প্যারালাল TEXT

(For HSC & Pre-Admission)

Higher Math 1st Paper

Chapter-02 : Vector

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Dear Students,

You are stepping into a significant part of your educational life. The higher secondary curriculum is much different and vast compared to the secondary curriculum. The specific NCTB 'textbooks' were the main focus of the SSC level, but there is no subjectwise specific book at the HSC level. But there are lots of NCTB approved books written by different authors available in the market. Because of this reason, many students face a dilemma while selecting textbooks. Besides, though the syllabus at the HSC level is much vast compared to the SSC level, the time for taking preparation is very limited. So, this Parallel Text has been designed to save the students from dilemma and thus save their valuable time in this crucial phase. One of the main causes of frustration of many HSC candidates is that they cannot understand the theoretical discussion in the textbooks. So, many students lose interest in studying by understanding the concepts. As a result, many students fail to secure good grades at the HSC exam and the admission test.

To make learning easier and interesting for you, the Parallel Text books have been written in easy-to-understand language accompanied by lots of practical examples, stories, cartoons, and figures. Mathematical examples have been incorporated after discussion on every topic; which will give an idea about the application of the topic and solving mathematical problems side by side with assisting to understand the next topics. For ease of understanding, the important definitions, characteristics, differences, etc. have been shown in a separate section. Besides, the most common mistakes have been highlighted under 'Caution!'.

But it is not enough to just understand the concepts, you also need enough practice. And to make this easier, the 'Topic-wise Question and Answer from Previous Years' section has been additionally incorporated at the end of some important topics from every chapter. And this sections consists of the solutions of previous years' Board questions along with the questions from the admission tests of BUET, RUET, KUET, CUET and University of Dhaka. If you practice step-by-step in this way, you will be able to take full preparation for the Board exams and also prepare yourself for the admission tests simultaneously. Besides, 'Important Practice Problems' and 'Mathematical Problems' has been added at the end of every chapter, which you can practice to enrich your preparation.

We hope our Parallel Text strengthen your basic idea about the concepts at HSC level and help you to secure A+ at the board exams in addition to being prepared for the admission tests.

Best wishes for a prosperous life and a bright future-

Udvash Math Team

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Higher Math 1st Paper

Chapter-02 : Vector

পারস্পরিক সহযোগিতা-ই পারে
পৃথিবীকে আরও সুন্দর করতে...

Dear learners,

We hope “HSC Parallel Text” will be more beneficial for you. Utmost efforts have been given to keep the book error free. However, if any error comes to your notice, do kindly inform us through the email address below, so that we may continue our efforts to the future editions.

Email: solutionpt.udvash@gmail.com

Following things have to be mentioned in the email:

(i) Name of subject of the “HSC Parallel Text”. (ii) Version (Bangla/English) (iii) Name of the Chapter (iv) Page Number (v) Question number (vi) What the error is (vii) What should it be according to your opinion.

Example: “HSC Parallel Text”, Math 1st Paper, English Version, Chapter-02, Page-32, Question-12, Given that, [6] but it should be [4].

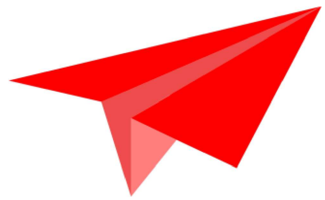
Any other suggestion to improve the quality will be appreciated sincerely. At last, we pray to almighty Allah for your success.

Best regards

Udvash Math Team

Chapter 02 Vector

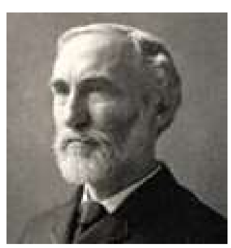
We are familiar with "origami" or paper crafts since childhood. Even if we can't build anything else, we can all build boats and planes. In our childhood we used to make boat and float it in a big bowl. But I am sure that all of us loved to fly the plane after making it . Ayon and Antor are such children .One day they made a plane and went to field to fly it .Ayon is standing infront of Antor facing him keeping a little bit distance . Aim of Antor is to fly up the plane and send it to Ayon.



