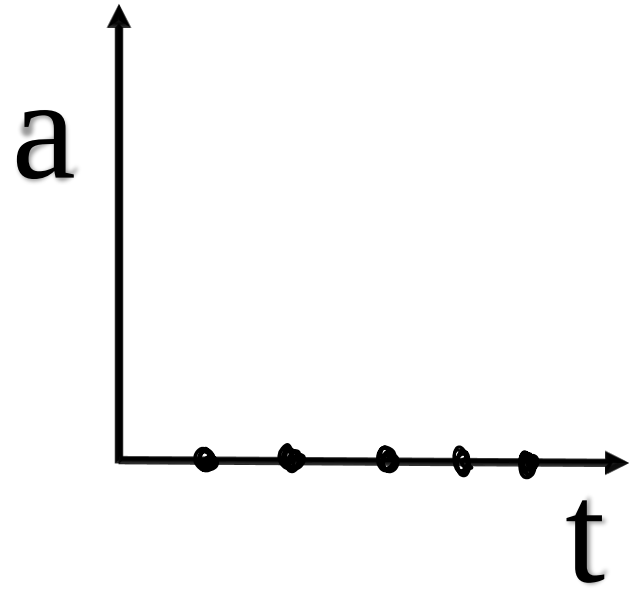
Graph theory:



Uniform-velocity

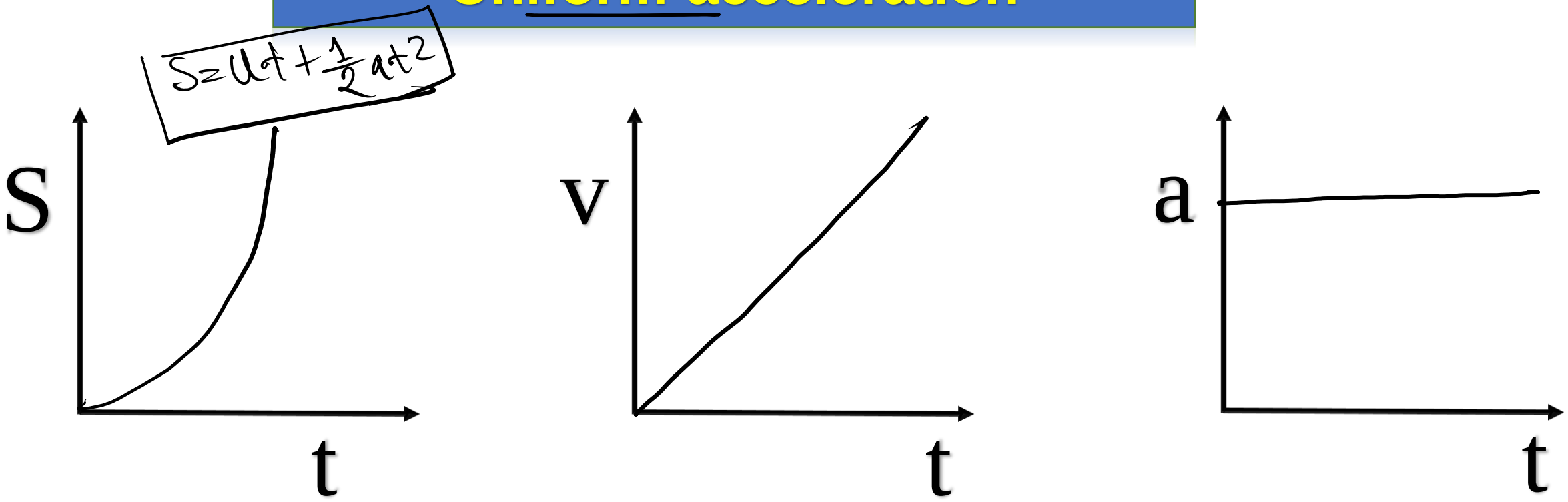






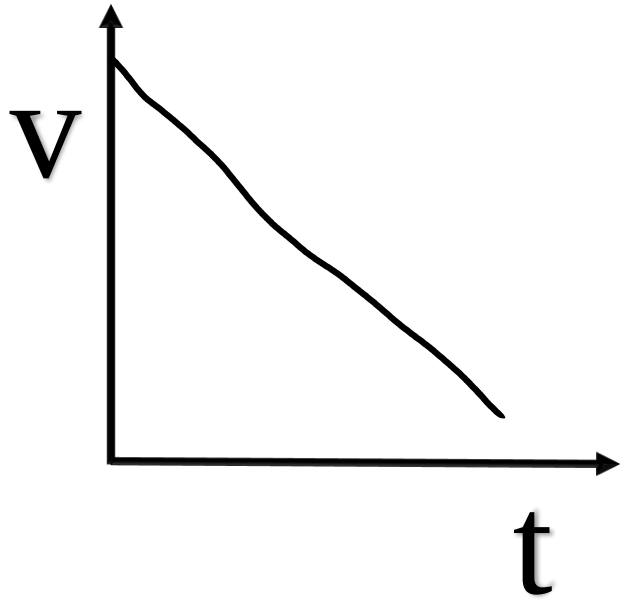
Graph theory:

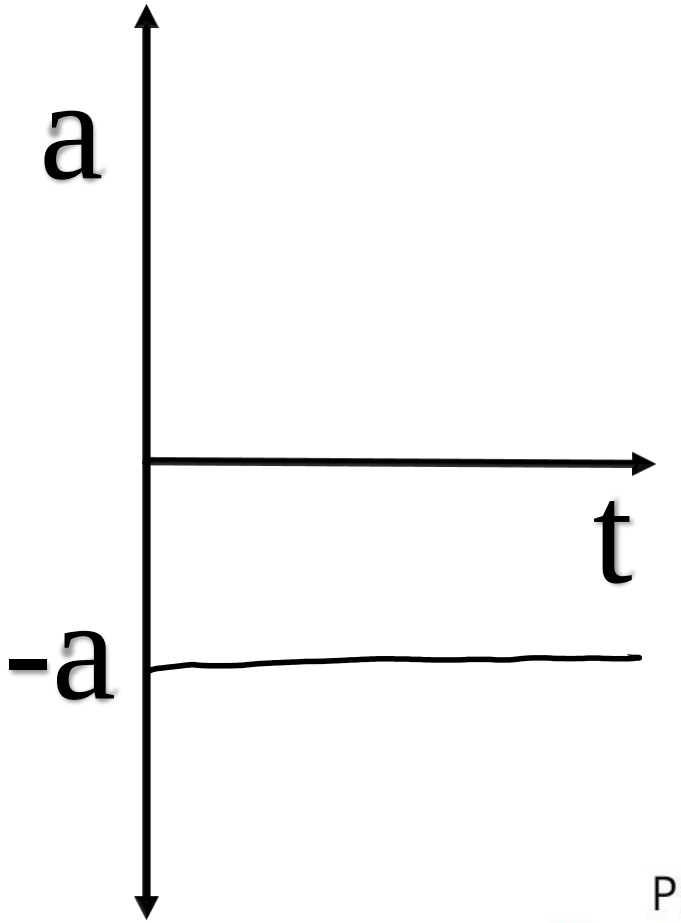
Uniform-acceleration



Graph theory:

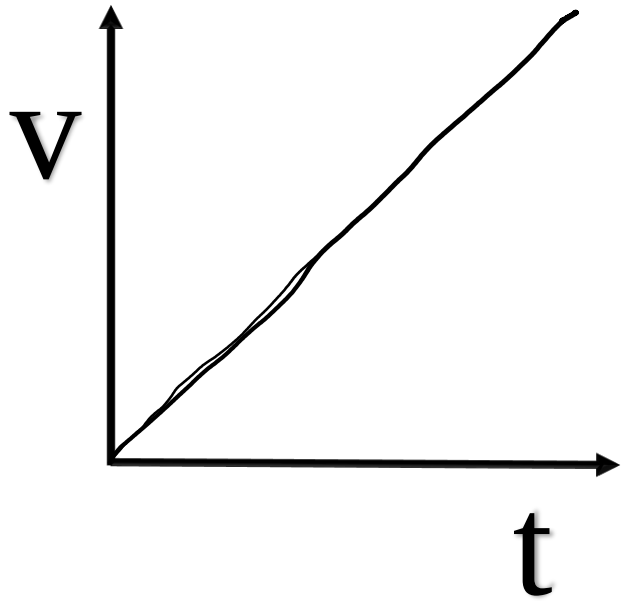
Deceleration

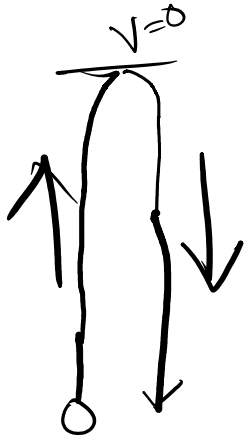


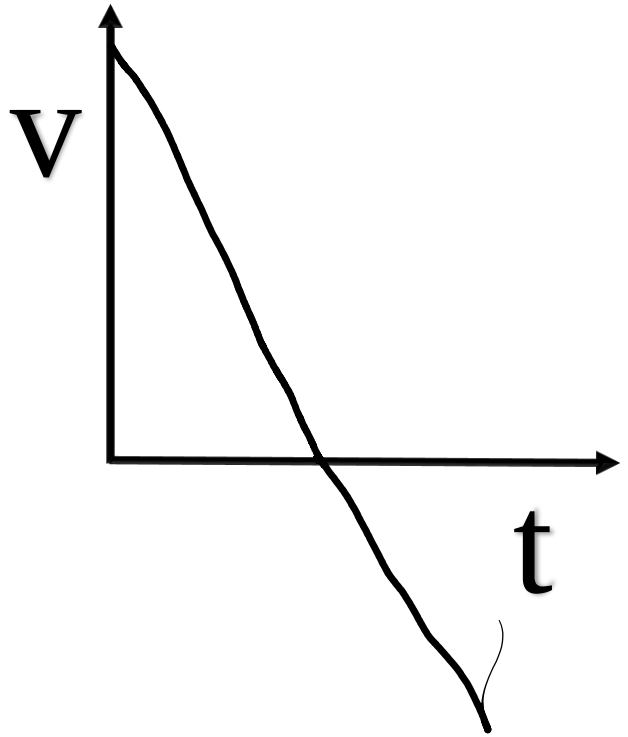


Graph theory:

Free falling and thrown objects







Mathematical problems to Graph:

What is the total displacement if the first 5s are parallel to the uniform acceleration, then the next 10s are in uniform-velocity, the last 2s are in deceleration, the speed is $4ms^{-1}$ after the 2s start of the journey? (Solve by drawing a graph)

$$a = \frac{V}{t} = \frac{4}{2} = 2ms^{-2}$$

$$a = \frac{V'}{t} \Rightarrow 2 = \frac{V'}{5} \Rightarrow V' = 10s$$

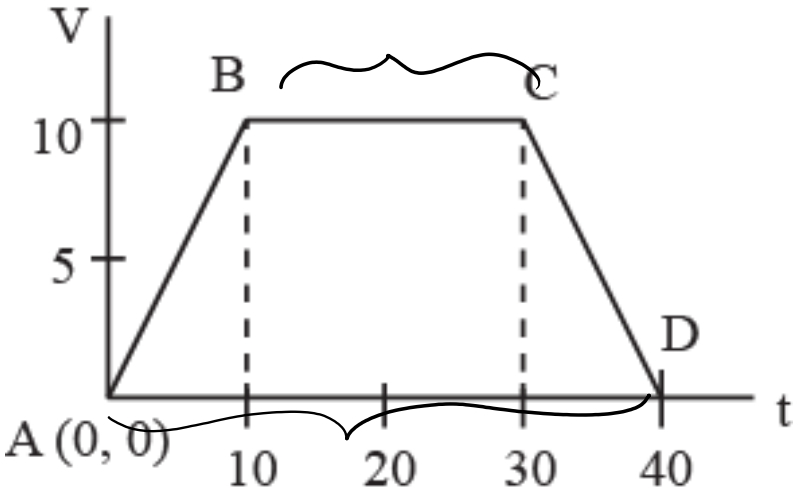
$$\# \text{ Displacement} = \frac{1}{2}(5\times10) + (10\times10) + \frac{1}{2}(2\times10)$$

$$= 25 + 100 + 10$$

$$= 135m$$

Mathematical problems to Graph:

Determine the total displacement.



Area: $\frac{1}{2}(40+20)\times 10$

$= \frac{1}{2} \times 600$

$= 300$ units

→ Displacement.