But the problem is, the wind is blowing between them like in the given picture. Now the question is, if Antor throws the plane directly towards Ayon, then will the plane reach to Ayon? What do you think ? Keep thinking. Within this time, let us go and know about vectors. It may happen that the answer is hidden within there.

Brief History

The concept of "vector" is very important in mathematics, physics and engineering. It is such a geometric concept that has both magnitude and direction. The concept of vector has came to its present state through about 200 years of gradual changes.



Josiah Willard Gibbs

Oliver Heaviside

The earliest traces of work with vectors date back to the early 19th century. In 1827, August Ferdinand Mobius published a book named "Barycentric calculus" in which he introduced the concept of "Directed line segments" which are denoted by English alphabet. Later, in the late 19th century, Josiah Williard Gibbs and Oliver Heaviside elaborated the concept of vector analysis and vector algebra separately. They did this to explain the first principle of electromagnetism given by James Cleark Maxwell. Josiah Williard Gibbs was a physicist. He found that his works on physics will become much easier by vector analysis. Later, Oliver Heaviside (1850-1925), another physicist, also played a role in explaining vector analysis. To understand what vector is, we need to understand some necessary quantities. Let's discuss about them now.

Vector

Quantity

Quantity: In this materialistic world the things which can be measured is known as quantity.
Example: Length, mass, force, speed, velocity etc.

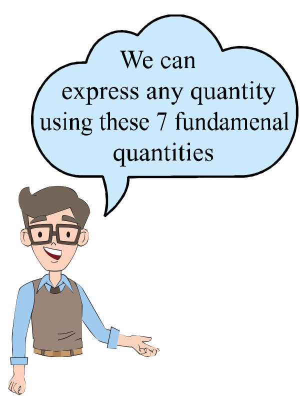
➲ **Classification of Quantities:** Quantities can be classified on different basis. To measure a quantity, it is needed to measure some fundamental quantities. Based on this, quantities are of 2 types:
(i) Fundamental Quantities
(ii) Derived Quantities

(i) Fundamental Quantities: Imagine I asked you, what is the distance between your house and your college? You replied that 1 km; How much time it would take to go there? You said 12 minutes. You only need one piece of data to answer these two questions. In 1^{st} case how much distance in km and in 2^{nd} case how much time in minutes. These are fundamental quantities.

The quantities that are independent or neutral or the quantities that does not require any other quantity to express itself, are called fundamental quantities.

➲ **There are total 7 fundamental quantities in the concern of science:**

Fundamental Quantities	Unit	Unit signal
(i) Length	Meter	m
(ii) Mass	Kilogram	kg
(iii) Time	Second	s
(iv) Temperature	Kelvin	K
(v) Electric current	Ampere	A
(vi) Luminous intensity	Candela	cd
(vii) Amount of Substance	Mole	mol



(ii) Derived Quantities: Now imagine you are going to college by cycle. In cycle's speedometer your speed is showing 4 ms^{-1}. That means, your cycle is crossing 4 m distance in 1s. Now observe, for measuring the speed, Speedometer has to measure 2 quantities. One of them is distance and the other one is time. Here speed is dependent on other two quantities or it is a derived quantity.

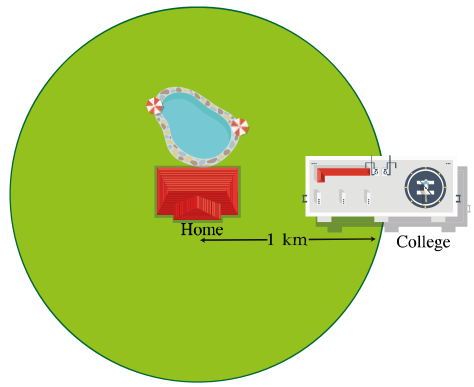
The quantities that are dependent on the fundamental quantities or the quantities that require more than one fundamental quantity to express itself are called derived quantities.

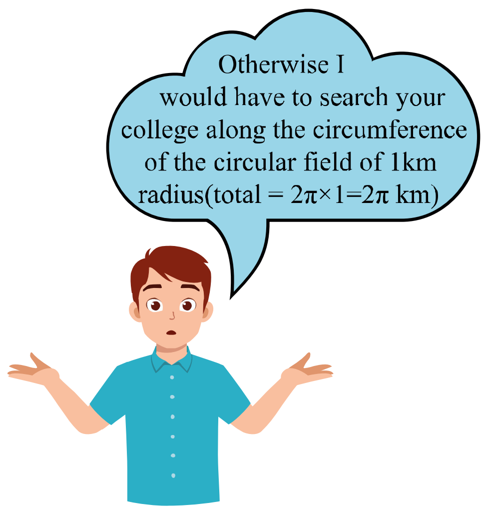
Example: Force = [mass × $\frac{\text{distance}}{\text{time}^2}$],

Speed/velocity = [$\frac{\text{distance}}{\text{time}}$],

Work = [mass × $\frac{\text{distance}^2}{\text{time}^2}$], Charge = [current × time] etc.

When I asked you some time ago how far is the college from your house, you said 1 km. which made me understand how far your college is. I don't need any more information. But now if I want to go to your college, is 1km distance information enough for me? No, it is not. Because you didn't tell me which direction to go for 1 km. But if you tell me that I have to go 1 km east then I will reach your college just fine.





That means for describing some quantities it is enough to know about only the magnitude (like distance). These are called scalar quantities. But there are some other quantities which can't be expressed completely only with the value, direction is also needed. Like in the previous example, in a 1 km east direction my displacement is needed, only then i will be able to reach your college. This displacement is vector quantities. That is, on the basis of direction, quantities can be divided into 2 types-

(i) Scalar Quantity

(ii) Vector Quantity

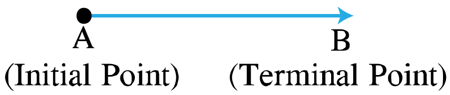
(i) Scalar Quantity: A scalar quantity is a quantity that can be expressed entirely by the value alone and changes only when the value changes.

Example: Length, Distance, Time, Volume etc

(ii) Vector Quantity: Quantities that require both magnitude and direction to be fully expressed and quantities for which, change in only magnitude or direction or both changes the quantity, are called vector quantities.

Example: Displacement, Force, Acceleration etc.

Expression of vector quantities:

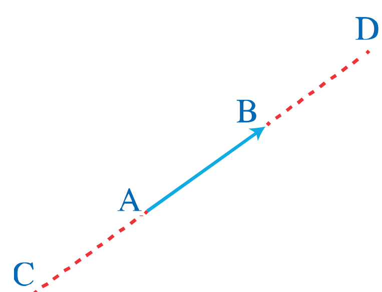


Taking one end of a straight line as the origin and the other end as the endpoint, the directed line segment expresses a vector quantity. The length of the line indicates the magnitude of the vector quantity, and the direction indicates the direction of the vector quantity. A vector quantity can be expressed in any of the following ways in writing: $\overrightarrow{AB}$, $\overline{AB}$, $\underline{AB}$

When typing on a computer, AB can also be expressed in bold letters **(AB)** to represent vector quantities. Also, the value of a vector quantity is generally expressed in the form of AB or $|\overrightarrow{AB}|$.

Support Line:

The segment of an infinite straight line directed by a vector is called its support line. [That is, the line containing a vector is called the support line]



In figure $\overrightarrow{AB}$ vector is a segment of an infinite straight line a portion of which CD is indicated in figure.

Direction:

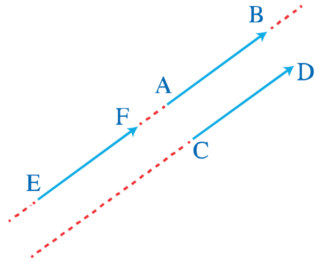
In figure the direction of vector $\overrightarrow{AB}$ is from point A to point B. Similarly, the direction of $\overrightarrow{BA}$ is from point B towards point A.

Warning!

Some students want to express $\overrightarrow{BA}$ as $\overleftarrow{AB}$, which is not acceptable.