Theory of rotational motion:

$$S = r\theta \Rightarrow \theta = \frac{S}{r}$$

$$1 \text{ rev} = 360^\circ = 2\pi^c = 2\pi \cdot \text{rad}.$$

$$\Rightarrow 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ \text{ (almost)}$$

On the Contrary, $1^\circ = \frac{2\pi}{360^\circ} = 0.0175 \text{ rad}$

$$\omega = \frac{2\pi}{T} = \frac{2\pi N}{t}$$

Unint of ω = rad/s

$$\# [\omega] = \frac{\theta}{t} = \frac{S/r}{t}$$

$$= \frac{S}{r \times t} = \frac{[L]}{[L] \times [T]}$$

$$= \frac{1}{[T]} = [T^{-1}]$$

Poll Question 08

Dimension of angular velocity:

(a) $[T^{-1}]$

(b) $[T^{1}]$

(c) $[T^{-2}]$

(d) $[L^{-1}T^{1}]$

Relation of quantities related to rotation and linear motion:

Linear	Rotational	Relation
s	θ	$S=r\theta$
v	ω	$\vec{v}=\vec{\omega}\times\vec{r}=r\omega[\theta=90^\circ]$
a	α	$\vec{a}=\vec{\alpha}\times\vec{r}=r\alpha[\theta=90^\circ]$
F	τ	$\vec{\tau}=\vec{r}\times\vec{F}=rF[\theta=90^\circ]$
m	I	$I=\sum mr^2$
$P=mv$	$L=I\omega$	
$E_K=\frac{1}{2}mv^2$	$E_K=\frac{1}{2}I\omega^2$	

$$E=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

Poll Question 09

The relationship between linear velocity, angular velocity and radius of a circle is:

(a) $r = v\omega$

(b) $v = \frac{\omega}{\mathrm{r}}$

(c) $v = \omega r$

(d) $r = \frac{\omega x \omega}{v}$

Mathematical problems related to rotational speed:

After rotating 50 times the number of rotations of the fan per minute decreased from 1050 times to 450 times.

(a) Angular deceleration of the fan

(b) Time to rotate 50 times

(c) Find the linear deflection of which point 0.2 m away from the center of the fan?

$\omega_0 = \frac{2\pi N}{t} = \frac{2\pi \times 1050}{60} = 109.96 \text{ rad/s}$

$\omega_f = \frac{2\pi N'}{t} = \frac{2\pi \times 450}{60} = 47.124 \text{ rad/s}$

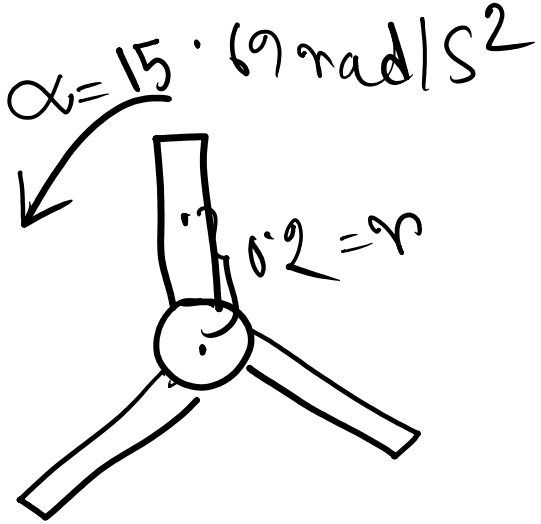
$\#$ 50 times rotation, $\theta = 2\pi N = 2\pi \cdot 50 = 314.16 \text{ rad}$

(a) We know, $\omega_f^2 = \omega_0^2 - 2\alpha\theta \Rightarrow \alpha = \frac{\omega_f^2 - \omega_0^2}{-2\theta} = \frac{\omega_0^2 - \omega_f^2}{2\theta} = \frac{109.96 - 47.124}{2 \times 314.16}$

$\therefore \alpha = 15.69 \text{ rad/s}^2$

(b) $\omega_f = \omega_0 - \alpha t \Rightarrow \frac{\omega_0 - \omega_f}{\alpha} = t$

$\therefore t = 4S$, [Am]



$a = \alpha r = 15.69 \times 0.2 = 3.138 \text{ ms}^{-2}$, [Am]

না বুঝে মুখস্থ করার অভ্যাস প্রতিভাকে ধ্বংস করে।