Types of vector quantities or different types of vectors

(i) Equal Vectors:



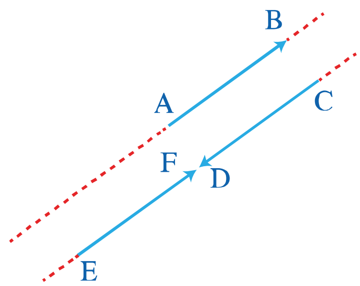
$\overrightarrow{AB} = \overrightarrow{CD} = \overrightarrow{EF}$

(i) Values are equal. [Length of line segment indicating direction is equal]

(ii) The support lines are same or parallel.

(iii) Direction is same.

(ii) Opposite Vectors:



$\overrightarrow{AB} = -\overrightarrow{CD}$ or, $\overrightarrow{EF} = -\overrightarrow{CD}$

(i) Values are equal. [Length of line segment indicating direction is equal]

(ii) The support lines are same or parallel.

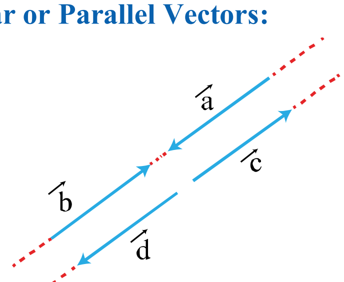
(iii) Directions are opposite.

Remember: $\overrightarrow{AB} = \overrightarrow{DC}$

Note

Here, $\overrightarrow{AB}$ is opposite vector of $\overrightarrow{CD}$, but equal to vector $\overrightarrow{DC}$.

(iii) Collinear or Parallel Vectors:



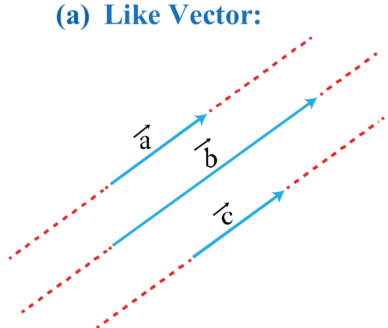
(i) Values are equal/not equal.

(ii) Having a parallel support line of a straight line.

(iii) Direction maybe same or opposite.

Collinear or Parallel Vectors are of 2 types:

(a) Like Vector:

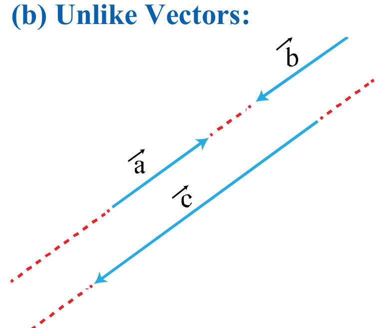


(i) Values are same or different.

(ii) Support line same or parallel.

(iii) Same direction.

(b) Unlike Vectors:



(i) Values are same or different.

(ii) Support line same or parallel.

(iii) Opposite direction.

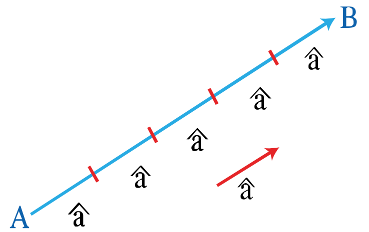
(iv) Proper Vector: All vectors are proper vectors except the zero vector.

(v) Zero or Null or Improper Vector: A vector whose value is zero is called a null vector.

Note

Direction of null vector is unspecified.

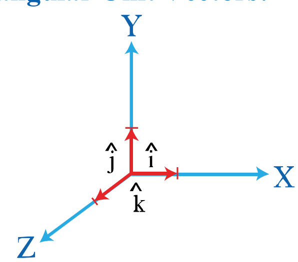
(vi) Unit Vector: A vector whose value is one is called a unit vector. Dividing a vector by its magnitude gives a unit vector in the direction or parallel to that vector. Unit vector of vector $\vec{A}$ is usually expressed as $\hat{a}$.



$$\overrightarrow{AB} = 5\hat{a} \Rightarrow \hat{a} = \frac{\overrightarrow{AB}}{5}$$

$$\therefore \hat{a} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

(vii) Rectangular Unit Vectors:



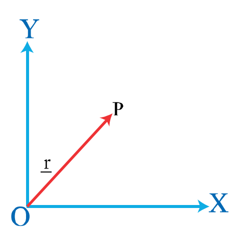
The three unit vectors along the three axes x, y and z in the three dimensional Cartesian coordinate system are collectively called rectangular unit vectors.

(i) Unit vector along x-axis: $\hat{i}$

(ii) Unit vector along y-axis: $\hat{j}$

(iii) Unit vector along z-axis: $\hat{k}$

(viii) Position Vector:



$\overrightarrow{OP} = \underline{r}$

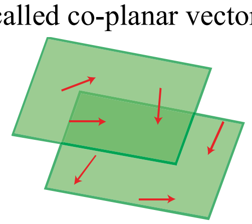
In three dimensional coordinate system, a vector expressing the position of a point with respect to the origin of the frame of reference is called a position vector. The position vector is also sometimes called the radius vector $(\underline{r})$. For example, we can express the position of point P in the figure by the vector $\overrightarrow{OP}$. $\therefore \overrightarrow{OP} = \underline{r}$ is the position vector or radius vector of point P. That is, the magnitude of the position vector of a point is the length and direction is along the line connecting that point from the origin.

Note

In two-dimensional coordinate geometry, a point is expressed by abscissa(x) & ordinate(y). A point in vector is denoted by position vector.

(ix) Co-planar Vector:

If the lines containing the vectors lie in the same plane [or lie in a plane parallel to the same plane] then they are called co-planar vectors.



The vectors indicated in the figure are Co-planar vectors.

(x) Reciprocal Vector:

If the value of one of two parallel vectors is reciprocal of other, then they are called reciprocal vectors.

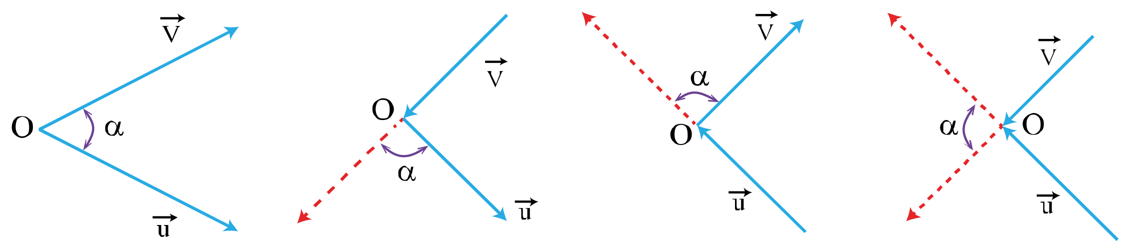
Let, $\overrightarrow{AB} = 5\hat{a}$ and $\overrightarrow{CD} = \frac{1}{5}\hat{a}$ [$\hat{a}$ is a unit vector]

$\therefore$ $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are mutually reciprocal vectors.

(xi) Free Vector:

The vector that has a fixed modulus and direction but no fixed position, i.e. if the position of vector can be moved without changing the modulus or direction then it is called free vector.

Angle Between Two Vector:



To determine the angle between two vectors, α, place the two vectors at a point and extend if necessary, so that the vectors appear to emanate from the point [O]. In this case the angle between them is the angle between the two vectors.

Limit of angle: $0^c \leq \alpha \leq \pi^c$

Note

$\alpha = 0^c$ Both vectors are like parallel vectors

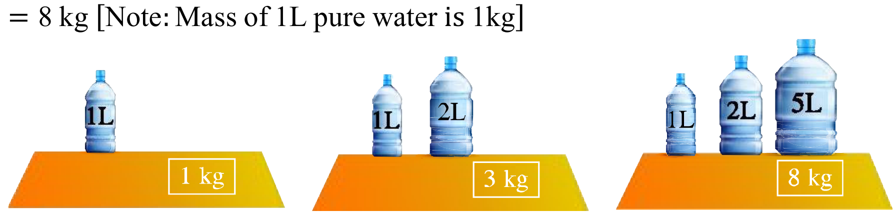
$\alpha = \pi^c$ Both vectors are unlike parallel vectors

$\alpha = \frac{\pi^c}{2}$ Both vectors are perpendicular to each other

Addition of Vectors

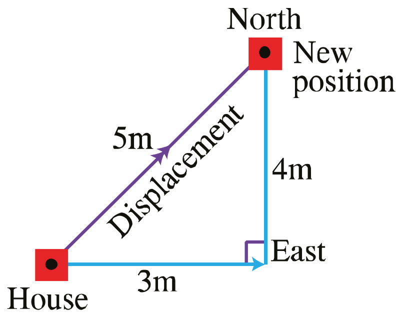
Suppose you have a mass measuring device at home. Again you have 3 bottles full of water. One of 1 L, another of 2 L and another of 5 L volume. The mass of the bottles is negligible. Now if you put the water bottle of 1 L volume in the mass measuring machine, you will see that the mass will be shown 1 kg. Then if the 2 L water bottle is also placed on the machine, the mass will be shown $(1 + 2) = 3$ kg Again, if the 5 L bottle is also placed on the machine, then the mass will be shown-

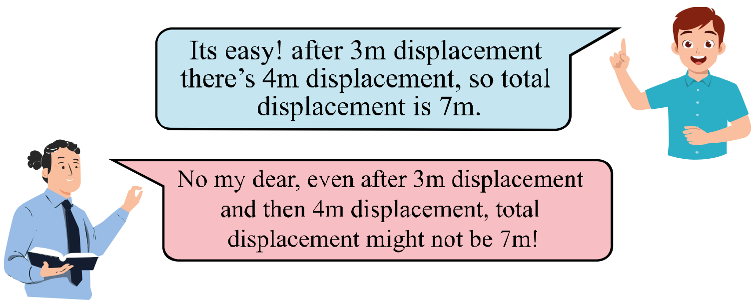
$(1 + 2 + 5) = 8$ kg [Note: Mass of 1L pure water is 1kg]



Note that mass is a scalar quantity and in case of addition, masses or scalar quantities are generally added. That is, scalar quantities can be added by general algebraic rules.

But suppose you went 3 m east from your house and then 4 m north from there, now if you are asked how many meters are you away from your house, will your answer be $3 + 4 = 7$m?





The answer is no, 7 m won't be the right answer because you have to think about direction here as well. From the figure it can be seen that you are now $\sqrt{3^2+4^2}=5$ m away from your house. In other words, your displacement here is 5 m, that is, vectors cannot be added according to normal algebraic rules. It requires vector algebra.

⚠ Warning!

In the case of addition of two vector quantities, the two quantities must be of same type. For example, only velocity can be added to velocity but addition of displacement to velocity is meaningless. The same is true for scalar quantities. Adding time to mass is meaningless.

The following methods are used for the addition of two vector quantities:

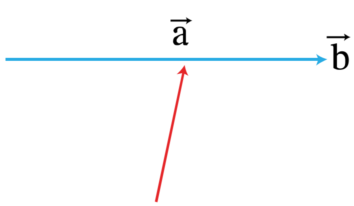
(i) Initial & Terminal Point Rule

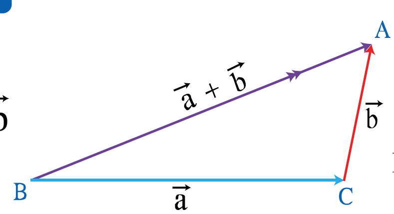
(ii) Law of Triangle

(iii) Law of Parallelogram

(iv) Law of Polygon

Initial & Terminal Point Rule:





In the figure, $\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{BA}=\vec{a}+\vec{b}$

If $\vec{a}$ and $\vec{b}$ are two same type of vector quantities, then for addition of $\vec{a}$ & $\vec{b}$ vectors:

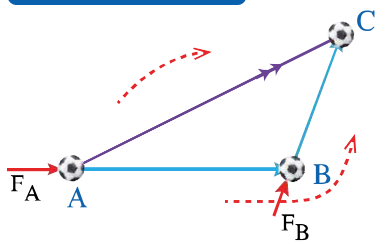
(i) Let vector $\overrightarrow{BC}$ is taken equal in magnitude and parallel to the vector $\vec{a}$.

(ii) Let us draw a vector $\overrightarrow{CA}$ parallel and equal to $\vec{b}$ at the vertex C of $\overrightarrow{BC}$ (or $\vec{a}$).

(iii) Then the vector $\overrightarrow{BA}$ obtained by adding the origin B of $\overrightarrow{BC}$ and the vertex A of $\overrightarrow{CA}$ is the sum or resultant of $\vec{a}$ and $\vec{b}$.

In simple words if you go from B to C and then from C to A then your resultant displacement is $\overrightarrow{BA}$. A triangle is obtained from the intial-terminal point rule through which the triangle formula can be derived. For ease of understanding we will read triangle formula/rule and formula of parallelogram together.

Law of triangle:



Suppose a person hits a football with a force $\vec{F}_A$ at point A and it stops moving from point A to point B. The ball stops at point C when another person hits it with force $\vec{F}_B$. That is, the resultant displacement of football is $\overrightarrow{AC}$.

Conditions:

(i) Two forces (vectors) act at different points

(ii) Two forces (vectors) act at different times

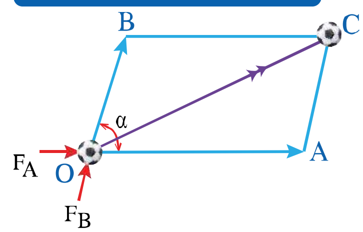
(iii) The forces/vectors can be represented in the same order by two adjacent sides AB and BC of a triangle. [anti-clock wise]

Result:

Then the third side of that triangle AC indicated in reverse [clock wise] will represent value and direction of the resultant.

Expression: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

Law of Parallelogram:



Suppose, two persons strike a ball with forces $\vec{F}_A$ & $\vec{F}_B$ at an angle of α at point O. The ball tends for displacement $\overrightarrow{OA}$ for force $\vec{F}_A$ and $\overrightarrow{OB}$ for force $\vec{F}_B$. But since two forces act together, the ball will move along the middle path $\overrightarrow{OC}$.

Conditions:

(i) Two vectors/forces act at the same point O.

(ii) Two vectors/forces act simultaneously.

(iii) Two adjacent sides of a parallelogram (OA and OB) will represent magnitude and direction of the forces/vectors.

Result:

Then the diagonal (OC) along that point of the parallelogram will indicate the magnitude and direction of the resultant.

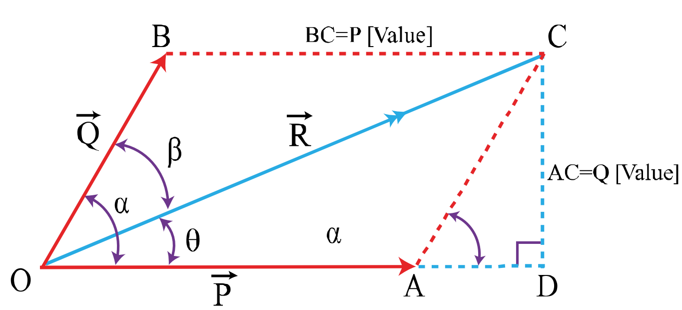
Expression: $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$

Mathematical Form of Law of Parallelogram:

Let forces $\vec{P}$ & $\vec{Q}$ act along OA and OB in both magnitude and direction respectively. Their resultant $\vec{R}$ acts along OC. Angle between $\vec{P}$ & $\vec{Q}$ is α. θ is the angle between $\vec{P}$ & $\vec{R}$. [β is the angle between $\vec{Q}$ & $\vec{R}$]

Draw CD perpendicular to OA extended from C that intersects the extended OA at point D.

In figure, $OA = |\vec{P}| = P$, $OB = AC = |\vec{Q}| = Q$, $OC = |\vec{R}| = R$



According to the law of Parallelogram:

$\vec{P} + \vec{Q} = \vec{R}$

Here, OB||AC and OD is the transversal

$\therefore \angle BOA = \angle CAD = \alpha$

Then, in ΔACD, $\frac{AD}{AC} = \cos\alpha$

$\Rightarrow AD = AC\cos\alpha$

$\therefore \boxed{AD = Q\cos\alpha}$

$\frac{CD}{AC} = \sin\alpha \Rightarrow CD = AC\sin\alpha$

$\therefore \boxed{CD = Q\sin\alpha}$

Determination of resultant value:

$OC^2 = OD^2 + CD^2$ [∵ ΔOCD is a right angled triangle]

$\Rightarrow R^2 = (OA + AD)^2 + CD^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$

$= P^2 + Q^2\cos^2\alpha + 2PQ\cos\alpha + Q^2\sin^2\alpha = P^2 + Q^2(\cos^2\alpha + \sin^2\alpha) + 2PQ\cos\alpha$

$\Rightarrow \boxed{R^2 = P^2 + Q^2 + 2PQ\cos\alpha}$ ∴ $R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

Determination of the direction of the resultant:

Angle between P and R, $\angle COD = \theta$, ΔCOD is a right angled triangle.

$\therefore \tan\theta = \frac{CD}{OD} = \frac{CD}{OA+AD}$

$\therefore \boxed{\tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha}}$ $\therefore \theta = \tan^{-1}\left(\frac{Q\sin\alpha}{P+Q\cos\alpha}\right)$

Note

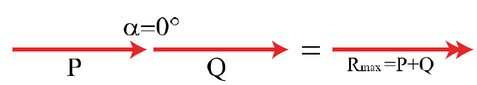
The vector that makes (P) θ angle with R remains below and nothing remains in multiplication with it.

Similarly, it can be proved by drawing a perpendicular on the extension of OB from C.

$\boxed{\tan\beta = \frac{P\sin\alpha}{Q+P\cos\alpha}}$ [β is the angle between $\vec{Q}$ and $\vec{R}$] $\therefore \beta = \tan^{-1}\left(\frac{P\,si\ \ \alpha}{Q+P\cos\alpha}\right)$

Maximum value of the resultant:

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$



R_{max} will be found when the value of cosα is maximum. $(\cos\alpha)_{max} = 1 \Rightarrow \boxed{\alpha = 0°}$

In that case, $R = \sqrt{P^2 + Q^2 + 2PQ} = \sqrt{(P+Q)^2}$

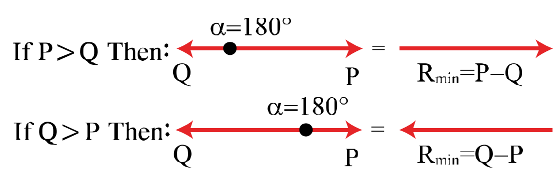
$\boxed{\therefore R_{max} = P + Q}$ [when $\alpha = 0°$]

Minimum value of the resultant:

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$

If the value of cosα is minimum, then the value of resultant will be minimum. $(\cos\alpha)_{min} = -1$

$\boxed{\therefore \alpha = 180°}$



In that case, $R = \sqrt{P^2 + Q^2 + 2PQ(-1)} = \sqrt{(P-Q)^2} = \sqrt{(Q-P)^2} = \sqrt{(P\sim Q)^2}$

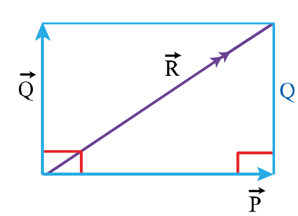
$\therefore R_{min} = P \sim Q$ [$\sim \Rightarrow$ Difference [Subtract smaller from larger] [when $\alpha = 180°$]

If $P > Q$ then, $R_{min} = P - Q$ and if $P < Q$ then, $R_{min} = Q - P$

If $\alpha = 90°$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°}$

$\therefore \boxed{R = \sqrt{P^2 + Q^2}}$



This can be proved by Pythagoras theorem. In the figure $R^2 = P^2 + Q^2$

Let's think back to our starting example.